

# ABSTRACT

Title of dissertation:      **TOPOLOGICAL DISPERSION RELATIONS IN  
SPIN-ORBIT COUPLED BOSE GASES**

Ana Valdés-Curiel,  
Doctor of Philosophy, 2019

Dissertation directed by:   **Professor Ian Spielman**  
Joint Quantum Institute,  
National Institute of Standards and Technology and  
the University of Maryland College Park

Quantum degenerate gases have proven to be an ideal platform for the simulation of complex quantum systems. Due to their high level of control it is possible to readily design and implement systems with effective Hamiltonians in the laboratory. This thesis presents new tools for the characterization and control of engineered quantum systems and describes their application in the realization of a topological system with Rashba-type spin-orbit coupling.

The underlying properties of these engineered systems depend on their underlying energies. I describe a Fourier transform spectroscopy technique for characterizing the single particle spectrum of a quantum system. We tested Fourier spectroscopy by measuring the dispersion relation of a spin-1 spin-orbit coupled Bose-Einstein condensate (BEC) and found good agreement with our predictions.

Decoherence due to uncontrolled fluctuations of the environment presents fundamental obstacles in quantum science. I describe an implementation of continuous

dynamical decoupling (CDD) in a spin-1 BEC. We applied a strong radio-frequency (RF) magnetic field to the ground state hyperfine manifold of Rubidium-87 atoms, generating a dynamically protected dressed system that was first-order insensitive to changes in magnetic field. The CDD states constitute effective clock states and we observed a reduction in sensitivity to magnetic field of up to four orders of magnitude. We additionally show that the CDD states can be coupled in a fully connected geometry and thus enable the implementation of new models not possible using the bare atomic states.

Finally, I describe a new realization of Rashba-type SOC using Raman coupled CDD states. Our system had non-trivial topology but no underlying crystalline structure that yields integer valued Chern numbers in conventional materials. We validated our procedure using Fourier transform spectroscopy to measure the full dispersion relation containing only a single Dirac point. We measured the quantum geometry underlying the dispersion relation and obtained the topological index using matter-wave interferometry. In contrast to crystalline materials, where topological indices take on integer values, our continuum system reveals an unconventional half-integer Chern number.



TOPOLOGICAL DISPERSION RELATIONS IN  
SPIN-ORBIT COUPLED BOSE GASES

by

Ana Valdés Curiel

Dissertation submitted to the Faculty of the Graduate School of the  
University of Maryland, College Park in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
2019

Advisory Committee:

Professor Alicia Kollar, Chair

Professor Gretchen Campbell

Professor Mohammad Hafezi, Dean's representative

Professor Norbert Linke

Professor Trey Porto

Professor Ian Spielman, Advisor

© Copyright by  
Ana Valdés Curiel  
2019

## Preface

A few weeks after I had started writing this thesis Ian came to me and asked if I had learned all the physics I wish I had known while I was doing my PhD. I felt a bit puzzled. I had been operating the lab, analyzing data and writing papers for a while, of course I already knew the physics relevant to my research! It finally hit me when I was writing the introductory chapters how many subtleties I had missed and how much I still didn't know. As experimental physicists being in the lab can give us some physical intuition and a sense of understanding but sometimes that is not enough. This was a very striking and unexpected side effect of the thesis and I invite experimentalists reading this to challenge their lab intuition.

In the end, I actually found it very enjoyable to look back at the history of our field and to put my research into a bigger context. I never thought the words thesis and enjoyable could go well together! My advice for a graduate student starting to write a thesis is to try to enjoy the ride, it will definitely be stressful and overwhelming but it is also a great learning opportunity.

A PhD thesis is not just a service for oneself but also a service to other students and new members of the lab. I had this in mind during the process of writing and I hope that future students find this document helpful.

*For my parents and in the  
memory of Dr. Ernesto Valdés Krieg*

## Acknowledgments

I was very lucky to be surrounded by an amazing group of people throughout my PhD. None of this would have been possible without them and I feel infinitely lucky and grateful for that.

I would like to start by thanking my advisor Dr. Ian Spielman for giving me the opportunity of joining his research group. Ian is one of the most brilliant scientists I have had the pleasure to meet and I was very fortunate to be able to work and learn with him. I am very grateful for all of his support, teaching, and for always being so patient and good-humored. Ian's unique advising style combining top-quality science with cheerful and laid back interactions makes the excitement for physics contagious!

I was exposed to a cold atoms lab for the first time when I spent a summer as an undergrad working in the cavity QED lab of Dr. Luis Orozco. I thank him for giving me this opportunity that would later define the course of my PhD. He is a great mentor who cares very deeply about his students. I am extremely grateful for his support, for all the delicious meals and bread, the concerts and plays, and the good stories. I also thank him for carefully reading my thesis and providing comments.

I would like to thank Dr. Bill Phillips for always being warm and welcoming and for taking the time to participate in 'paper torture' sessions with me. I learned a lot of physics (and history of physics) from these interactions. I would also like to thank him for being an endless source of chocolate.

My research was performed at the RbLi lab at UMD, where I was lucky that my path crossed with many wonderful physicists and also now good friends. First I would like to thank Daniel Campbell, Ryan Price and Andika Putra for welcoming me to RbLi and patiently teaching me about the lab. A very special thank you goes to our former postdoc Dimitri Trypogeorgos for taking me under his wing and becoming my lab brother. He is a brilliant scientist who really helped to shape my PhD. I am extremely grateful for all his physics and life teachings as well as his friendship. I would also like to thank and acknowledge my current lab mates. Qi-Yu Liang for her brutal honesty and for the physics discussions that tended to start with confusion and end with enlightenment. I also thank her for the rabbit pictures and videos. I admire her determination and her ability to stand up for what she believes in. Mingshu Zhao and Junheng Tao for their enthusiasm and diligently taking over the tasks of building a new BEC apparatus. I am also grateful for their encouraging words when I was writing this thesis and my last paper. They are very smart physicists and I am sure there will be great things to come in the new lab.

Our lab was also very fortunate to have a couple brilliant visitors. I thank Nathan Lundblad for all his help when the lab was feeling a bit understaffed. I would also like to thank Russ ‘Mr. Broadband’ Anderson for all the helpful experimental diagnostics gadgets, the very thorough help on calculations and comments on papers, and for always being in such a good mood. It was great fun to have him around.

Other great visitors not directly working with me but that had an impact on my PhD include Marcell Gall, Pierre Dussarat, Max Schemmer, Daniel Ohl de Melo, and Pierre Bataille. I thank them for their friendship and for the good times we

shared.

I have been away from the lab many months now and instead spending a great deal of time in my office. I would like to thank my office mates for making it all more fun. Hector Soza Martínez for being absolutely ridiculous. His sense of humor reminds me that we shouldn't take ourselves too seriously. Joao 'the champ' Braz for being a great theory buddy. I really enjoyed our physics discussions and am thankful for all the thorough comments on the Rashba paper. I am also thankful that he made me leave my desk to get a beer when he thought I needed a break from working too hard. Dina Genkina for always listening and offering encouraging words, also for the constant flow of Kit Kats.

I was very lucky to find a great community of Latin American physicists (and non-physicists) at UMD (thanks again Luis) who became some of my closest friends and kind of a family away from home. Having them around made everything more fun and helped me stay sane throughout my PhD. A big thank you to Paco, Ale, Pablo, Dalia, David, Israel, Kristel, Andres, Hector, Maggie, Carlos, Monica and Carla for their friendship. A special shout out to Dalia who has been like my academic sister ever since we both joined Luis' labs. Having a good friend like her sharing this PhD path really makes things better.

Outside of the university, I was also very fortunate to run into a wonderful group of people, especially those affiliated to the extended International House of Physicists (IHoP) family: Juraj, Kanu, Yigit, Shanon, Anna, Marko, Aydn, Seyit, Daniel, Rory, Dina, Chris, Ankit, Ray. Having them around made life more fun and enjoyable!

Besides the people I met in grad school, I had a network of good friends back home who have always been very supportive throughout the years. I thank Andrea, Adriana, Elsa, Guillermo, and Bernardo for always cheering me and making me feel like I was doing something cool.

A common denominator in many aspects of my PhD has been my partner Paco. He has always been there for me through the best and worst times and I am very grateful for that. Paco is a very kind and caring person and also a great physicist. I also thank him for listening, making me laugh, and for all his help with physics and non-physics issues. I was very lucky to find a partner, friend, and colleague in him.

I was deeply touched by the support I received for my thesis defense. Friends getting together to cook delicious tacos, surprise visits from Europe, family members visiting from various places, emails and text messages from friends and colleagues with good wishes. A special thanks to Kanu for also being my presentation coach. Her advise and ‘cups of tea’ really helped me calm down when the nerves were kicking in hard. I don’t want to sound repetitive but I am very lucky to be surrounded by such a nice community.

Finally and most importantly, I would like to thank my family. My parents always made my education a priority and none of this would have been possible without their unconditional support. I also thank my brothers for their support and all the good (and bad) times that we have shared. They try hard to annoy me sometimes but I wouldn’t change them for anyone else.



# Table of Contents

Preface	ii
Dedication	iii
Acknowledgements	iv
List of Tables	xii
List of Figures	xiii
List of Abbreviations	xvi
1 Introduction	1
1.1 Thesis overview . . . . .	5
2 Overview of Bose-Einstein condensation	7
2.1 Bose-Einstein condensation of an ideal gas . . . . .	8
2.1.1 Critical temperature . . . . .	9
2.1.2 Condensate fraction . . . . .	11
2.1.3 Bose gas in a harmonic trapping potential . . . . .	12
2.2 Bose-Einstein condensation with atomic interactions . . . . .	13
2.2.1 Gross-Pitaevskii equation . . . . .	15
2.2.2 Multiple component BECs . . . . .	17
2.2.3 Thomas-Fermi approximation . . . . .	18
2.3 Density profiles . . . . .	20
3 Manipulation and detection of ultra-cold atoms	25
3.1 Electronic structure of $^{87}\text{Rb}$ . . . . .	26
3.2 Interaction between atoms and magnetic fields . . . . .	28
3.2.1 Magnetic trapping . . . . .	31
3.3 Interaction between atoms and electric fields . . . . .	32
3.3.1 Scalar polarizability . . . . .	34
3.3.1.1 Optical trapping . . . . .	36

3.3.2	Vector polarizability and effective magnetic fields	38
3.3.2.1	Raman coupling	39
3.3.2.2	Spin-orbit coupling	42
3.4	Detection: Resonant absorption imaging	43
3.4.1	High intensity absorption imaging	46
3.5	Coherent manipulation	47
3.5.1	Rabi oscillations	48
3.5.2	Adiabatic rapid passage	51
3.5.3	Magnetic field stabilization with microwave assisted partial transfer absorption imaging	53
3.5.4	Ramsey interferometer	55
3.5.5	Floquet theory	56
4	Making BECs in the Rubidium Lithium apparatus	62
4.1	Overview of the RbLi apparatus	63
4.1.1	Laser systems	63
4.1.2	Magnetic field control	65
4.2	Experimental sequence to make BECs	67
4.3	Upgrades to the RbLi Machine	71
4.3.1	Master laser system	71
4.3.2	Raman laser system	71
4.3.3	High power RF system	74
4.3.4	6.8 GHz microwave system	77
4.4	Computer control and data acquisition	79
5	Fourier Transform Spectroscopy	83
5.1	Operating principle of Fourier spectroscopy	84
5.2	Measuring the SOC dispersion with Fourier transform spectroscopy	87
5.2.1	System	87
5.2.2	Tunable SOC	89
5.2.3	Application of Fourier spectroscopy	92
5.2.4	Effective mass	96
5.2.5	Measured dispersions	99
6	Synthetic clock transitions through continuous dynamical decoupling	103
6.1	Basic principles of CDD	104
6.2	CDD of a spin-1 system	105
6.3	The $ xyz\rangle$ states	106
6.3.1	State decomposition	107
6.3.2	Energies	108
6.3.3	Transition matrix elements	109
6.4	$ xyz\rangle$ state preparation	111
6.5	Initial characterization of $\Omega$	113
6.6	Spectroscopy	114
6.7	Robustness	115

6.7.1	Optimal response to noise	117
6.8	Driving dressed state transitions	118
6.9	Concatenated CDD	121
6.10	Conclusions	125
7	Topological order in quantum systems	128
7.1	Topology in mathematics	129
7.2	Topological order in condensed matter	131
7.3	Berry phase and Berry curvature	133
7.3.1	Aharonov-Bohm phase as an example of a Berry's phase	136
7.3.2	Chern number	138
7.4	The bulk-edge correspondence principle	140
7.5	Example: two-level model	140
7.6	Monopoles and Dirac strings	142
7.7	Conclusions	144
8	Unconventional topology in a quantum gas with Rashba SOC	145
8.1	Rashba spin-orbit coupling	146
8.2	Rashba SOC for neutral atoms	148
8.3	Experimental implementation of Rashba SOC	152
8.3.1	Raman coupling the $ xyz\rangle$ states	153
8.3.1.1	Floquet and off-resonant coupling effects	155
8.3.1.2	Lifetime	156
8.3.2	Measuring quasimomentum distributions	157
8.3.2.1	Correcting shears from Stern-Gerlach gradient	159
8.4	Fourier spectroscopy of the Rashba dispersion	161
8.5	Quantum state tomography with Ramsey interferometer	163
8.5.1	Wave function evolution in Ramsey interferometer	166
8.5.2	Combining phases from different measurements	170
8.5.3	Measuring the topological index	171
8.6	Conclusion	174
9	Conclusions and outlook	175
A	The good, the bad and the ugly of RbLi	178
A.1	The good	178
A.2	The bad	182
A.3	The ugly	185
B	New apparatus	187
B.1	Water cooling	187
B.2	Electrical installation	189
B.3	Coil winding	190
B.4	Rb source and 'oven'	191
B.5	Table enclosures	194

C Full derivation of the Raman coupled $ xyz\rangle$ states	195
Bibliography	205

## List of Tables

4.1	List of AOM driver components . . . . .	74
4.2	6.8 GHz microwave system components . . . . .	78
4.3	Camera technical specifications . . . . .	81
6.1	Summary of CDD energies . . . . .	122

## List of Figures

1	The Bose-Einstein distribution . . . . .	9
2	The Thomas-Fermi density distribution . . . . .	20
3	Momentum distribution of atoms near $T_c$ . . . . .	24
1	$^{87}\text{Rb}$ level structure . . . . .	26
2	Zeeman splitting of the $5^2S_{1/2}$ manifold of $^{87}\text{Rb}$ . . . . .	30
3	Raman coupling with two-photon transitions . . . . .	40
4	Electric realizabilities and scattering rates as a function of wavelength . . . . .	41
5	The Beer-Lambert law . . . . .	44
6	Resonant absorption imaging . . . . .	45
7	High intensity absorption image . . . . .	47
8	The Rabi cycle . . . . .	50
9	Adiabatic rapid passage . . . . .	53
10	Magnetic field stabilization using microwave assisted PTAI . . . . .	55
11	A Ramsey interferometer . . . . .	57
12	Magnetic phases of a spin-1 SOC system . . . . .	61
1	The RbLi vacuum system . . . . .	64
2	Laser cooling frequencies . . . . .	66
3	Diagram of the RbLi apparatus . . . . .	67
4	Magnetic coil geometry in the RbLi apparatus . . . . .	68
5	Master laser system . . . . .	72
6	Ti:Sapphire laser output as a function of pump power . . . . .	73
7	Optical layout of Raman and optical lattice lasers . . . . .	75
8	AOM driving electronics . . . . .	76
9	High power RF system . . . . .	77
10	Impedance matching of high power RF antenna . . . . .	78
11	Schematic of 6.8 GHz microwave system. . . . .	80
12	Comparison of probe images for Flea3 and Mako cameras . . . . .	82
1	Operating principle of Fourier spectroscopy . . . . .	85
2	Experimental setup for engineering a tunable system with equal contributions of Rashba and Dresselhaus-type spin-orbit coupling . . . . .	88

3	Floquet quasi-energy spectrum of a three-level system with spin-orbit coupling and periodic coupling strength . . . . .	90
4	Time evolution and Fourier transforms of a SOC system . . . . .	94
5	Power spectral density of the time dependent occupation probability for each state in the measurement basis for three coupling regimes . .	95
6	Dipole oscillations of a spin-orbit coupled BEC in a dipole trap . . .	98
7	Converting energy differences into absolute energies . . . . .	100
8	Spin-dependent SOC dispersion for three different coupling regimes .	101
1	Implementing CDD. . . . .	105
2	State decomposition of the $ xyz\rangle$ states . . . . .	107
3	Experimental CDD protocol . . . . .	112
4	Calibration of $\Omega$ . . . . .	113
5	Spectroscopy of the $ xyz\rangle$ states . . . . .	116
6	The $ z\rangle \rightarrow  x\rangle$ transition as a function of $\Omega_{\text{RF}}$ . . . . .	117
7	Optimal response of the $zx$ transition . . . . .	118
8	Coherent driving of dressed state transitions . . . . .	119
9	Loss of contrast of coherent oscillations . . . . .	120
10	Concatenated CDD protocol . . . . .	123
11	Concatenated CDD . . . . .	127
1	The Gaussian curvature of a surface . . . . .	130
2	The quantum Hall effect . . . . .	132
3	The Aharonov-Bohm experiment . . . . .	137
4	Graphical representation of Chern number . . . . .	142
5	Graphical representation of Chern number . . . . .	143
6	Different topological invariants . . . . .	144
1	Rashba dispersion relation . . . . .	147
2	The Rashba ring coupling . . . . .	149
3	Rashba ring coupling eigenenergies . . . . .	150
4	Rashba ring coupling ground state dispersion . . . . .	151
5	Experimental setup for implementing Rashba spin-orbit coupling . . .	153
6	Effect of neighboring Floquet manifolds on Rashba dispersion . . . .	156
7	Lifetime of Raman dressed states . . . . .	157
8	Mapping momentum into quasimomentum . . . . .	159
9	Correcting shears in momentum distribution from magnetic field gradients . . . . .	160
10	Fourier spectroscopy of a Rashba SOC syste: Experimental protocol and representative data . . . . .	162
11	Sample power spectral densities for all states used in the measurement of the Fourier spectrum . . . . .	164
12	Predicted dispersion relation and Fourier spectrum . . . . .	165
13	Experimental protocol for three-arm Ramsey interferometer (not to scale . . . . .	166

14	Dragging the Dirac point through the atoms . . . . .	168
15	Representative data of the probabilities measured after the three out- put ports of Ramsey interferometer . . . . .	170
16	Weight arrays used for combination of interferometer phases . . . . .	171
17	Topological invariants obtained from quantum state tomography . . . . .	172
1	New MOSFET bank . . . . .	179
2	The RbLi oven chamber . . . . .	181
3	Water cooling manifold schematic . . . . .	183
1	Water cooling . . . . .	188
2	Configuration of the welding cables connecting the power supplies to equipment in the lab. . . . .	189
3	Electrical installation . . . . .	190
4	Lathing of magnetic transport coils . . . . .	192
5	Rubidium oven assembly . . . . .	193
1	Raman laser layout . . . . .	197
2	Raman coupling of the $ xyz\rangle$ states . . . . .	200



## List of Abbreviations

Abbreviation	Full meaning
BEC	Bose-Einstein condensate
SOC	spin-orbit coupling
RbLi	Rubidium-Lithium
DD	dynamical decoupling
CDD	continuous dynamical decoupling
CCDD	concatenated continuous dynamical decoupling
TOF	time of flight
OD	optical depth
TF	Thomas-Fermi
MOT	magneto optical trap
RF	radio frequency
ARP	adiabatic rapid passage
RWA	rotating wave approximation
PTAI	partial transfer absorption imaging
PSD	power spectral density
TTL	transistor-transistor logic
CCD	charge-coupled device
AOM	acousto-optic modulator
SG	Stern-Gerlach
DDS	direct digital synthesizer
1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
DC	direct current
VNA	vector network analyzer
FWHM	full width at half maximum
NIST	National Institute of Standards and Technology
JQI	Joint Quantum Institute

## Chapter 1: Introduction

Bose statistics were first developed in 1924 to describe the quantum behavior of photons [1] and generalized by Einstein to include massive particles [2]. Today, we routinely produce atomic Bose-Einstein condensates (BECs) in the laboratory and even use them as a platform for the analog simulation of complex quantum systems. The field has certainly come a long way!

A connection between Bose-Einstein condensation and the onset of superfluidity in liquid  $^4\text{He}$  was first made in 1938 [3]. However, due to the strong interactions, the occupancy of the lowest energy state is dramatically reduced, leading to the search of this phase for weakly interacting Bose gases. Bose-Einstein condensation in dilute atomic gases was observed for the first time in 1995 in vapors of  $^{87}\text{Rb}$  [4],  $^{23}\text{Na}$  [5] and  $^7\text{Li}$  [6]. The dilute nature of these gases required cooling down to temperatures never achieved before and it wasn't until the experimental development of laser cooling and trapping together with evaporative cooling that bosonic atoms were cooled to the critical condensation temperature<sup>1</sup>. The experimental realization of this new phase of matter opened new possibilities for studying macroscopic quantum phenomena such as the propagation of collective modes [8, 9] and interference

---

<sup>1</sup>For a more in depth story of the BEC field I highly recommend reading [7].

of coherent matter waves [10] and jump-started an ever-growing field of research. For this achievement, Eric Cornell, Carl Wieman, and Wolfgang Ketterle received the 2001 Nobel Prize in physics.

It has been more than 20 years (and many more in the learning of atomic physics) and the field of quantum degenerate gases has expanded to include degenerate Fermi gases [11] and spinor gases [12]. As the field continues to grow, new control and detection techniques are being constantly developed, enabling the use of BECs and ultracold atomic systems in general not just as an object of study by themselves but as tools for a wide range of scientific endeavors, from precision measurements [13] to the analog simulation of complex systems.

Quantum degenerate gases are an ideal platform for quantum simulation [14]. A straightforward example comes from the use of optical lattices, where the periodic potential imparted by standing waves of light serves as an analogue to the crystal structure in a solid. Perhaps the first iconic realization of a quantum simulation was the study of the Bose-Hubbard model in three-dimensional optical lattices [15], the bosonic analogue of a model which is believed to be relevant to high- $T_c$  superconductors.

The development of light-induced gauge-fields [16] has been another important milestone in the field of quantum simulation. Such fields can be used to mimic the effect of magnetic [17, 18] and electric [19] fields with potential applications to the realization of quantum Hall materials with large magnetic fluxes [17, 20] and fractional quantum Hall states [21]. Furthermore, light-induced gauge fields can be used to engineer spin-orbit coupling (SOC) interactions [22] as those present in

two-dimensional materials, a necessary ingredient for the spin quantum Hall effect and certain kinds of topological insulators [23].

The precise level of control and tunability of ultracold atomic systems allow us to readily implement important physical models in the laboratory. Furthermore, we can go beyond conventional materials existing in nature, and create new exotic atomic ‘materials’, with interaction-dominated or topologically non-trivial band structures that can help deepen our understanding of the physical consequences of these effects on materials.

This thesis focuses on the development of new tools for the characterization and control of engineered quantum systems and their application for creating and characterizing a topological system with Rashba-type [24] SOC.

The creation of new engineered materials requires the ability to characterize their single-particle energies. We developed a Fourier transform spectroscopy technique which allows us to probe the single particle spectrum, and verify our quantum engineering, by only looking at the quantum coherent evolution. We have used Fourier transform spectroscopy to characterize the spectra from variety of engineered quantum systems with SOC [25,26] and fractional period adiabatic superlattices [27].

Atomic systems are susceptible to coupling to the environment, leading to undesired effects such as the loss of coherence. In particular laboratories such as ours greatly suffer from noise in ambient magnetic fields and go through great efforts to diminish their effects. We implemented continuous dynamical decoupling (CDD) on a set of internal atomic states which renders them first order insensitive to magnetic field changes, effectively turning them into clock states. These CDD

states are a robust basis for performing experiments, however, the most important feature is the fact that they can be coupled in a fully connected geometry unlike the hyperfine atomic states. The CDD states gave us access to new matrix elements which were essential for the engineering of Rashba SOC [26] as well as other novel lattice systems [27] not presented in this thesis.

Rashba-type SOC is a relativistic effect present in two-dimensional materials subject to an intrinsic out-of-plane electric field. The dispersion relation of Rashba systems is characterized by a Dirac point at zero momentum and an infinitely degenerate ring-shaped ground state. The presence of the Dirac point gives rise to non-trivial topology while the ring-shaped single particle ground state opens new possibilities of studying strongly correlated phases in the presence of interactions for systems of both fermions and bosons [28–30]. Using ultracold atomic systems to engineer Rashba-type SOC has been a longstanding goal [22].

In the last part of this thesis we describe a new experimental realization of Rashba SOC using Raman coupled CDD states. Our system is certainly condensed matter inspired. However, part of the beauty of engineered quantum systems is we can depart from conventional materials, for example, by considering a system with Rashba SOC but without an underlying crystalline structure. We studied the quantum state geometry of such a system with non-trivial topology in a Brillouin zone that extends to infinity. We find that our system is characterized by half-integer valued invariants, defying our conventional understanding of the topology of Bloch bands.

## 1.1 Thesis overview

This thesis describes both the standard experimental control and measurement techniques used to create BECs of  $^{87}\text{Rb}$  as well as new techniques developed in our laboratory for the engineering of Rashba SOC.

Chapter 2 describes the basic theory of Bose-Einstein condensation in dilute gases. I focus on the properties of gases confined to harmonic potentials and their density and momentum distributions as they are most relevant to the experiments presented here.

Chapter 3 describes the properties of Alkali atoms and their interactions with magnetic and electric fields which are used as standard tools for the creation, manipulation and detection of ultracold atomic systems.

Chapter 4 summarizes the experimental apparatus and mentions the most important upgrades that were not reported previously.

Chapter 5 describes a Fourier transform spectroscopy technique that exploits the relation between quantum coherent evolution and the underlying spectrum of a system and that was used to characterize experiments described later in the thesis.

Chapter 6 describes an implementation of continuous dynamical decoupling using a strong radio-frequency magnetic field that helped to both make our system more robust against environmental magnetic field noise. The implementation of CDD allows us to couple the internal states of the atoms in new ways that were not possible before, opening the path for new kinds of quantum simulations described in Chapters 8.

Chapter 7 presents concepts of topology in physics and its application to the band theory of solids. These concepts will be important for a better understanding of the topological properties of our Rashba spin-orbit coupled system.

Chapter 8 describes a new experimental realization of Rashba SOC using a combination of laser beams that couple a set of CDD states. This system has a topological dispersion relation but no underlying crystalline structure which allows for topological invariants to take non-integer values.

Appendix A summarizes the best and worst aspects of the experimental apparatus as a guide for future generations working in BEC labs.

Appendix B describes my work related to the design and construction of a dual species apparatus for the production of BECs of  $^{87}\text{Rb}$  and  $^{39}\text{K}$ .

Appendix C shows the derivation of the full time-dependent Hamiltonian describing the Raman dressing of the RF dressed states (Chapter 6) used to generate Rashba-type SOC (Chapter 8).

## Chapter 2: Overview of Bose-Einstein condensation

Bose-Einstein condensate (BEC) is a quantum state of matter in which particles with integer valued spin all tend to occupy or ‘condense’ into the ground state. In dilute gases, condensation occurs when the temperature of the system goes below a critical temperature where quantum statistics become relevant.

BECs enable the study, both theoretically and experimentally, of macroscopic quantum phenomena. There have been a number of fascinating experiments studying the properties of these systems, from measuring interference fringes of a macroscopic wave function to studying collective effects such as the propagation of sound [7]. In our experiments, however, BECs are not the primary object of study, instead, they are used as a platform enabling the simulation of analog physical systems.

In this Chapter I give an overview of Bose-Einstein condensation in dilute atomic gases and I describe the properties most relevant to our experiments. I start by describing the case of an ideal gas and then consider the effects of interactions and trapping potentials as are present in our case. A reader interested in learning about Bose-Einstein condensation in dilute gases in more depth should read [31] and [32].



## 2.1 Bose-Einstein condensation of an ideal gas

At low temperatures and in thermodynamic equilibrium, the mean occupation number of non-interacting identical bosons in the state with energy  $E$  is given by the Bose-Einstein distribution

$$n(E_j) = \frac{1}{e^{(E_j - \mu)/k_B T} - 1} \quad (2.1)$$

where  $T$  is the temperature,  $\mu$  is the chemical potential (the energy cost of adding or removing a particle) and  $k_B$  is the Boltzmann constant. In the limit of large temperatures the Bose distribution approximates the Maxwell-Boltzmann distribution

$$n(E_j) \approx e^{-(E_j - \mu)/k_B T} \quad (2.2)$$

which applies to classical, distinguishable particles. The chemical potential is determined by the condition that the total number of particles  $N$  is equal to the sum over all states in the distribution  $N = \sum_j n(E_j)$  and is a function of  $N$  and  $T$ . Additionally, in order for  $n(E_j)$  to be positive definite we must have  $\mu \leq E_0$  where  $E_0$  is the energy of the ground state. From the Bose distribution, we can see that the occupation number of the ground state is unbounded when  $\mu \rightarrow 0$  as in Figure 2.1. The number of particles occupying the excited states is bounded and when that number is reached, the remaining particles can occupy the ground state and thus Bose-Einstein condensation occurs.

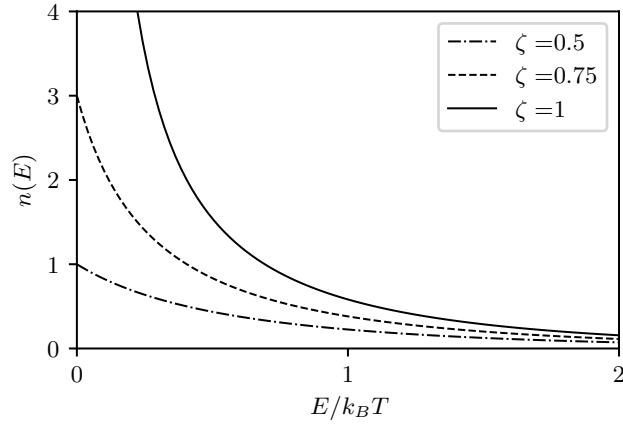


Figure 1: The Bose-Einstein distribution. Occupation number as a function of energy for different values of fugacity  $\zeta = \exp(\mu/k_B T)$ . Condensation occurs when  $\mu = 0$  ( $\zeta = 1$ ) and the occupation number in the ground state diverges.

### 2.1.1 Critical temperature

Bose-Einstein condensation can be understood in terms of the de Broglie waves associated with particles. The thermal de Broglie wavelength is

$$\lambda_{\text{th}} = \left( \frac{2\pi\hbar^2}{mk_B T} \right)^{1/2} \quad (2.3)$$

and it characterizes the spatial extension of the wave packet an individual particle at temperature  $T$ . Condensation occurs when  $\lambda_{\text{th}}$  becomes comparable with the inter-particle separation  $n^{-1/3}$ , where  $n = N/V$  and  $V$  is the volume. The quantity  $n\lambda_{\text{th}}^3$  is known as the phase space density which describes the number of particles contained in a box with volume  $\lambda_{\text{th}}^3$ .

An analytical expression for the critical temperature at which atoms condense can be derived using the Bose-Einstein distribution. For closely spaced energy lev-

els (compared to  $k_B T$ ) the sum representing the total number of particles can be replaced by the integral

$$N = \int_0^\infty n(E)g(E)dE \quad (2.4)$$

where  $g(E)$  is the density of states and  $g(E)dE$  corresponds to the number of available states with energy between  $E$  and  $E+dE$ . For a free particle in three dimensions the density of states is

$$g(E) = \frac{Vm^{3/2}}{\sqrt{2}\pi^2\hbar^2}E^{1/2}, \quad (2.5)$$

and in general the density of states can be expressed as a power of energy  $g(E) = C_\alpha E^{\alpha-1}$ .

The integral in Equation 2.4 is not analytically solvable, however, we can make the simplifying assumption  $\mu = 0$ . The critical temperature  $T_c$  is determined by the condition that all particles are in the excited states

$$\begin{aligned} N &= N_{\text{ex}}(T_c, \mu = 0) \\ &= \int_0^\infty \frac{g(E)dE}{e^{E/k_B T_c} - 1} \\ &= C_\alpha (k_B T_c)^\alpha \int_0^\infty \frac{x^{\alpha-1}}{e^x - 1} \\ &= c_\alpha (k_B T_c)^\alpha \Gamma(\alpha) \zeta(\alpha) \end{aligned} \quad (2.6)$$

where  $x = E/k_B T_c$ ,  $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$  is the Gamma function and  $\zeta(\alpha) = \sum_{n=1}^\infty n^{-\alpha}$  is the Riemann zeta function. From Equation 2.6 we find that the critical

temperature for Bose-Einstein condensation is

$$k_B T_c = \left( \frac{N}{C_\alpha \Gamma(\alpha) \zeta(\alpha)} \right)^{1/\alpha}. \quad (2.7)$$

If we compute the phase space density for free particles in 3D with density of states given by Equation 2.5 in combination with the expression for the critical temperature (Equation 2.6) we find that when  $T = T_c$

$$n \lambda_{\text{th}}^3 = \zeta\left(\frac{3}{2}\right) \approx 2.612, \quad (2.8)$$

the inter-particle spacing and the thermal wavelength are comparable. In order to experimentally produce BECs, we deploy a combination of laser and evaporative cooling techniques such that we can increase the density while minimizing the temperature, maximizing the phase space density. The densities for BECs of Alkali atoms typically range between  $10^{13}$  and  $10^{15}$  atoms/cm<sup>-3</sup>.

### 2.1.2 Condensate fraction

The total number of particles is given by  $N = N_0 + N_{\text{ex}}$ , where  $N_0$  is the number of particles in the condensate. The number of particles in the excited state will be given by the integral in Equation 2.4. For  $g(E) = C_\alpha E^{\alpha-1}$  and  $\alpha > 0$  the integral converges, we can then evaluate the integral in Equation 2.6 for  $T < T_c$  and

get

$$\begin{aligned} N_{\text{ex}} &= c_\alpha (k_B T)^\alpha \Gamma(\alpha) \zeta(\alpha) \\ &= N \left( \frac{T}{T_c} \right)^\alpha, \end{aligned} \tag{2.9}$$

since for  $T = T_c$  the total number  $N = N_{\text{ex}}$ . The number of particles in the ground state is

$$\begin{aligned} N_0 &= N - N_{\text{ex}} \\ &= N \left[ 1 - \left( \frac{T}{T_c} \right)^\alpha \right] \end{aligned} \tag{2.10}$$

### 2.1.3 Bose gas in a harmonic trapping potential

Our experiments are performed in optical dipole traps that can be described as harmonic potentials

$$V(\mathbf{r}) = \frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \tag{2.11}$$

The density of states for this case is

$$g(E) = \frac{E^2}{2\hbar^2 \omega_x \omega_y \omega_z}, \tag{2.12}$$

which corresponds to  $\alpha = 3$  and  $C_3 = (2\hbar^3\omega_x\omega_y\omega_z)^{-1}$ . Using Equation 2.6, we find that the transition temperature is

$$k_B T_c = \frac{\hbar\bar{\omega}N^{1/3}}{\zeta(3)^{1/3}} \approx 0.94\hbar\bar{\omega}N^{1/3} \quad (2.13)$$

where  $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$  is the geometric mean of the oscillation frequencies. Similarly we find that the condensed fraction is

$$N_0 = N \left[ 1 - \left( \frac{T}{T_c} \right)^3 \right] \quad (2.14)$$

Condensates in harmonic traps have some striking features that will be further explored in more detail in the following sections. The confining potential makes the BECs both finite-sized and inhomogeneous which means that the BEC can be observed both in momentum space and in coordinate space. Another consequence of the inhomogeneity of these systems is the role of two-body interactions, which gets enhanced and leads to noticeable effects in measurable quantities [33, 34] such as interaction driven expansion when they are released from the confining potential.

## 2.2 Bose-Einstein condensation with atomic interactions

Even though atomic BECs are made from very dilute gases, the system is far from being an ideal gas and a complete treatment requires taking into account interactions.

The collisional properties of particles at low energies, such as cold atoms in

a condensate, are dominated by  $s$ -wave scattering which can be described in terms of a single parameter the scattering length  $a$  that determines both the scattering cross-section  $\sigma = 4\pi a^2$  and the phase shift of the scattered wave function.

The magnitude of the scattering length is determined by the interatomic interaction potentials. For Alkali atoms at large distances, the two-body interactions are dominated by an attractive Van der Waals interaction  $U(r) = -C_6/r^6$  that arises from dipole-dipole interactions. At smaller distances, the attractive potential is replaced by a strong repulsive electron-exchange interaction  $U(r) \rightarrow \infty$ . This minimal model captures the most important properties of the inter-atomic potential and can be solved analytically [35].

If the range of the interaction is much shorter than the mean inter-atomic distance the interaction can be approximated by an effective pseudo-potential  $U_{\text{eff}}(\mathbf{r} - \mathbf{r}')$  such that

$$a = \frac{m}{4\pi\hbar^2} \int U_{\text{eff}}(\mathbf{r} - \mathbf{r}') d\mathbf{r} \quad (2.15)$$

which determines

$$U_{\text{eff}}(\mathbf{r} - \mathbf{r}') = \frac{4\pi\hbar^2 a}{m} \delta(\mathbf{r} - \mathbf{r}') = g\delta(\mathbf{r} - \mathbf{r}'). \quad (2.16)$$

This approximation allows us to model the scattering between atoms as a hard-sphere scattering process instead of considering the more complicated inter-atomic potentials. The sign of the scattering length determines the attractive or repulsive nature of the interactions and it plays an important role in the experimental produc-

tion of BECs as it determines the rate at which atoms thermalize during evaporative cooling.

### 2.2.1 Gross-Pitaevskii equation

The full Hamiltonian describing  $N$  identical bosons with contact interactions is

$$\hat{H} = \sum_{i=1}^N \left[ \frac{\mathbf{p}_i^2}{2m} + V(\mathbf{r}_i) \right] + g \sum_{i<j} \delta(\mathbf{r}_i - \mathbf{r}_j), \quad (2.17)$$

where  $V(\mathbf{r})$  is an external potential and  $\mathbf{p}_i = -i\hbar\nabla_i$  is the individual momentum of each atom. We consider a normalized eigenstate of the Hamiltonian  $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  that satisfies the Schrödinger equation. We can simplify this state by taking a mean field approach; we assume that the system has undergone condensation so that the majority of the particles share the same single particle ground state  $\phi(\mathbf{r})$ , then the wavefunction can be approximated by a symmetrized product

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \prod_{i=1}^N \phi(\mathbf{r}_i), \quad (2.18)$$

where  $\phi$  is normalized to unity. The energy of the state from Equation 2.18 is given by the expectation value

$$\begin{aligned} E &= \int \Psi^* \hat{H} \Psi d\mathbf{r} \\ &= N \int \phi^*(\mathbf{r}) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + \frac{(N-1)}{2} g |\phi(\mathbf{r})|^2 \right] \phi(\mathbf{r}) d\mathbf{r}, \end{aligned} \quad (2.19)$$



where  $N(N - 1)/2 \approx N^2/2$  counts the number of terms in the interaction energy. We introduce the wave function of the condensate  $\psi(\mathbf{r}) = N^{1/2}\phi(\mathbf{r})$ , and inserting it in Equation 2.19 makes the  $N$  factors cancel out. The optimal form of  $\psi$  should minimize the energy, introducing a Lagrange multiplier  $\mu$ , subject to the normalization condition  $N = \int |\psi(\mathbf{r})|^2 d\mathbf{r}$

$$\frac{\delta}{\delta\psi^*(\mathbf{r})} \left( E - \mu \int |\psi|^2 d\mathbf{r} \right) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g|\psi(\mathbf{r})|^2 - \mu \right] \psi(\mathbf{r}) = 0, \quad (2.20)$$

and we find that the condensate wave function obeys a non-linear Schrödinger equation known as the Gross-Pitaevskii (GP) equation

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g|\psi(\mathbf{r})|^2 \right] \psi(\mathbf{r}) = \mu\psi(\mathbf{r}), \quad (2.21)$$

where  $\mu$  plays the role of the chemical potential. The dynamics of the condensate will similarly be described by the time-dependent GP equation

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g|\psi(\mathbf{r}, t)|^2 \right] \psi(\mathbf{r}, t) \quad (2.22)$$

The GP equation describes the relevant phenomena associated with the propagation of collective excitations and the expansion of the condensate when released from a trap. The crucial assumption when deriving these equations is the mean field approximation. This should be valid for dilute BECs where the condensate fraction is close to unity. The excitations of the system can be described by a set of equations similar to those of classical hydrodynamics derived from the GP equation

or alternatively using Bogoliubov theory for weakly interacting bosons [31].

## 2.2.2 Multiple component BECs

So far the discussion has been limited to single component BECs but most of our experiments are performed using a combination of multiple atomic internal states. In general, for a multiple component condensate the scattering lengths characterizing the interactions depend on the internal states of the incoming and outgoing scattering channels. Two spin- $f$ <sup>1</sup> particles colliding particles will be characterized by  $2f$  scattering lengths  $a_F$ . For bosons the total spin  $F$  takes even values and in particular for  $^{87}\text{Rb}$  atoms in the  $f = 1$  hyperfine ground state there are two scattering lengths  $a_0$  and  $a_2$  corresponding to the two-particle total angular momentum states of  $F = 0$  and  $F = 2$  respectively. The values of scattering lengths are  $a_0 = 101.8a_0$  and  $a_2 = 100.4a_0$  [12] where  $a_0 = 5.29 \times 10^{-11}$  is the Bohr radius.

From the scattering lengths we can calculate two interaction coefficients

$$\begin{aligned} c_0 &= \frac{4\pi\hbar^2}{m} \frac{a_0 + 2a_2}{3} = 100.84a_0 \frac{4\pi\hbar^2}{m} \\ c_2 &= \frac{4\pi\hbar^2}{m} \frac{a_0 - a_2}{3} \approx -4.7 \times 10^{-3} c_0. \end{aligned} \quad (2.23)$$

Here  $c_0$  represents a spin-independent interaction strength that depends only on the total density while  $c_2$  is a spin-dependent energy that is relevant only where there is non-zero density of both atoms in  $m_F = \pm 1$  and is much smaller than the spin-

---

<sup>1</sup>Here I use the symbol  $f$  to denote the angular momentum of the individual particles and  $F$  to denote the total angular momentum of the two particles.

independent energy. Similar to the case of single component BECs, the dynamics of multiple component BECs is governed by a spinor GP equation (see [12, 31]). The spin-dependent interaction strength gives rise to processes like coherent spin-mixing oscillations and domain formation and coarsening which was previously studied in the our lab [36].

The time scale at which interactions become relevant is set by the interaction energies  $n|c_i|$ . The most noticeable effect of interactions in our system is the density profile of the condensate and its anisotropic expansion after it is released from a trap which I will describe in the following sections. For the typical densities and timescales of our experiments as well as the relatively high magnetic fields that we operate at, we do not observe noticeable effects from interactions in the dynamics of the system and in the remaining chapters I will describe the dynamics of the BEC using single particle physics (i.e. the regular time-dependent Schrödinger equation).

### 2.2.3 Thomas-Fermi approximation

For systems with large  $N$ , the interaction term in the GP equation is very large compared to the kinetic energy<sup>2</sup>. As the kinetic energy becomes less important we enter the Thomas-Fermi (TF) regime where the energy of the system is given only by the external potential and the mean field energy and the GP equation is considerably simplified

$$\left[ V(\mathbf{r}) + g|\psi(\mathbf{r})|^2 \right] \psi(\mathbf{r}) = \mu\psi(\mathbf{r}). \quad (2.24)$$

---

<sup>2</sup>It can be shown that the ratio of kinetic energy to interactions scales as  $N^{-4/5}$

In the TF regime the density distribution of the condensate  $n(\mathbf{r}) = |\psi(\mathbf{r})|^2$  reflects the shape of the external potential

$$n(\mathbf{r}) = g^{-1}[\mu - V(\mathbf{r})], \quad (2.25)$$

when  $\mu - V(\mathbf{r}) > 0$  and is otherwise zero. For a harmonic confining potential (Equation 2.11) as is typical in our experiments we find that the length scale that characterizes the size of the condensate is the Thomas-Fermi radius

$$R_j = \sqrt{\frac{2\mu}{m\omega_j^2}}, \quad j = x, y, z. \quad (2.26)$$

The density of the condensate is described by an inverted parabola

$$n(\mathbf{r}) = \frac{\mu}{g} \left( 1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right). \quad (2.27)$$

as is shown in Figure 2a. By integrating over Equation 2.27 we find that

$$N = \frac{8\pi}{15} \frac{\mu}{g} R_x R_y R_z, \quad (2.28)$$

which allows to determine the number of atoms in the condensate based on the density profile. In practice, in-situ BECs are very dense which can lead to some technical difficulties when trying to image directly their density profiles (see Section 3.4) so instead our images are taken after the atoms are released from the trap and allowed to expand for some time.

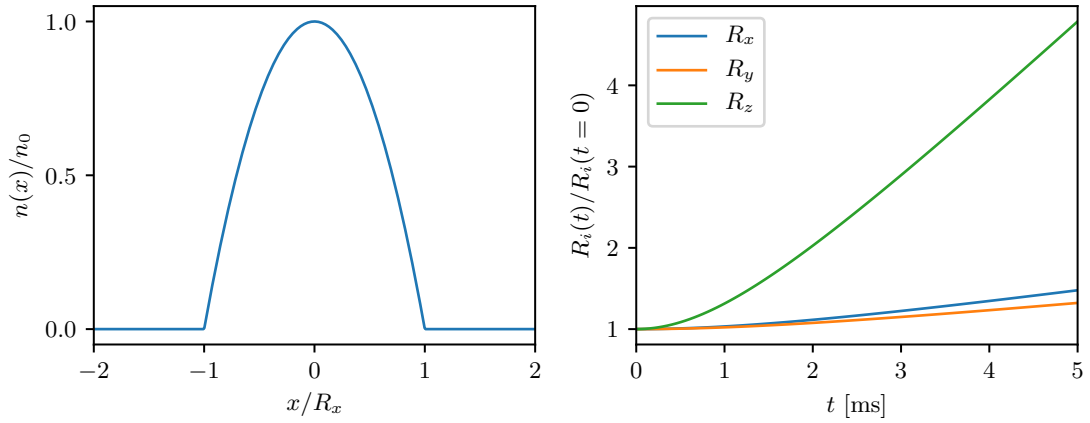


Figure 2: In the Thomas-Fermi regime where interactions are large compared to kinetic energy the density profile is determined by the external potential. **a.** Density profile along  $\mathbf{e}_x$  of a BEC in a harmonic potential. **b.** Interaction driven expansion of a BEC in time-of-flight for a trap with trapping frequencies  $(\omega_x, \omega_y, \omega_z) = 2\pi(42, 34, 133)$  Hz obtained by numerically integrating Equation 2.30.

### 2.3 Density profiles

Direct imaging of the atoms probes most ultracold atom experiments. (e.g. with absorption imaging, Section 3.4). If the atoms are imaged in-situ we access their spatial density profiles. If the atoms are released from the trap and allowed to expand in time of flight (TOF) we image their momentum distribution.

For the case of a BEC confined in a harmonic potential at zero temperature (no thermal fraction) and in the Thomas-Fermi regime discussed in Section 2.2.3,

the in-situ density distribution is described by

$$\begin{aligned} n(\mathbf{r}) &= n_0 \left( 1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right) \\ &= \frac{15N}{8\pi R_x R_y R_z} \left( 1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right). \end{aligned} \quad (2.29)$$

Even though the BEC is in the motional ground state, it will expand during TOF as a consequence of interactions. The expansion can be determined using the time dependent GP equation. A detailed account of the procedure can be found in [34], the procedure relies on using the ansatz that the TF radii expand as

$$R_i(t) = \lambda_i(t) R_i(t=0), \quad (2.30)$$

where the condensate is in the trap at  $t = 0$  which implies that  $\lambda_i(0) = 1$  and it is then suddenly turned off at  $t > 0$ . If we insert the condensate wave function with TF radii given by Equation 2.30 into the time-dependent GP equation we find a series of differential equations

$$\frac{d^2 \lambda_i}{dt^2} = \frac{\omega_i^2}{\lambda_i \lambda_x \lambda_y \lambda_z} \quad (2.31)$$

whose solutions determine the density profile of the BEC in TOF. Alternatively, if the density profile of the BEC is known from an image, these relations can be used to back-propagate the original TF radii of the confined condensate. This is helpful for example to calculate the atom number in the condensate using Equation 2.28.

Figure 2b shows the scaling factors  $\lambda_i$  as a function of TOF that were obtained by numerically integrating Equation 2.30 for a harmonic potential with frequencies close to those characterizing the optical dipole trap used in most of our experiments.

For a thermal gas in a harmonic potential at temperatures higher than the level spacing  $k_B T > \hbar \omega_{x,y,z}$  the density is given by [7]

$$n_{\text{th}}(\mathbf{r}) = \frac{1}{\lambda_{\text{th}}^3} g_{3/2}(z(\mathbf{r})) \quad (2.32)$$

where  $z(\mathbf{r}) = \exp(\mu - V(\mathbf{r})/k_B T)$ ,  $V(\mathbf{r})$  is given by Equation 2.11,  $\mu$  is the chemical potential and  $g_j(z) = \sum_i z^i / i^j$  is the Bose function. The Bose function introduces effects of quantum statistics and compared to the Maxwell-Boltzmann distribution of distinguishable particles, the peak density of a Bose gas increases by  $g_{3/2}(z)/z$ , a phenomenon known as Bose-enhancement.

The distribution after TOF is calculable considering that the trapped atoms fly ballistically from their position in the trap. An atom starting initially at the point  $\mathbf{r}_0$  moves to the point  $\mathbf{r}$  after a time  $t$  if its momentum is given by  $\mathbf{p} = m(\mathbf{r} - \mathbf{r}_0)/t$ , then

$$\begin{aligned} n_{\text{tof}} &= \frac{1}{\lambda_{\text{th}}} \prod_{i=1}^3 g_{3/2} \left( \exp \left[ \mu - \frac{m}{2} \sum_{i=1}^3 x_i^2 \left( \frac{\omega_i^2}{1 + \omega_i^2 t^2} \right) \right] \right) \\ &\approx \frac{1}{\lambda_{\text{th}}} g_{3/2} \left( \exp \left[ (\mu - \frac{mr^2}{2t^2}) / k_B T \right] \right) \end{aligned} \quad (2.33)$$

where the approximation in the second line is valid for  $t \gg \omega_i^{-1}$ . The temperature of the atoms can be estimated by looking at the wings of the density distribution after

TOF. Even with the case of Bose enhancement, the density of the wings still decays exponentially as  $\exp(-x_i^2/2\sigma_i^2)$ . The temperature of the cloud can be determined using

$$\begin{aligned} k_B T &= \frac{m}{2} \left( \frac{\omega_i^2}{1 + \omega_i^2 t^2} \sigma_i^2 \right) \\ &\approx \frac{m}{2} \left( \frac{\sigma_i}{t} \right)^2 \end{aligned} \tag{2.34}$$

For partially condensed clouds the density profiles will be given by a combination of the thermal density profiles and the Thomas-Fermi density profile. Figure 3 shows the density distributions of atoms extracted from images taken after a 21 ms TOF and therefore the position is mapped to momentum. The images summarize some of the main concepts discussed in this Chapter. Above  $T_c$  the density profile of the atoms corresponds to the thermal distribution (Equation 2.33). When  $T < T_c$  a small peak in the center of the thermal distribution appears indicating condensation. As the temperature decreases, the fraction of atoms in the condensed state (and therefore the height of the peak) increases. Equation 2.29 gives density distribution of the condensed atoms, where the TF radius increases due to interactions and the scaling factors can be found using Equation 2.30.



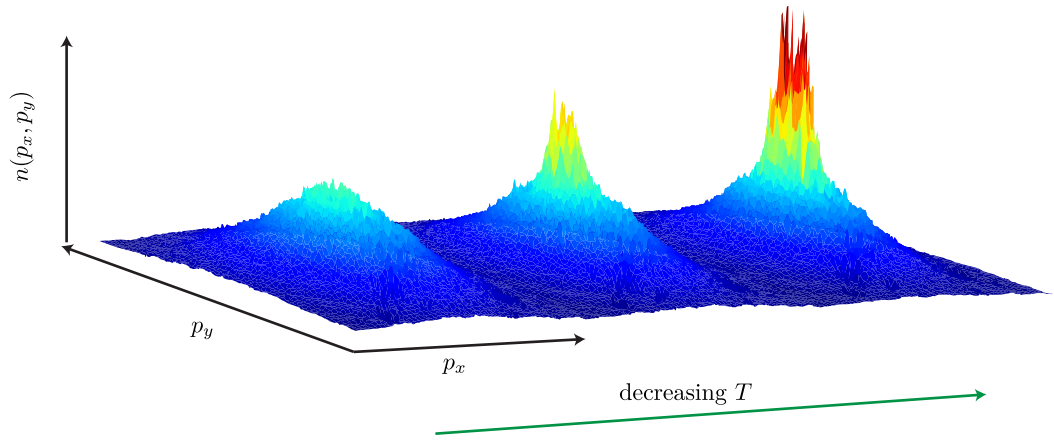


Figure 3: Momentum distribution of atoms near  $T_c$  after a 21 ms TOF. As the atoms cool below  $T_c$  a sharp peak in the momentum distribution appears indicating condensation.

## Chapter 3: Manipulation and detection of ultra-cold atoms

All of the experiments described in this thesis were performed using ultracold clouds of  $^{87}\text{Rb}$ . In this Chapter I describe the techniques and interactions that make our experiments possible. This Chapter is not an extensive survey of atomic physics but rather covers the topics that are most relevant to my experiments. The references I included are helpful if the reader is interested in learning the details of the derivations or wants to expand on a given topic.

In Section 3.1 I start by describing the electronic structure of  $^{87}\text{Rb}$ . In Sections 3.2 and 3.3 I describe atomic interactions with external fields that are necessary for the creation, manipulation and detection of ultracold atoms. First I review the interactions of atoms with magnetic fields and its application to magnetic trapping. Then I describe the foundations of atom-light interactions that make possible both laser cooling and trapping of atoms and give rise to Raman induced transitions. In Section 3.4 I discuss the resonant absorption imaging technique that we use to detect atoms after all our experiments are performed. Finally, in Section 3.5 I discuss coherent processes that use the magnetic and electric dipole interaction and are relevant to the experiments presented in Chapters 5, 6, and 8.

### 3.1 Electronic structure of $^{87}\text{Rb}$

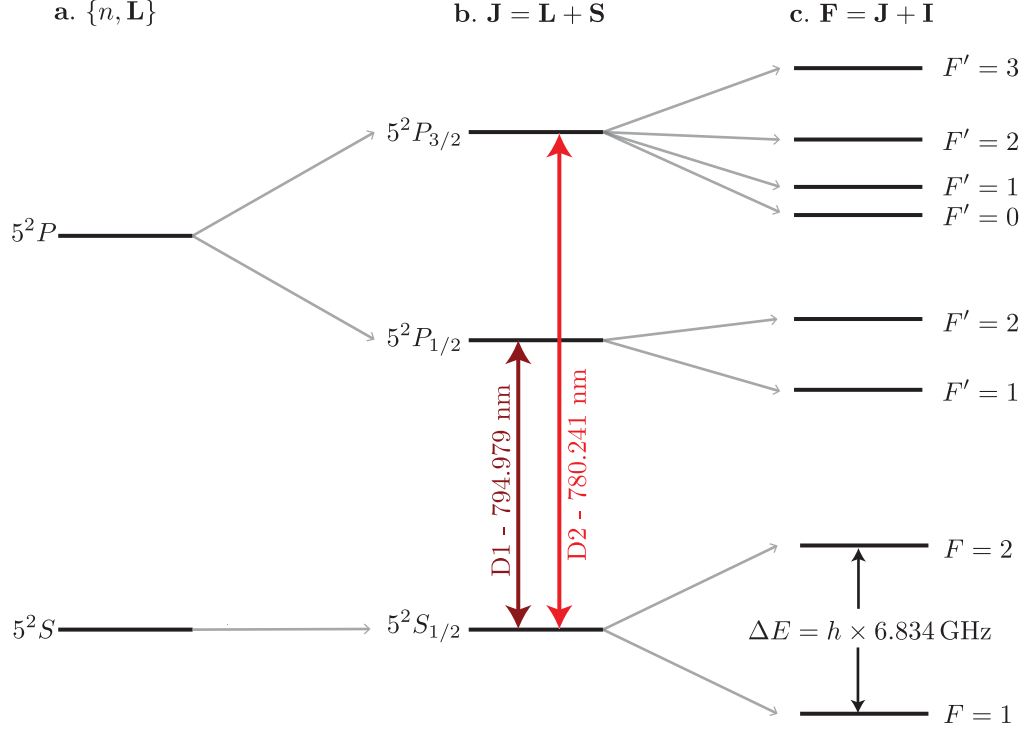


Figure 1: Level structure of the two lowest electronic energy levels of  $^{87}\text{Rb}$  (not to scale). **a.** Ground and first excited state electronic configuration of  $^{87}\text{Rb}$  given by the  $\{n, \mathbf{L}\}$  quantum numbers. **b.** The interaction between the orbital angular momentum and the spin of the electron leads to the fine structure splitting of orbitals with  $L > 0$ . The splitting of the  $5^2P$  line gives rise to the D1 and D2 lines. **c.** The interaction between the total angular momentum and the nuclear spin causes the fine structure levels to split further into states characterized by the quantum number  $F$ .

Rubidium is an Alkali metal. Alkali metals correspond to the first group (leftmost column) of the periodic table and are characterized by having a single valence electron, which makes the description of their internal structure simpler than that of other elements. The state of an electron in an atom is described by its angular momentum  $\hat{\mathbf{L}}$  and its spin  $\hat{\mathbf{S}}$ . Due to Pauli's exclusion principle there

can not be two electrons with the same quantum numbers and in multi-electron atoms they tend to fill ‘shells’ of different angular momentum values, historically labeled by the letters  $S$ ,  $P$ ,  $D$ ,  $F$ , ...<sup>1</sup> (corresponding to  $L = 0, 1, 2, 3, \dots$ ). In particular, Rb has 4 filled shells and one electron in the  $5S$  shell, where the number 5 corresponds to the principal quantum number  $n$ . Figure 1 shows the energy levels of the ground state  $5S$  and its closest  $5P$  orbital.

The relativistic treatment of the electron’s motion gives rise to an interaction between the electron’s intrinsic magnetic moment (the spin)  $\hat{\mathbf{S}}$  and the orbital angular momentum  $\hat{\mathbf{L}}$ . This spin-orbit coupling interaction  $\hat{H}_{\text{fs}} = A_{\text{fs}} \mathbf{L} \cdot \mathbf{S}$  causes the fine structure splitting of the electronic orbitals into levels with different total electronic angular momentum  $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$ . Figure 1b shows the  $5^2S_{1/2}$ ,  $5^2P_{1/2}$  and  $5^2P_{3/2}$  electronic configurations that arise from this splitting, using the spectroscopic notation  $n^{2S+1}L_J$  that indicates the values of the relevant quantum numbers. For  $S$  ( $L = 0$ ) orbitals,  $J = 1/2$  is the only possible value and the levels do not split. For the  $P$  orbital ( $L = 1$ ) and a single electron with  $S = 1/2$ ,  $J$  can be  $1/2$  or  $3/2$  and the  $P$  orbital splits into two levels. The  $5^2S_{1/2} \rightarrow 5^2P_{1/2}$  is known as the D1 line and has wavelength  $\lambda = 794.979 \text{ nm}$  and  $5^2S_{1/2} \rightarrow 5^2P_{3/2}$  transition is known as the D2 line and has  $\lambda = 790.241 \text{ nm}$  [37].

The atomic level structure gets further modified by the magnetic interaction of the electron with the nuclear spin  $\mathbf{I}$ . This is another kind of spin-orbit interaction that gives rise to the hyperfine splitting of the atomic levels which can be described

---

<sup>1</sup>These terms were used to describe the lines in the emission spectra when they were first discovered.  $S$  stands for sharp,  $P$  for principal  $D$  for diffuse and  $F$  for further noted

by the Hamiltonian  $\hat{H}_{\text{hfs}} = A_{\text{hfs}} \mathbf{I} \cdot \mathbf{J}$ . A complete derivation of  $\hat{H}_{\text{hfs}}$  can be found in [38]. The hyperfine levels correspond to different values of the total angular momentum  $\hat{\mathbf{F}} = \hat{\mathbf{J}} + \hat{\mathbf{I}}$ . For  $^{87}\text{Rb}$   $I = 3/2$  [37] which results in the level structure shown in Figure 1c, where the ground state is split into two levels with total angular momentum  $F = 1$  and  $F = 2$ .

### 3.2 Interaction between atoms and magnetic fields

Atoms have an intrinsic magnetic moment that is given by the sum of nuclear and electronic moments

$$\hat{\boldsymbol{\mu}} = -\frac{\mu_B}{\hbar}(g_S \hat{\mathbf{S}} + g_L \hat{\mathbf{L}} + g_I \hat{\mathbf{I}}) \quad (3.1)$$

where  $\mu_B$  is the Bohr magneton and  $g_S$ ,  $g_L$  and  $g_I$  are the ‘ $g$ -factors’ corresponding to the spin, orbital and nuclear angular momentum. In the presence of an external magnetic field  $\mathbf{B}$ , the internal levels of an atom get modified due to the Zeeman [39] interaction

$$\hat{H}_{\text{Zeeman}} = -\hat{\boldsymbol{\mu}} \cdot \mathbf{B}. \quad (3.2)$$

If the energy shift due to the Zeeman interaction is small compared to the hyperfine splitting then  $F$  is a good quantum number and we can write

$$\hat{H}_{\text{Zeeman}} = \frac{\mu_B g_F}{\hbar} \hat{\mathbf{F}} \cdot \mathbf{B} \quad (3.3)$$

where  $g_F$  is the hyperfine Landé  $g$ -factor and is given by

$$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + g_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)}, \quad (3.4)$$

and

$$g_J \approx 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}. \quad (3.5)$$

is the Landé  $g$ -factor associated to the total electronic angular momentum  $J$ . The total energy shifts can be calculated by diagonalizing the full atomic Hamiltonian including the fine and hyperfine structure terms. Figure 2 shows the energies of the  $|m_F\rangle$  levels in the  $F = 1$  and  $F = 2$  manifolds of the  $5^2S_{1/2}$  electronic ground state of  $^{87}\text{Rb}$  as a function of magnetic field. If the magnetic field is small then the Zeeman term can be treated as a perturbation to the atomic Hamiltonian and the energy split is linear with the magnitude of the field  $E_{m_F} = g_F \mu_B m_F B$ , what is known as the ‘linear Zeeman regime’ where  $F$  and  $|m_F\rangle$  are good quantum numbers. In contrast, in the ‘Pachien-Back regime’ at large magnetic fields<sup>2</sup> the Zeeman term dominates over the fine and hyperfine terms and therefore the good quantum numbers of the system are  $J$  and  $m_J$ . Our experiments typically operate in an intermediate regime ( $B \sim 10 - 30$  G, the gray box in Figure 2) where the externally imposed magnetic field starts to be comparable to the magnetic field produced by the valence electron at the nucleus and the energy of  $|m_F = 0\rangle$  gets a small shift in energy that is quadratic in  $B$ . For atoms in  $F = 1$  we define this quadratic Zeeman

---

<sup>2</sup>A couple orders of magnitude larger than the fields we operate at.

shift as  $\epsilon = E_0 - (E_{+1} - E_{-1})/2$ , where  $E_{m_F}$  is the Zeeman shift for state  $|m_F\rangle$ .

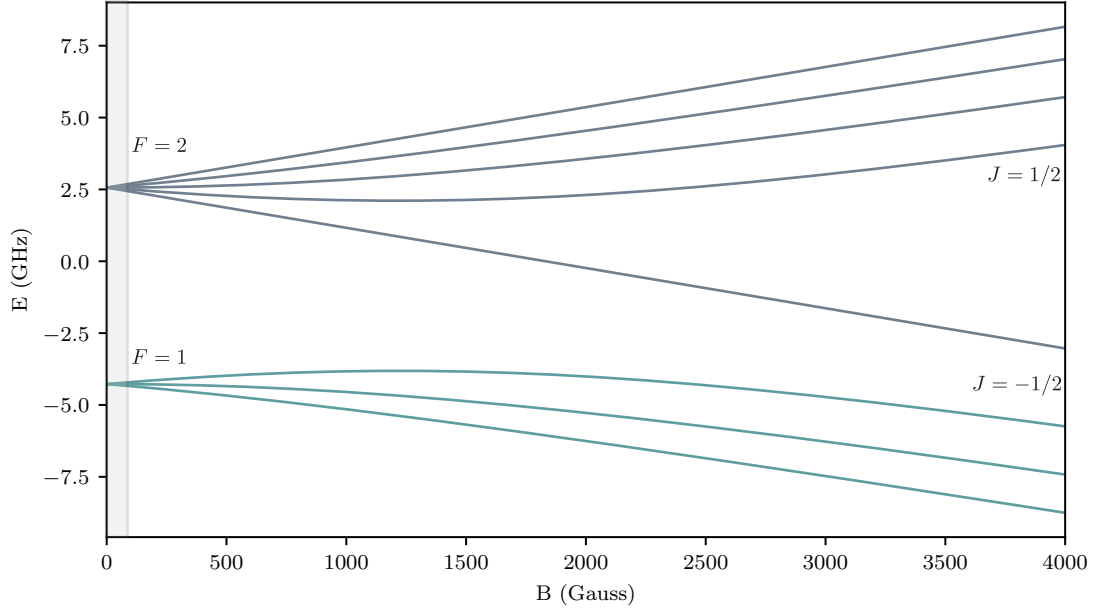


Figure 2: Zeeman splitting of the  $5^2S_{1/2}$  manifold of  $^{87}\text{Rb}$ . At small magnetic fields  $F$  and  $|m_F\rangle$  are good quantum numbers describing the system and at large magnetic fields (Paschen-Back regime) the states are described by the  $J$  and  $m_J$ . Our experiments operate in the regime marked by the small gray box ( $B < 35$  G).

For the particular case of states with  $J = 1/2$  (like the ground state of Alkalies), the Zeeman energies can be found analytically using the Breit-Rabi formula [40]

$$E_{m_F} = -\frac{1}{2(2I+1)} + \frac{\mu_B g_I m_F B}{\Delta E_{\text{hf}}} + \frac{1}{2} \sqrt{1 + \frac{4m_F}{2I+1}x + x^2}, \quad (3.6)$$

where  $\Delta E_{\text{hf}} = A_{\text{hf}}(J + 1/2)$  and  $x = (g_J - g_I)\mu_B B_z / \Delta E_{\text{hf}}$ .

### 3.2.1 Magnetic trapping

The sign of the Zeeman energy for different  $|m_F\rangle$  states can be used to create state-dependent traps for atoms. We implement magnetic traps using quadrupole magnetic fields

$$\mathbf{B} = B'(x\mathbf{e}_x + y\mathbf{e}_y - 2z\mathbf{e}_z) + \mathbf{B}_0, \quad (3.7)$$

where  $\mathbf{B}_0$  is a constant magnetic field, for simplicity I will assume that  $\mathbf{B}_0 = B_0\mathbf{e}_z$ .

The Zeeman Hamiltonian gives a trapping potential

$$\begin{aligned} U(\mathbf{r}) &= g_F\mu_B m_F B \\ &= g_F\mu_B m_F B' \sqrt{x^2 + y^2 + 4\left(z - \frac{B_0}{2B'}\right)^2} \\ &\approx g_F\mu_B m_F B' \left(\rho + 2\left|z - \frac{B_0}{2B'}\right|\right) \end{aligned} \quad (3.8)$$

where  $\rho^2 = x^2 + y^2$  and the approximation on the second line is valid for small displacements from the trap center.

The sign of the magnetic moment determines which states can be trapped. For  $^{87}\text{Rb}$  the  $|F=1, m_F=-1\rangle$ ,  $|F=2, m_F=2, 1\rangle$  are magnetically trappable. The state  $|F=2, m_F=0\rangle$  is also weakly magnetically trappable due to the quadratic Zeeman shift.

In addition to generating trapping potentials, we use quadrupole fields before imaging the atoms to generate state-dependent forces that separate the different  $|m_F\rangle$  states in a similar way as the Stern-Gerlach (SG) experiment [41].



### 3.3 Interaction between atoms and electric fields

In the presence of an electric field  $\mathbf{E}$ , an atom can become polarized and its energy levels get modified by the Stark effect [42]. If the electric field is spatially uniform with respect to the atom's size we consider the electric field as a classical object and its effect on the atom can be described by the Hamiltonian [43]

$$\hat{H}_{\text{dip}} = -\hat{\mathbf{d}} \cdot \mathbf{E}, \quad (3.9)$$

where  $\hat{\mathbf{d}} = -e \sum_j \mathbf{r}_j$  is the atomic dipole operator,  $e$  is the electron charge and  $\mathbf{r}_j$  are the position operators of the atom's electrons relative to the center of mass of the atom. This approximation, known as the dipole approximation, is valid for electromagnetic radiation when the wavelength is much larger than the size of an atom  $\lambda \gg r_{\text{atom}}$  [44].

Consider first the simplified case of a two-level system interacting with a coherent electromagnetic field  $\mathbf{E} = \mathbf{E}^{(+)}e^{-i\omega t} + \mathbf{E}^{(-)}e^{i\omega t}$ , where  $\mathbf{E}^{(\pm)} = \hat{\mathbf{e}}E^{(\pm)}$  are the positive/negative frequency components of the field,  $\hat{\mathbf{e}}$  the polarization, and  $\omega$  is the angular frequency. The shift in energy of the ground state using second order perturbation theory is

$$\begin{aligned} \Delta E_g &= -\frac{2\omega_{eg} |\langle g | \hat{\mathbf{e}} \cdot \mathbf{d} | e \rangle|^2 |E^{(+)}|^2}{\hbar(\omega_{eg}^2 - \omega^2)} \\ &= -\frac{1}{2}\alpha(\omega)E^2 \end{aligned} \quad (3.10)$$

where  $\omega_{eg} = (E_e - E_g)/\hbar$  is the angular frequency associated to the energy splitting of the two states and  $\alpha(\omega)$  is a dynamic polarizability. Things are a bit more complicated with real atoms though, and we need to take into account all the atomic levels. Furthermore, there are degeneracies associated to the different angular momentum states as well as different light polarizations so we have to be more careful with the orientation of the atom and the field. To take these effects into account one can introduce a generalization of the polarizability [44, 45] which takes the form

$$\begin{aligned}\alpha_{\mu\nu}(\omega) &= \sum_j \frac{2\omega_{jg} \langle g | d_\mu | e_j \rangle \langle e_j | d_\nu | g \rangle}{\hbar(\omega_{jg}^2 - \omega^2)} \\ &= \sum_{F', m_{F'}} \frac{2\omega_{F'F} \langle F, m_F | d_\mu | F', m_{F'} \rangle \langle F', m_{F'} | d_\nu | F, m_F \rangle}{\hbar(\omega_{F'F}^2 - \omega^2)}.\end{aligned}\quad (3.11)$$

Here  $|e_j\rangle$  represent the excited states and  $\omega_{jg} = (E_j - E_g)/\hbar$  and the expression in the second line corresponds to the polarizability of the hyperfine levels of an atom in the ground state  $|F, m_F\rangle$  coupled to excited states  $|F', m_{F'}\rangle$ . We can write an effective Hamiltonian for the Stark shift as

$$\hat{H}_{\text{Stark}} = -\alpha_{\mu\nu}(\omega) E_\mu^{(+)} E_\nu^{(-)}.\quad (3.12)$$

The polarizability is a rank-2 tensor operator and can be represented by 3 irreducible tensor operators (see [44] for a complete derivation). In the limit of small magnetic fields so that  $F$  and  $|m_F\rangle$  are good quantum numbers describing the state of the atom  $|n, F, m_F\rangle$  the dipole Hamiltonian in this representation takes a

convenient form

$$\begin{aligned}\hat{H}_{\text{Stark}} = & \alpha^{(0)}(\mathbf{E}^{(-)} \cdot \mathbf{E}^{(+)}) + i\alpha^{(1)}(\mathbf{E}^{(-)} \times \mathbf{E}^{(+)}) \cdot \hat{\mathbf{F}} \\ & + \alpha^{(2)} E_i^{(-)} E_j^{(+)} \left( \frac{1}{2}(F_i F_j + F_j F_i) - \frac{1}{3} \hat{\mathbf{F}}^2 \delta_{i,j} \right) \Big],\end{aligned}\quad (3.13)$$

where  $\alpha^{(0)}$ ,  $\alpha^{(1)}$  and  $\alpha^{(2)}$  are the scalar, vector and tensor polarizabilities respectively and  $\hat{\mathbf{F}}$  is the total angular momentum operator. For all our experiments  $\alpha^{(2)}$  is very small so I limit the discussion to the effect of the first two terms. The scalar term is responsible for the dipole force that allow us to trap atoms using off-resonant light and the vector component is necessary for engineering spin-orbit coupling and other spin-dependent potentials through two-photon processes. Finally, I want to emphasize that it is because of the fine and hyperfine structure that the treatment using a tensor polarizability is necessary and the vector and tensor polarizabilities arise.

### 3.3.1 Scalar polarizability

The scalar polarizability takes the form

$$\alpha^{(0)} = \sum_j \frac{2\omega_{jg} |\langle g | \mathbf{d} \cdot \hat{\mathbf{e}} | e_j \rangle|^2}{\hbar(\omega_{jg}^2 - \omega^2)}, \quad (3.14)$$

where the matrix element can be expressed in terms of the Clebsch-Gordan coefficients and the reduced matrix element using the Wigner-Eckart theorem [46]. For the ground state of an Alkali atom ( $J = 1/2$ ) and if the detuning is large compared

to the hyperfine splitting the expression above gets simplified to

$$\alpha^{(0)} \approx \sum_{J'} \frac{2\omega_{JJ'} |\langle J = 1/2 \| \mathbf{d} \| J' \rangle|^2}{3\hbar(\omega_{JJ'}^2 - \omega^2)}. \quad (3.15)$$

Due to the second order-perturbation theory treatment, the scalar polarizability can be interpreted as arising from a two-photon process where the atom absorbs an off-resonant (virtual) photon and then returns to its initial state by emitting another (virtual) photon.

The dipole matrix elements needed to compute the polarizability are related to the transition scattering rate via Fermi's golden rule [44, 46]

$$\Gamma_{JJ'} = \frac{\omega_{JJ'}^2}{3\pi\epsilon_0\hbar c^3} \frac{2J+1}{2J'+1} |\langle J \| \mathbf{d} \| J' \rangle|^2, \quad (3.16)$$

and combining this with the expression for the intensity of the electric field  $I(\mathbf{r}) = 2\epsilon_0 c |\mathbf{E}(\mathbf{r})|^2$  it can be shown that the energy of the ground state manifold is shifted by

$$U(\omega, \mathbf{r}) = -\frac{\pi c^2 I(\mathbf{r})}{2} \left[ \frac{\Gamma_{D1}(1 - qg_F m_F)}{\omega_{D1}^3} \left( \frac{1}{\omega + \omega_{D1}} - \frac{1}{\omega - \omega_{D1}} \right) + \frac{2\Gamma_{D2}(1 + qg_F m_F)}{\omega_{D2}^3} \left( \frac{1}{\omega + \omega_{D2}} - \frac{1}{\omega - \omega_{D2}} \right) \right], \quad (3.17)$$

where  $q = 0, \pm 1$  for linearly and circularly polarized  $\sigma^\pm$  light and only the most significant contribution from the closest transitions (the D1 and D2 lines) are included. Here  $U(\mathbf{r})$  is related to the real part of the polarizability which is a complex

valued number. So far I have only considered a real valued polarizability by assuming the excited states have an infinitely long lifetime. However, in reality the atom can spontaneously emit photons and decay. This exponential decay can be accounted for by adding an imaginary contribution to the energies  $\omega_D \rightarrow \omega_D + i\Gamma_D\omega^3/\omega_D^3$  of the D1 and D2 transitions [47]. The scattering rate is related to the imaginary part of the polarizability and is given by

$$\Gamma(\omega, \mathbf{r}) = \frac{\pi c^2 I(\mathbf{r})}{2\hbar} \left[ \frac{\Gamma_{D1}\omega^3}{\omega_{D1}^6} \left( \frac{1}{\omega + \omega_{D1}} - \frac{1}{\omega - \omega_{D1}} \right)^2 + \frac{2\Gamma_{D2}\omega^3}{\omega_{D2}^6} \left( \frac{1}{\omega + \omega_{D2}} - \frac{1}{\omega - \omega_{D2}} \right)^2 \right] \quad (3.18)$$

The energy shift  $U(\omega, \mathbf{r})$  is a conservative term and is related to dipole trapping, while the scattering term  $\Gamma(\omega, \mathbf{r})$  is dissipative and is important for laser cooling. In the context of engineering potentials for ultracold atoms with off-resonant light, the scattering is translated into heating because every time an atom emits a photon with angular frequency  $\omega_L$  it gets a recoil momentum  $\hbar\mathbf{k}_L$ . If the frequency  $\omega$  satisfies the relation  $\omega + \omega_D \gg \omega - \omega_D$ , as is often the case, we can neglect the terms proportional to  $1/(\omega + \omega_D)$ , an approximation typically known as the rotating wave approximation (RWA). If the RWA is valid then the frequency dependence of both the energy shifts and the scattering rates are given by the detuning from the D1 and D2 transitions.

### 3.3.1.1 Optical trapping

One important application of the scalar light-shift is to create optical traps for clouds of ultracold atoms. An optical field with non-uniform spatial intensity gen-

erates traps (and anti-traps) for the atoms which experience a force proportional to the intensity gradient  $F_{\text{dip}} = -\nabla U(\mathbf{r})$ . The attractive or repulsive nature of the trap depends on the sign of  $U(\mathbf{r})$  which is determined by the sign of the detuning (blue-detuned traps are repulsive and red-detuned traps are attractive). The production of BECs in our lab relies on the use of focused Gaussian laser beams of wavelength  $\lambda = 1064 \text{ nm}$ . The intensity profile of a focused Gaussian beam propagating along  $\mathbf{e}_z$  is given by

$$I(x, y, z) = \frac{2P}{\pi\omega^2(z)} e^{-\frac{x^2+y^2}{\omega^2(z)}} \quad (3.19)$$

where  $P$  is the total power of the beam and the  $1/e^2$  radius is given by  $w(z) = w_0\sqrt{1 + z^2/z_R^2}$  where the minimum radius  $w_0$  is known as the waist and  $z_R = \pi\omega_0^2/\lambda$  is the Rayleigh range<sup>3</sup>. If the extent of an atomic cloud is small compared to the size of the beam we can perform a Taylor expansion around  $\mathbf{r} = 0$  to obtain a harmonic trapping potential

$$U(\mathbf{r}) = -U_0 \left( 1 - 2\frac{x^2 + y^2}{\omega_0^2} - \frac{z^2}{z_R^2} \right). \quad (3.20)$$

The oscillation frequencies of the trap along the radial direction are  $\omega_r = (4U_0/m\omega_0^2)^{1/2}$  and along the axial direction  $\omega_z = (2U_0/mz_R)^{1/2}$ . The beam waist is usually much smaller than the Rayleigh range ( $\omega_0 \sim 50 - 150 \mu\text{m}$  for my experiments) and the trap is much stronger along the axial direction. To get around this we use a ‘crossed’

---

<sup>3</sup>If the waist of a Gaussian beam is comparable to the wavelength  $\lambda$ , a term with longitudinal polarization appears. The waist of the Gaussian beams used in the lab typically exceed  $\lambda$  by at least one order of magnitude

dipole trap which is formed by a combination of two cross-polarized<sup>4</sup> and frequency shifted focused Gaussian beams propagating along perpendicular directions, ensuring that we get good confinement of atoms along all spatial directions.

### 3.3.2 Vector polarizability and effective magnetic fields

The general expressions for the vector polarizability are quite complicated and depend both on reduced matrix elements and Wigner  $6 - j$  symbols (see [44] for example for the complete expressions). For the particular case of Alkali atoms and for large detuning compared to the hyperfine splitting, the vector polarizability takes a simplified form [16]

$$\alpha^{(1)} = \frac{2\Delta_{\text{fs}}\alpha^{(0)}}{3(\tilde{E} - \hbar\omega)} \quad (3.21)$$

where  $\Delta_{\text{fs}} = 3A_{\text{fs}}/2$  and  $\tilde{E} = (2E_{\text{D}_1} + E_{\text{D}_2})$ .

If we recall the Zeeman Hamiltonian introduced in Section 3.2, the term proportional to the vector polarizability in Equation 3.13 looks very similar to Equation 3.2 for an effective magnetic field

$$\mathbf{B}_{\text{eff}} = -\frac{i\hbar}{\mu_B g_J} \alpha^{(1)} (\mathbf{E}^* \times \mathbf{E}). \quad (3.22)$$

For the intensities that we typically operate at, the vector light shift is small and can be treated as a perturbation so the Hamiltonian resulting from this effective

---

<sup>4</sup>The beams are cross-polarized to avoid interference between them

magnetic field can be written as

$$\hat{H}_{\text{eff}} = \frac{\mu_B g_F}{\hbar} \mathbf{B}_{\text{eff}} \cdot \hat{\mathbf{F}} \quad (3.23)$$

### 3.3.2.1 Raman coupling

The vector light shift enables the realization of various spin dependent-potentials in the laboratory. The results of Chapters 5 and 8 use a combinations of cross-polarized laser beams such that the total electric field  $\mathbf{E}^* \times \mathbf{E} \neq 0$  could induce two-photon Raman transitions. A Raman transition is also a two-photon process but instead, we consider two ground states and an intermediate state that is off-resonantly coupled as is shown in Figure 3a. Due to the large detuning, the population transferred into the intermediate state is negligible and the state can be adiabatically eliminated [48]. In our experiments, we typically couple the  $|m_F\rangle$  levels of the  $F = 1$  manifold after applying a bias magnetic field such that  $\epsilon$  is non-negligible.

Consider two laser beams counter-propagating along  $\mathbf{e}_x$  and with polarizations along  $\mathbf{e}_y$  and  $\mathbf{e}_z$  as in Figure 3b. The electric field from the Raman beams is

$$\mathbf{E}(x, t) = E_a \cos(k_a x - \omega_a t) \mathbf{e}_y + E_b \cos(k_b x + \omega_b t) \mathbf{e}_z, \quad (3.24)$$

and consequently

$$\mathbf{E}^* \times \mathbf{E} = 2iE_a E_b \cos(2k_L x - \omega_{a,b} t) \mathbf{e}_x, \quad (3.25)$$



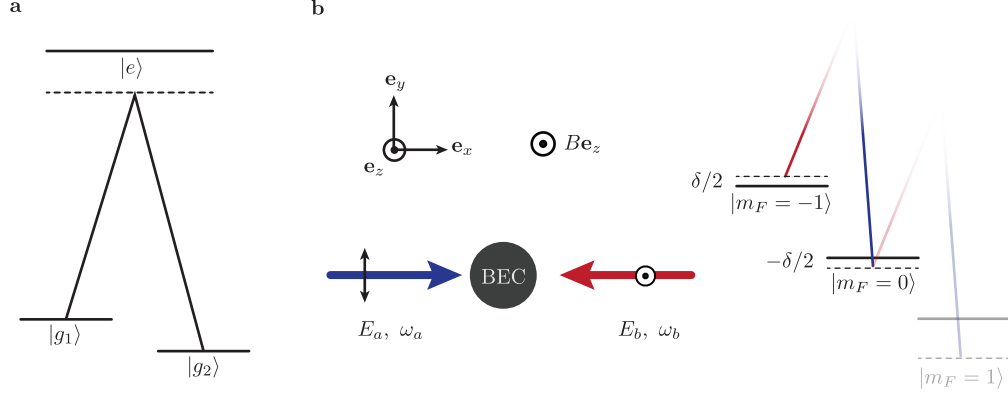


Figure 3: **a.** A Raman transition is a two photon process that couples two ground states through and intermediate far detuned state. **b.** We induce Raman transitions using a pair of cross-polarized laser beams and we set the difference in their angular frequencies close to the Zeeman splitting between two consecutive  $|m_F\rangle$  states.

where  $\omega_{a,b} = \omega_a - \omega_b$ . The Raman Hamiltonian is given by

$$\hat{H}_R = \Omega \cos(2k_L x - \omega_{a,b} t) \hat{F}_x \quad (3.26)$$

where  $\Omega = \alpha^{(1)} g_F E_a E_b / g_J \propto \sqrt{I_a I_b}$  is the Raman coupling strength. The geometry and wavelength of the Raman fields determine the natural units of the system: the single photon recoil momentum  $k_L = 2\pi/\lambda_R$  and its associated recoil energy  $E_L = \hbar^2 k_L^2 / 2m$ , as well as the direction of the recoil momentum  $\mathbf{k}_L = k_L \mathbf{e}_x$ . For most experiments we tune to what is known as the ‘magic wavelength’ or tune-out wavelength [49]  $\lambda_R = 790.034$  nm, at which the ground-state scalar polarizability vanishes and the scattering rate is reduced (Figure 4a,c). We occasionally had to tune away from the magic wavelength, for example when we were starving for laser power and wanted to increase our Raman coupling strength. An important metric for us is Raman coupling strength and Figure 4b shows its dependence on

wavelength; tuning closer to resonance allows us to decrease the laser intensity for the same intensities but comes with increased scattering rates and reduced lifetime as can be seen in Figure 4d which shows the decay in number of Raman dressed atoms as a function of time for different wavelengths.

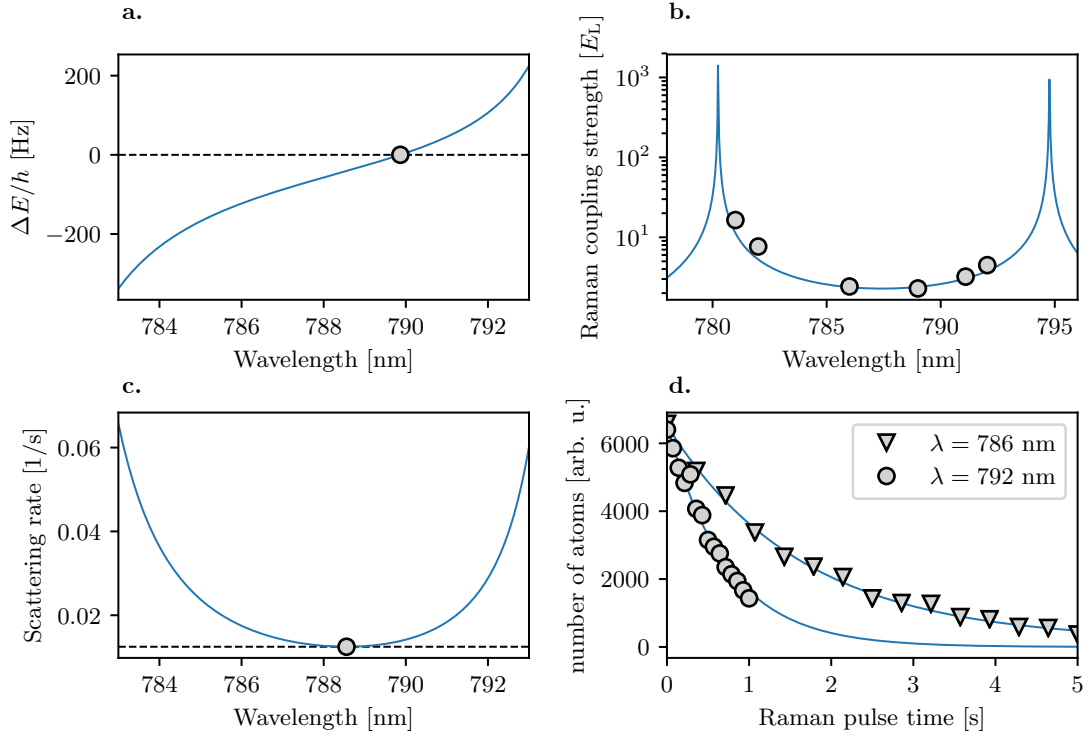


Figure 4: **a.** Scalar polarizability as a function of wavelength near the D1 and D2 lines of  $^{87}\text{Rb}$ . We typically tune our Raman laser beams near the magic wavelength  $\lambda = 790.034\text{ nm}$ . **b.** Raman coupling strength as a function of wavelength measured for a pair of Raman beams with waist  $w_0 \sim 150\text{ }\mu\text{m}$  and powers of 50, 10 mW. **c.** Scattering rate as a function of wavelength, while it is not minimized at 790 nm its value is kept relatively low. **d.** Decay in number of Raman dressed atoms as a function of hold time for the same beam parameters as in **b.**. At  $\lambda = 786\text{ nm}$  the  $1/e$  lifetime is  $\tau = 1.64\text{ s}$  and for  $\lambda = 792\text{ nm}$  it is reduced to  $\tau = 0.72\text{ s}$ .

In a frame rotating with angular frequency  $\omega_{a,b}$  corresponding to applying the unitary transformation  $\hat{U}(t) = \exp(-i\omega_{a,b}t\hat{F}_z)$  and neglecting the fast terms rotating

at frequency  $2\omega_{a,b}$  (applying a RWA) the transformed Hamiltonian is

$$\hat{U}^\dagger \hat{H}_R \hat{U} - i\hbar \hat{U}^\dagger \partial_t \hat{U} = \omega_{a,b} \hat{F}_z + \frac{\Omega}{2} \cos(2k_L x) \hat{F}_x - \frac{\Omega}{2} \sin(2k_L x) \hat{F}_y, \quad (3.27)$$

which describes a helically precessing magnetic field with period  $\lambda_R/2$ .

### 3.3.2.2 Spin-orbit coupling

The Raman Hamiltonian from Equation 3.27 can be further arranged to make it look like a spin-orbit coupled (SOC)<sup>5</sup> Hamiltonian that is familiar to condensed matter physicists. If we apply a spin-dependent momentum boost which is described by the unitary operator  $\hat{U}(k_L) = \exp(i2k_L x \hat{F}_z)$  the full Hamiltonian including the Raman coupling and the free particle energies is transformed into

$$\hat{H}_{\text{SOC}} = \frac{\hbar^2}{2m} (\hat{q}_x - 2k_L \hat{F}_z)^2 + \frac{\Omega}{2} \hat{F}_x + \delta \hat{F}_z + \hbar\epsilon \left( \mathbb{1} - \frac{\hat{F}_z^2}{\hbar^2} \right), \quad (3.28)$$

where  $\delta = E_{-1} - \omega_{a,b}$ . We can go from a 3 level system to an effective spin-1/2 system if we set  $\omega_{a,b} = E_{-1} - E_0$  and consider a sizable quadratic Zeeman shift  $\epsilon$  so the  $|m_F = 1\rangle$  state can be adiabatically eliminated [50] giving an effective two-level system. After performing a global rotation  $\hat{\sigma}_z \rightarrow \hat{\sigma}_y$ ,  $\hat{\sigma}_y \rightarrow \hat{\sigma}_x$ , and  $\hat{\sigma}_x \rightarrow \hat{\sigma}_z$  of the Pauli matrices describing the pseudo-spin of the two-level system, the Hamiltonian becomes

$$\hat{H}_{\text{SOC}} = \frac{\hbar^2}{2m} (q_x - k_L \hat{\sigma}_y)^2 + \frac{\hbar}{2} \Omega \hat{\sigma}_z + \frac{\hbar}{2} \delta \hat{\sigma}_y \quad (3.29)$$

---

<sup>5</sup>Not to be confused with the spin-orbit coupling giving rise to the fine and hyperfine structure mentioned earlier, perhaps a better name could be spin-momentum coupling

which corresponds to a spin-orbit coupled Hamiltonian with an equal superposition of Rashba-type [24] ( $\propto \hat{\sigma}_x k_y - \hat{\sigma}_y k_x$ ) and Dresselhaus-type [51] ( $\propto -\sigma_x k_y - \sigma_y k_x$ ) SOC and effective magnetic field  $\propto \Omega$  in the  $\mathbf{e}_y - \mathbf{e}_z$  plane [22, 50]. In Chapter 8 I discuss the Rashba term in more detail and introduce a way of engineering a system with only Rashba-type SOC using multiple internal levels and Raman transitions.

### 3.4 Detection: Resonant absorption imaging

Ultracold atom experiments rely on optical imaging as the main method to probe and characterize the system. We use resonant absorption imaging by shining a probe laser at the atomic cloud and then imaging it into a camera. From the absorption of the light, we can infer properties from the atoms such as the number, temperature, integrated column density, and momentum distribution if we allow the clouds to expand.

Consider a laser beam with intensity  $I(x, y, z)$  and angular frequency  $\omega$  propagating along  $\mathbf{e}_z$  through a cloud of atoms with density  $n(x, y, z)$  as in Figure 5a. We define a (frequency-dependent) scattering cross-section  $\sigma(\omega)$  which characterized the probability of an atom absorbing a probe photon and is given by the Lorentzian function

$$\sigma(\omega) = 3A_{eg} \frac{\pi^2 c^2}{\omega_0^2} \frac{1}{2\pi} \frac{\Gamma}{\delta^2 + \Gamma^2/4} \quad (3.30)$$

plotted in Figure 5b, where  $\Gamma$  is the scattering rate,  $\omega_0$  is the transition frequency,  $\delta = \omega - \omega_0$  is the detuning, and  $A_{eg}$  is the Einstein coefficient associated to spontaneous emission. As the beam travels through the cloud is absorbed and its intensity

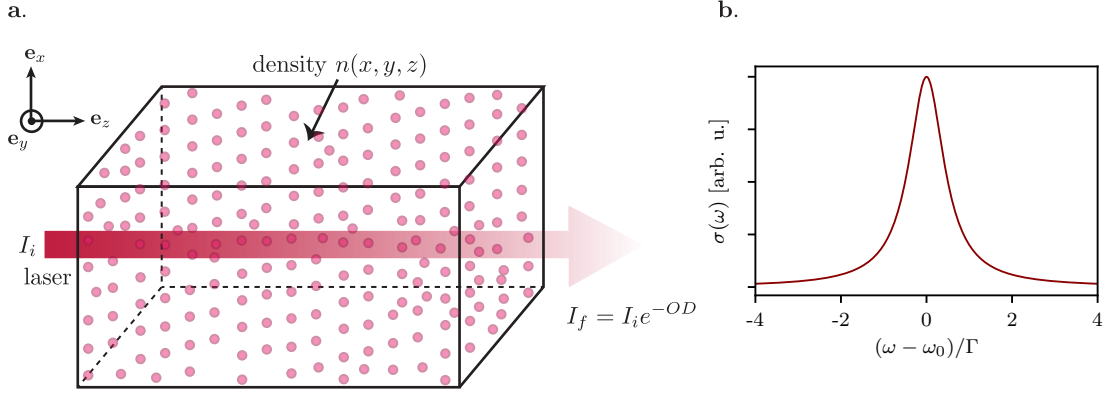


Figure 5: **a.** A laser beam traveling along  $\mathbf{e}_z$  through a medium with density  $n(x, y, z)$ . The intensity decays exponentially with the integrated column density and the scattering cross section  $\sigma\omega$ . **b.** The scattering cross section has a Lorentzian line shape with a full width half maximum equal to  $\Gamma$ .

reduced at a rate given by

$$\frac{dI}{dz}(x, y, z) = -n(x, y, z)\sigma(\omega)I(x, y, z). \quad (3.31)$$

In the limit of small intensities, we can integrate this expression over the thickness of the cloud and find that the intensity decays exponentially with the density and the scattering cross section

$$I(x, y, z) = I(x, y, 0)e^{-\int_0^z n(x, y, z')\sigma(\omega)dz'}, \quad (3.32)$$

where  $\text{OD} = \int_0^z n(x, y, z')\sigma(\omega)dz'$  is the optical depth (OD) of the medium. If we measure the OD of the cloud it is then straightforward to obtain the integrated column density  $n(x, y)$ , a result known as the Beer-Lambert law.

In the experiment we measure the optical depth of a cloud by imaging the probe into a CCD camera under two different conditions: first in the presence of atoms to measure the attenuated intensity  $I_f = I(x, y, z)$  and then without any atoms to get a measure of the initial intensity  $I_i = I(x, y, 0)$ . The optical depth can then be computed as

$$OD = \ln \left( \frac{I_f}{I_i} \right). \quad (3.33)$$

Figure 6 shows the different images used to compute the OD. In practice we take a third image of the background intensity  $I_{bg}$  and subtract it from the other two images.

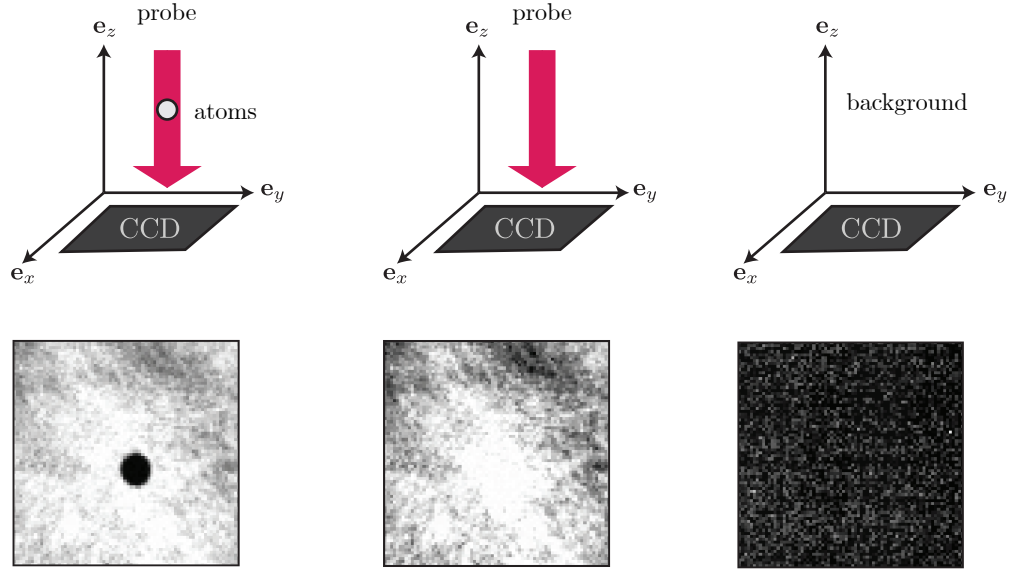


Figure 6: Resonant absorption imaging. An atomic sample is illuminated with a resonant probe whose intensity is later recorded on a CCD camera. Two additional images of the unabsorbed probe intensity and the background intensity are captured in order to reconstruct the optical density of the atoms.

### 3.4.1 High intensity absorption imaging

The use of the OD to infer the atomic density works well if we assume that the intensity of the probing laser is low such that the atoms mostly stay in the ground state. However, at high intensities a significant fraction of the atoms can become excited and effects such as stimulated emission of light have to be taken into account. As a result of this the scattering cross-section gets an additional dependence on intensity (see [52] for a complete derivation)

$$\sigma(\omega, I) = \sigma(\omega) \frac{1}{1 + I/I_{\text{sat}}}, \quad (3.34)$$

where  $I_{\text{sat}} = \pi \hbar c \Gamma / 3 \lambda_0^3$  is the saturation intensity, and when  $I = I_{\text{sat}}$  the population in the ground and excited state are equal. Integrating Equation 3.31 using the modified expression for  $\sigma(\omega, I)$  gives

$$n(x, y) \sigma(\omega) = -\alpha^* \ln(I_f/I_i) + \frac{I_i - I_f}{I_{\text{sat}}}, \quad (3.35)$$

where I have also added an additional dimensionless parameter  $\alpha^*$  which can account for imperfections in the imaging process (see [53]).

It is hard to reliably measure atomic clouds at low intensity when the OD is of the order of 3 or 4 (such as our BECs) and a significant fraction of the imaging light is absorbed. Due to the limited dynamic range of CCD cameras, the measured OD saturates resulting, for example, in imaging ‘flat-top’ BECs rather than the

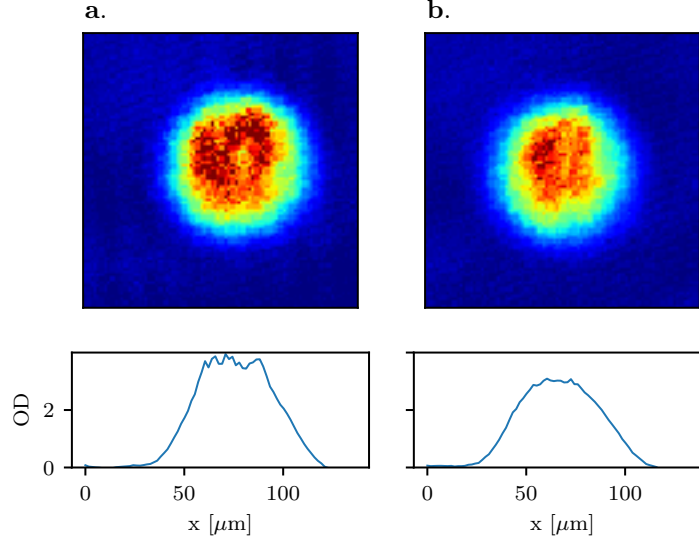


Figure 7: **a.** A BEC imaged at low intensity shows a ‘flat-top’ density profile. **b.** In order to recover the Thomas-Fermi profile it is necessary to image high density BECs with intensities larger than  $I_{\text{sat}}$ .

usual Thomas-Fermi distribution as shown in Figure 7. To get around this issue we typically image using intensities  $I > I_{\text{sat}}$ . In order to correctly compute the column density including saturation effects we need a conversion of  $I_{\text{sat}}$  from  $\text{mW}/\text{cm}^2$  to counts per pixel on the CCD camera. We follow the procedure described in [53] to find the values of  $\alpha^*$  and  $I_{\text{sat}}$  in counts per pixel. To learn about other effects such as the recoil momentum from the imaging light that could affect absorption images see [54].

### 3.5 Coherent manipulation

We rely on driven quantum coherent processes within the electronic grounds state using magnetic and electric dipole interactions techniques both for state preparation and characterization of our system. In all of the cases I consider a system



described by the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_I(t) \quad (3.36)$$

where  $\hat{H}_0$  describes unperturbed atomic levels and  $\hat{H}_I(t)$  is a time dependent interaction. For simplicity I consider only a two-level system

$$\hat{H}_0 = \hbar \begin{pmatrix} \omega_g & 0 \\ 0 & \omega_e \end{pmatrix} \quad (3.37)$$

where with  $|g\rangle$  and  $|e\rangle$  are the unperturbed ground and excited states where  $\hbar\omega_i$  are the energies of the unperturbed states.

### 3.5.1 Rabi oscillations

Consider an interaction term that oscillates with frequency  $\omega$  close to the transition energy  $\omega_{ge} = \omega_g - \omega_e$

$$\hat{H}_I = \hbar \begin{pmatrix} 0 & \Omega \cos(\omega t) \\ \Omega^* \cos(\omega t) & 0 \end{pmatrix}. \quad (3.38)$$

The coupling strength  $\Omega$  here could be related to an electric dipole transition  $\Omega \propto \langle g | \mathbf{d} \cdot \mathbf{E} | e \rangle$ <sup>6</sup> or magnetic dipole  $\Omega \propto \langle g | \boldsymbol{\mu} \cdot \mathbf{B} | e \rangle$  transition matrix element. The

---

<sup>6</sup>For our system intensities  $\Gamma \gg \Omega$  and we do not observe Rabi oscillations from (single photon) electric dipole transitions.

state of the system at any given time is given by

$$|\Psi\rangle = c_g(t)e^{-i\omega_g t} |g\rangle + c_e(t)e^{-i\omega_e t} |e\rangle, \quad (3.39)$$

and substituting this expression into the time dependent Schrödinger equation we find that

$$\begin{aligned} \dot{c}_g(t) &= \frac{\Omega}{2} \left( e^{i(\omega - \omega_{ge})t} + e^{-i(\omega + \omega_{ge})t} \right) c_e \\ \dot{c}_e(t) &= \frac{\Omega^*}{2} \left( e^{i(\omega - \omega_{ge})t} + e^{-i(\omega + \omega_{ge})t} \right) c_g. \end{aligned} \quad (3.40)$$

We can apply a RWA if the term  $\omega + \omega_{ge}$  is large compared to  $\omega - \omega_{ge}$ . If we assume that at  $t = 0$  the system is prepared in  $|g\rangle$ , the population in  $|e\rangle$  describes what is known as a Rabi oscillation [55]

$$|c_e(t)|^2 = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2 \left( \frac{\sqrt{\Omega^2 + \delta^2}}{2} t \right) \quad (3.41)$$

where  $\delta = \omega - \omega_{ge}$  is a detuning and  $\tilde{\Omega} = \sqrt{\Omega^2 + \delta^2}$  is known as the generalized Rabi frequency. The Hamiltonian after applying the RWA is

$$\hat{H}_0 = \hbar \begin{pmatrix} -\delta/2 & \Omega/2 \\ \Omega^*/2 & \delta/2 \end{pmatrix}, \quad (3.42)$$

and its eigenenergies correspond to  $E_{\pm} = \pm\tilde{\Omega}/2$ . Notice that the difference between the eigenenergies  $E_+ - E_-$  is exactly equal to the frequency at which the populations

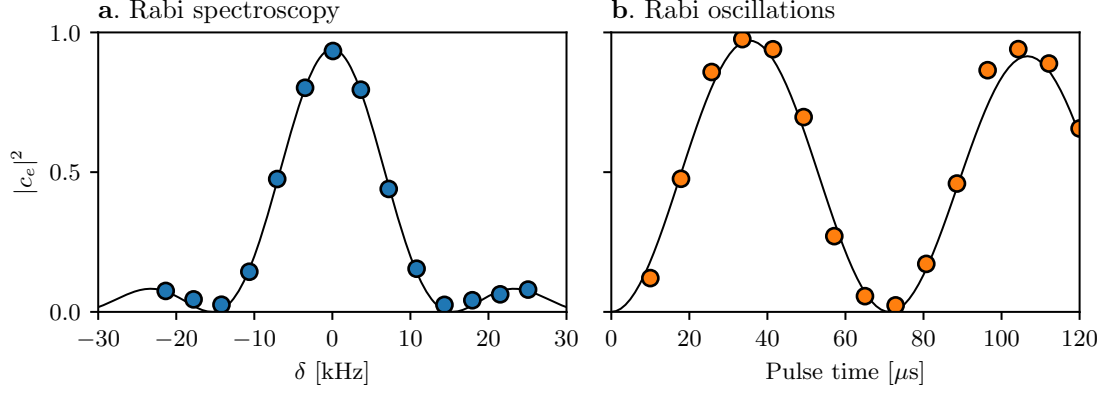


Figure 8: The Rabi cycle. Population transferred from  $|F = 1, m_F = -1\rangle$  into  $|F = 1, m_F = 0\rangle$  using an RF magnetic field with  $\Omega = 7.1$  kHz. The markers indicate experimental data points and the lines correspond to fits to the model in Equation 3.41a. **a.** Population transferred for a  $60 \mu$ s pulse as a function of detuning  $\delta$ . **b.** Population transferred as a function of time close to resonance.

in  $|g, e\rangle$  oscillate, this will come up again in Chapter 5. Figure 8 shows an example of this process where we coupled an initial state  $|g\rangle = |F = 1, m_F = -1\rangle$  to  $|e\rangle = |F = 1, m_F = 0\rangle$  using a radio-frequency (RF) magnetic field with  $\Omega = 7.1$  kHz. Figure 8a shows the population in  $|e\rangle$  as a function of  $\delta$  for a  $\pi$  pulse of duration  $\tau$  such that  $\Omega\tau = \pi$ . The location of the peak in this curve is as a way to find the transition frequency (we use this method in Chapter 6). Figure 8b shows the population transferred into  $|m_F = 0\rangle$  from  $|m_F = -1\rangle$  as a function of time for  $\delta \approx 0$ ; we typically look at the frequency of these Rabi oscillations to calibrate the coupling strength of an effective two-level system.

### 3.5.2 Adiabatic rapid passage

The Hamiltonian in Equation 3.42 can in principle be used to transfer all the population from the initial state  $|g\rangle$  into  $|e\rangle$  if we set the detuning of our oscillating field to  $\delta = 0$  and apply a  $\pi$  pulse. Unfortunately we are susceptible to noise in both  $\delta$  and  $\Omega$  and pulsing is not the most reliable technique for state preparation. To transfer atoms within different  $|F, m_F\rangle$  states within the  $5^2S_{1/2}$  hyperfine manifold we use instead an adiabatic rapid<sup>7</sup> passage (ARP) protocol which is based on the Landau-Zener model [56].

ARP relies on preparing dressed states; eigenstates of the atomic Hamiltonian (Equation 3.42) which I label using the symbols  $|\pm\rangle$  and dynamically changing the detuning  $\delta = \delta(t)$ . We start at a large and negative detuning  $\delta \ll -\Omega$  where the ground eigenstate  $|-\rangle \approx |g\rangle$  and therefore by slowly turning  $\Omega$  on we adiabatically prepare  $|-\rangle$ . We consider the rate of change in detuning  $\partial_t \delta > 0$ , and as  $\delta$  increases the state decomposition of  $|\pm\rangle$  changes. When  $\delta = 0$  the dressed states correspond to equal superpositions of the bare states

$$|\pm\rangle = \frac{1}{2}(|g\rangle \pm |e\rangle), \quad (3.43)$$

and when  $\delta \gg \Omega$  we have  $|-\rangle \approx |e\rangle$ . If the change in detuning is slow enough that the system can adiabatically follow the ground eigenstate  $|-\rangle$  then at the end of this process the state can be successfully transferred from  $|g\rangle$  into  $|e\rangle$  by sweeping

---

<sup>7</sup>The term rapid is with respect to the spontaneous emission rate of the excited state being coupled.

the detuning. It can be shown that the fraction that does not adiabatically follow the ground state is given by the Landau-Zenner tunnel probability [44]

$$P_{\text{lost}} = \exp\left(-\frac{\pi\Omega^2}{2|\partial_t\delta|}\right). \quad (3.44)$$

In the limit of large coupling strength compared to the detuning sweep rate, or slow detuning sweep rate compared to the smallest energy gap ( $\Omega \gg |\partial_t\delta|$ ) all the population adiabatically follows the ground state.

We mostly use ARP with RF magnetic fields to transfer between different  $|m_F\rangle$  states within the  $F = 1$  manifold. We set the frequency of the field  $\omega_{\text{RF}}$  so that it matches the Zeeman splitting between the  $|m_F = -1\rangle$  and  $|m_F = 0\rangle$  states for a target bias field  $B_0\mathbf{e}_z$ . We start with atoms in  $|m_F = -1\rangle$  and at a bias field  $B_i \approx B_0 - 380 \text{ mG}$  ( $\delta \approx -30 \text{ kHz}$ ). We ramp an  $\Omega = 20 \text{ kHz}$  RF field with angular frequency  $\omega_{\text{RF}}$  in 50 ms. We then sweep the detuning by linearly changing the bias field in 50 ms. Finally, the RF field is abruptly turned off, projecting the RF eigenstates back into the  $|m_F\rangle$  basis. In Figure 3.5.2 we set  $\omega_{\text{RF}} = 23 \text{ MHz}$  and when the Zeeman splitting between  $|m_F = -1\rangle$  and  $|m_F = 0\rangle$  is equal to  $\omega_{\text{RF}}$  we observe an equal superposition of both states and if the detuning is swept beyond resonance we can reliably prepare the  $|m_F = 0\rangle$  state.

In general it is necessary to consider the eigenstates of the three-level RF

Hamiltonian for a full description of this process

$$\hat{H}_{\text{RF}} = \begin{pmatrix} -\delta & \Omega_{\text{RF}}/2 & 0 \\ \Omega_{\text{RF}}/2 & -\epsilon & \Omega_{\text{RF}}/2 \\ 0 & \Omega_{\text{RF}}/2 & \delta \end{pmatrix}, \quad (3.45)$$

but for large quadratic Zeeman shifts as is usually the case in our experiments we can only look at an effective two-level system.

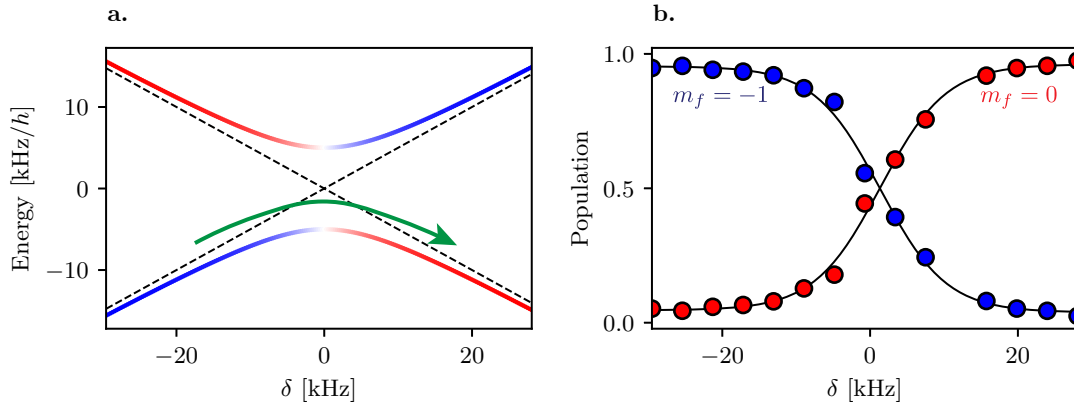


Figure 9: **a.** Eigenenergies and eigenstate decomposition of an RF dressed Hamiltonian for a two-level system (Equation 3.42) with  $\Omega = 10$  as a function of detuning. The eigenstates are linear combinations of  $|m_F = -1\rangle$  and  $|m_F = 0\rangle$  (red and blue respectively). **b.** Population in the  $|m_F\rangle$  states for different values of detuning

### 3.5.3 Magnetic field stabilization with microwave assisted partial transfer absorption imaging

Most experiments are performed in the  $F = 1$  ground hyperfine manifold with some bias field  $B_0 \mathbf{e}_z$  that shifts the energies of the different  $|m_F\rangle$  states. Due to the linear dependence of the energies of the  $|m_F = \pm 1\rangle$  and the constant changes

in the ambient magnetic field we use microwave assisted partial transfer absorption imaging (PTAI) to monitor and stabilize the magnetic field.

The method relies on transferring a small fraction of atoms into the  $5^2S_{1/2}$   $F = 2$  manifold using an oscillating magnetic field with frequency close to the 6.8 GHz ground hyperfine splitting. The atoms in  $F = 2$  can be imaged without the use of repump light and therefore minimally disturbing the remaining atoms in  $F = 1$ . We apply two microwave pulses for a total time  $\tau$  with frequency  $\omega_0 - \delta_{\pm}$  where  $\delta_{\pm} = \pm 1/(2\tau)$ . We typically set  $\omega_0$  equal to the Zeeman splitting between the  $|F = 1, m_f = -1\rangle$  and  $|F = 2, m_f = -2\rangle$  states at a target magnetic field and we set the coupling strength  $\Omega_0 \ll 1/\tau$  such that only about 5% of the atoms are transferred by each pulse. We image the transferred atoms following each pulse using absorption imaging and from the measured densities we calculate the imbalance or error

$$n_{\text{imb}} = \frac{n(\delta_+) - n(\delta_-)}{n(\delta_+) + n(\delta_-)} \quad (3.46)$$

signal that is both insensitive to fluctuations in the number of atoms and linearly sensitive to changes in magnetic field<sup>8</sup>. We use this error signal both to monitor the magnetic field before performing experiments and to cancel long term drifts in the field. In most cases, we chose the states  $|F = 1, m_F = -1\rangle$  and  $|F = 2, m_f = -2\rangle$  as their relative energies are the most sensitive to changes in magnetic field. Figure 10a shows the number of atoms transferred by each microwave pulse for different values of bias magnetic field and Figure 10b shows the imbalance. The microwave frequency

---

<sup>8</sup>A single pulse on resonance is quadratically sensitive to detuning (see Equation 3.41)

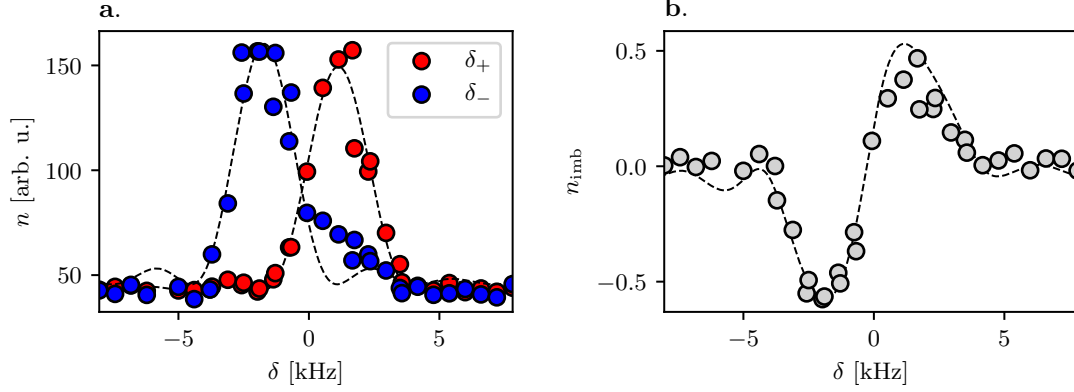


Figure 10: Magnetic field stabilization using microwave assisted PTAI. **a.** Population transferred into  $|F = 2, m_F = -1\rangle$  from  $|F = 1, m_F = -1\rangle$  as a function of bias magnetic field (global detuning  $\delta$ ). Each microwave pulse was  $\tau = 250 \mu\text{s}$  and detuned by  $\delta_{\pm} = \pm 1/(2\tau)$  transfer a small fraction of atoms from  $|F = 1, m_F = -1\rangle$  into  $|F = 2, m_F = -1\rangle$ . **b.** Error signal calculated using the transferred atoms by each pulse. We lock the magnetic field to the  $\sim 5 \text{ kHz}$  ( $\sim 7 \text{ mG}$ ) wide linear portion of the signal.

$\omega_0$  is on resonance with the  $|F = 1, m_F = -1\rangle \rightarrow |F = 2, m_F = -2\rangle$  transition when both pulses transfer the same number of atoms.

In [57] we studied partial transfer absorption imaging as a minimally destructive technique for imaging ultracold atoms. See Chapter 6 for an alternative solution for dealing with magnetic field noise.

### 3.5.4 Ramsey interferometer

A Ramsey interferometer [58] is a setup relevant to Chapters 6 and 8. The interaction Hamiltonian is the same as in the previous section but rather than being on continuously it is pulsed on for a time  $\tau = \pi/2\Omega$  (a  $\pi/2$  pulse), the system is then let to evolve only under  $\hat{H}_0$  for a variable time  $t_{\text{dark}}$  and finally a second



$\pi/2$  pulse is applied. Figure 11a illustrates this protocol: the  $\pi/2$  pulses can be visualized as  $\pi/2$  rotations on the Bloch sphere along  $\mathbf{e}_x$  that transform the initial state  $|g\rangle \rightarrow (|g\rangle + |e\rangle)/\sqrt{2}$ . For the duration of the dark time, the system rotates along the equator of the Bloch sphere (about the  $\mathbf{e}_z$  axis in general) by an angle  $\delta t_{\text{dark}}$  and finally the second pulse rotates the state along the  $\mathbf{e}_x$  axis again. The probability of measuring the excited state is related to the phase accumulated during the dark time is given by

$$|c_e(2\tau + t_{\text{dark}})|^2 = \left| \frac{\Omega\tau}{2} \right| \left[ \frac{\sin(\delta\tau/2)}{\delta\tau/2} \right]^2 \cos^2 \left( \frac{\delta t_{\text{dark}}}{2} \right). \quad (3.47)$$

In a Ramsey interferometer the oscillation frequency is only determined by the detuning unlike the generalized Rabi frequency where the detuning adds in quadrature with  $\Omega$ . The coupling strength dependence in the generalized Rabi frequency suppresses the effect of the detuning  $\Omega$  becomes large compared to  $\delta$ , something that will be relevant in Chapter 6. Additionally in Chapter 8 I rely on a variation of the Ramsey interferometer to perform quantum state tomography. Figure 11b shows an example of a Ramsey fringe as a function of detuning  $\delta$  that we measured using two states coupled with  $\Omega = 1$  kHz.

### 3.5.5 Floquet theory

The RWA has been used multiple times throughout this Chapter so that the Hamiltonian describing a driven system can effectively be viewed as time-independent. This approximation is valid most of the time for our experiments,

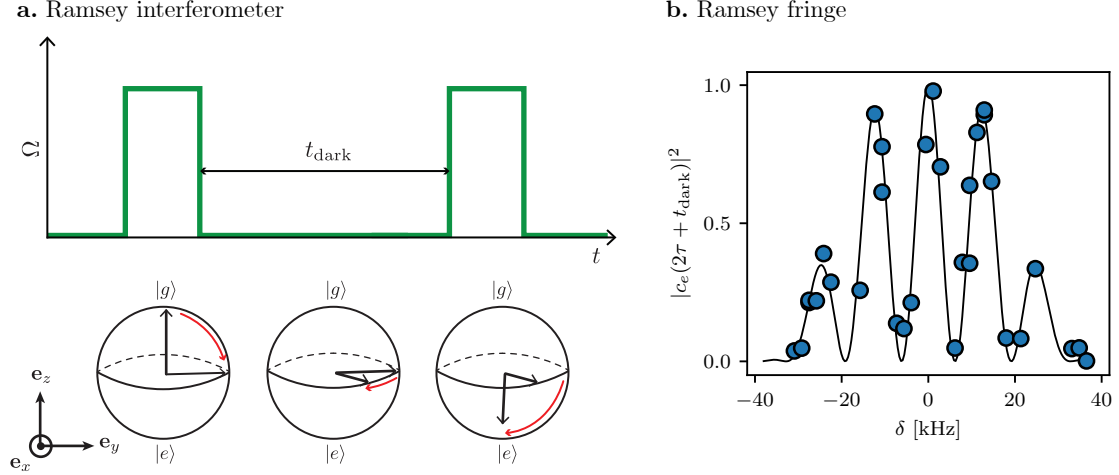


Figure 11: **a.** A Ramsey interferometer: Two  $\pi/2$  pulses are separated by a time  $t_{\text{dark}}$ . The phase accumulated in the interferometer is equal to the detuning multiplied by the dark time. **b.** A Ramsey interference fringe. Data (blue dots) and fit (black line) obtained from coupling two levels using an RF field with  $\Omega = 1$  kHz. We applied a pair of  $\tau = 25 \mu\text{s}$  pulses separated by a  $50 \mu\text{s}$  wait and varied the detuning by changing the bias magnetic field.

however, if we want to give a complete description of a time periodic system Floquet theory can be helpful. I will give a brief overview of Floquet theory using a matrix approach that is particularly useful for numerical computations.

An arbitrary time periodic Hamiltonian  $\hat{H}(t) = \hat{H}(t + T)$  can be written in terms of its Fourier components

$$\hat{H}(t) = \sum_{j=-\infty}^{\infty} \exp[ij\omega t] \hat{H}_j, \quad (3.48)$$

with  $\omega = 2\pi/T$  and because  $\hat{H}$  and  $\hat{H}_j = \hat{H}_{-j}^\dagger$  due to the Hermitian nature of  $\hat{H}$ . The eigenstates of the Hamiltonian can be written in a terms of quasi periodic

functions<sup>9</sup>

$$|\psi_\epsilon(t)\rangle = \exp(-i\epsilon t/\hbar) \sum_{k=-\infty}^{\infty} \exp[-ik\omega t] |\psi_{\epsilon,k}\rangle \quad (3.49)$$

where the term  $\epsilon$  is known as the quasi-energy. Inserting this expression into the time-dependent Schrödinger equation gives

$$\sum_k (\epsilon + \hbar\omega k) \exp[-k\omega t] |\psi_{\epsilon,k}\rangle = \sum_{j,j'} \exp[i(j-j')\omega t] \hat{H}_{j'} |\psi_{\epsilon,j}\rangle. \quad (3.50)$$

In order for the equality to be true we must have  $j' - j = -k$  because the complex exponentials form an orthonormal basis and we can write

$$\epsilon |\psi_{\epsilon,k}\rangle = \sum_j \left( \hat{H}_{j-l} - \hbar\omega k \delta_{j,k} \times \hat{\mathbb{1}} \right) |\psi_{\epsilon,j}\rangle, \quad (3.51)$$

where  $\hat{\mathbb{1}}$  is the identity matrix. The expression can be recast into a matrix form

$$\epsilon \begin{pmatrix} \dots \\ |\psi_{\epsilon,-1}\rangle \\ |\psi_{\epsilon,0}\rangle \\ |\psi_{\epsilon,1}\rangle \\ \dots \end{pmatrix} = \begin{pmatrix} \hat{H}_0 + 2\hbar\omega & \hat{H}_1 & \hat{H}_2 & \dots & \dots \\ \hat{H}_{-1} & \hat{H}_0 + \hbar\omega & \hat{H}_1 & \hat{H}_2 & \dots \\ \hat{H}_{-2} & \hat{H}_{-1} & \hat{H}_0 & \hat{H}_1 & \dots \\ \dots & \hat{H}_{-2} & \hat{H}_{-1} & \hat{H}_0 - \hbar\omega & \hat{H}_1 \\ \dots & \dots & \hat{H}_{-2} & \hat{H}_{-1} & \hat{H}_0 - 2\hbar\omega \end{pmatrix} \begin{pmatrix} \dots \\ |\psi_{\epsilon,-1}\rangle \\ |\psi_{\epsilon,0}\rangle \\ |\psi_{\epsilon,1}\rangle \\ \dots \end{pmatrix}. \quad (3.52)$$

The quasienergies  $\epsilon$  can be computed by truncating and then diagonalizing the matrix, and they are grouped into repeating manifolds separated in energy by

---

<sup>9</sup>Very much like Bloch wave functions

$\hbar\omega$ . The quasienergies within a manifold can be interpreted as the eigenenergies of an effective time-independent Hamiltonian  $\hat{H}_{Fl}$  that describes the evolution of the system sampled stroboscopically at an integer number of driving periods, with the time evolution operator  $\hat{U}(t_0, t_0 + T) = e^{-iT\hat{H}_{Fl}}$ .

Floquet theory plays an important role in the engineering of different dispersion relations for atoms in Chapters 5 and 8. I will give an example based on the experiments described in [59] (not included in this Thesis), where we considered a pair of Raman beams driving transitions between the  $|F = 1, m_F\rangle$  states of the ground hyperfine manifold with two different frequencies  $\omega_{-1,0}$  and  $\omega_{0,+1}$  set to the  $m_F = -1 \rightarrow m_F = 0$  and  $m_F = 0 \rightarrow m_F = 1$  transitions. By performing independent RWAs with respect to each of these transitions we found that the system could be described by a magnetic Hamiltonian

$$\hat{H} = \frac{\hbar k^2}{2m} + \boldsymbol{\Omega}(x) \cdot \hat{\mathbf{F}} + \Omega_2 \hat{F}_{zz}^{(2)} \quad (3.53)$$

with helical effective magnetic field  $\boldsymbol{\Omega}_1(x)/\Omega_1 = \cos(2k_R x)\mathbf{e}_x - \sin(2k_R x)\mathbf{e}_y$ , an effective quadratic Zeeman shift  $\Omega_2 = (\omega_{-1,0} - \omega_{0,1})/2$ , and  $\hat{F}_{zz}^{(2)}\hbar = \hat{F}_z^2/\hbar^2 - 2/3$  an element of the quadrupole tensor. The competing contributions between kinetic and magnetic ordering energies gave rise to different magnetic phases. Figure 12a. shows the ground branch of the dispersion relation for small  $\Omega_1 < 4E_L$  (top) and large  $\Omega_1 > 4E_L$  (bottom). As the value of  $\Omega_2$  was decreased, the magnetization in the system changed along with the location of the global minima in the dispersion. The experimental parameters  $\Omega_1$  and  $\Omega_2$  spanned a two-dimensional phase

diagram shown in Figure 3.53b that we experimentally mapped. The eigenenergies of Equation 12 are plotted in Figure 12c. However, in order to get a good agreement between the experiment and the phase diagram we had use the full Floquet Hamiltonian which results in having modified parameters in Equation 3.53  $\Omega_2^{(\text{eff})} = \Omega_2 + \mathcal{O}(\Omega_1^2/\epsilon)$  (red dotted line in Figure 12b). Figure 12d shows three manifolds of Floquet quasienergies for this system, illustrating their periodic nature.

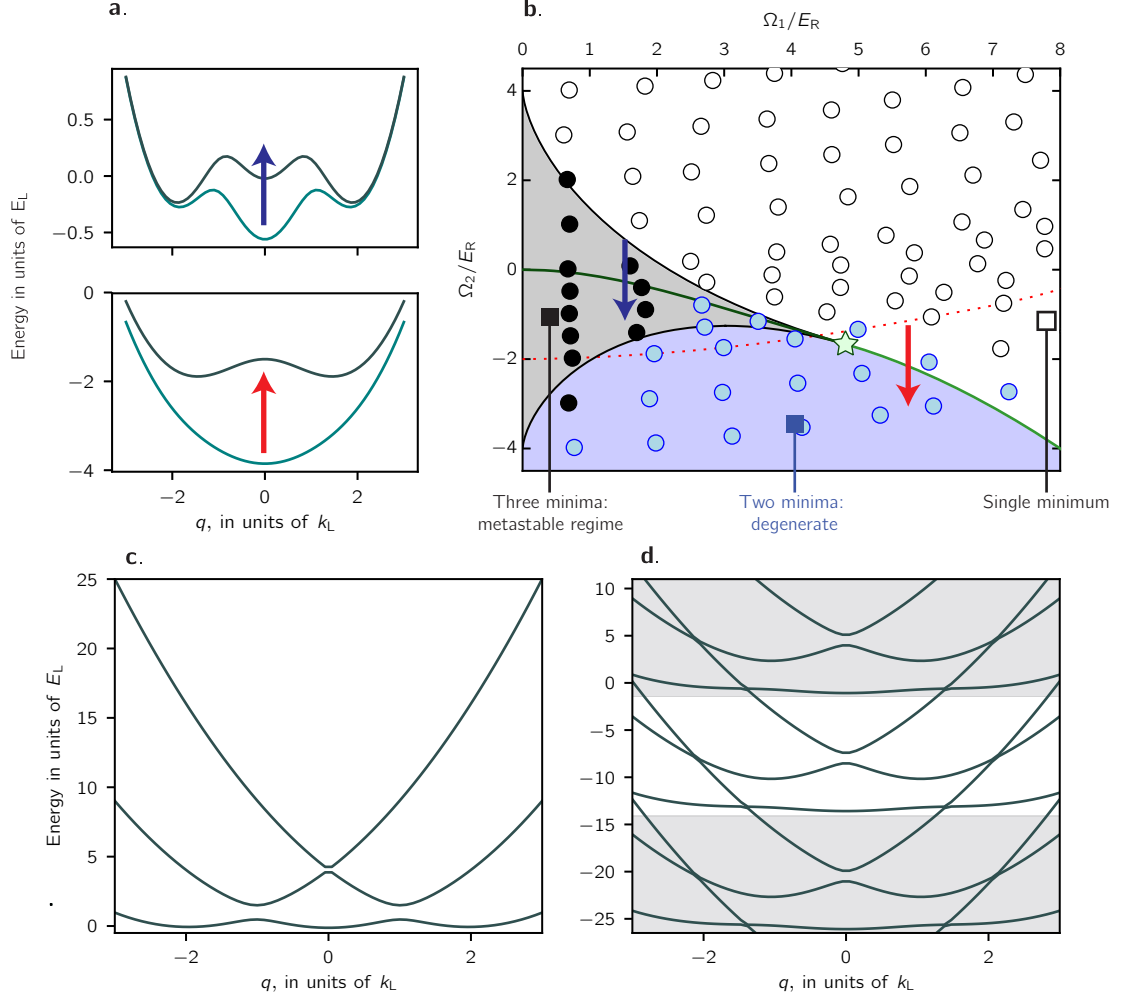


Figure 12: Magnetic phases of a spin-1 SOC system. **a.** Ground state energies of a spin-1 SOC system for  $\Omega_1 = 1.5 E_L$  (top) and  $\Omega_2 = 1.5 E_L$  (bottom). By changing  $\Omega_2$  we moved the location of the central minima. **b.** Phase diagram of a spin-1 SOC system. The green line corresponds to a line of phase transitions where the system goes from magnetized to unmagnetized. **c.** Dispersion relation of effective Hamiltonian computed for  $\Omega_1 = 2 E_L$ ,  $\Omega_2 = 0$ . **d.** Floquet quasienergy dispersion relation for the same parameters. The magnitude of  $\Omega_1$  effectively modifies  $\Omega_2$  in the RWA Hamiltonian.

## Chapter 4: Making BECs in the Rubidium Lithium apparatus

All the experiments presented in this thesis were performed in the Rubidium-Lithium (RbLi) apparatus at the University of Maryland. The apparatus was designed to produce mixtures of quantum degenerate gases of bosons and fermions. The original plan was abandoned because the cross-species scattering length was found to be repulsive and small ( $a_s \approx 20 a_B$ ) [60] and the nearest Feshbach resonance was measured to occur at the unexpectedly large magnetic field of 1066 G [61]. All our experiments were performed using only  $^{87}\text{Rb}$  instead.

The RbLi apparatus is scheduled to be shut down and the construction of a new dual-species apparatus for  $^{87}\text{Rb}$  and  $^{39}\text{K}$  is underway. The RbLi apparatus has been thoroughly described in [62, 63] so I only give a brief overview of the apparatus. I additionally discuss in detail the new elements that have been added to the setup and implemented changes not previously reported. In Appendix A I discuss the best and the worst aspects of the apparatus and in Appendix B I discuss some aspects of the construction of the new apparatus for those interested to look deeper into the technical aspects of experimental apparatus.

This Chapter is divided into three sections. In Section 4.1 I give a brief overview of the RbLi apparatus and describe its basic capabilities. In Section 4.2 I

describe the experimental sequence used to produce BECs. Finally in Section 4.3 I describe changes to the RbLi apparatus that were not previously reported.

## 4.1 Overview of the RbLi apparatus

The RbLi apparatus is divided into two optical tables. One table contains laser systems that are fiber-coupled into the main experiment optical table, shown in Figure 1, containing a vacuum system where atoms are cooled to degeneracy. The vacuum system can be divided into three regions: an oven region where Rb and Li atoms are heated up, a Zeeman slower that acts as a differential pumping stage and an ultra-high vacuum (UHV) region with a glass cell where all the experiments are performed.

### 4.1.1 Laser systems

We use three lasers to perform laser cooling and imaging of atoms: a cooling laser that addresses the  $F = 2 \rightarrow F' = 3$  transition, a repump laser that takes atoms that have decayed into the  $F' = 1$  state back to  $F = 2$  via the  $F' = 1$  state and a master laser that provides a frequency reference for both lasers. The frequency of the master laser is locked using saturation absorption spectroscopy to the  $F = 3 \rightarrow F' = 3$  and  $F = 3 \rightarrow F' = 4$  crossover of the D2 line of  $^{85}\text{Rb}$ . A fraction of repump and cooling light is used in beam for performing laser-cooling in the Zeeman slower and most of the remaining light is combined in six beams used to create a magneto-optical trap (MOT). We have two additional beams with cooling



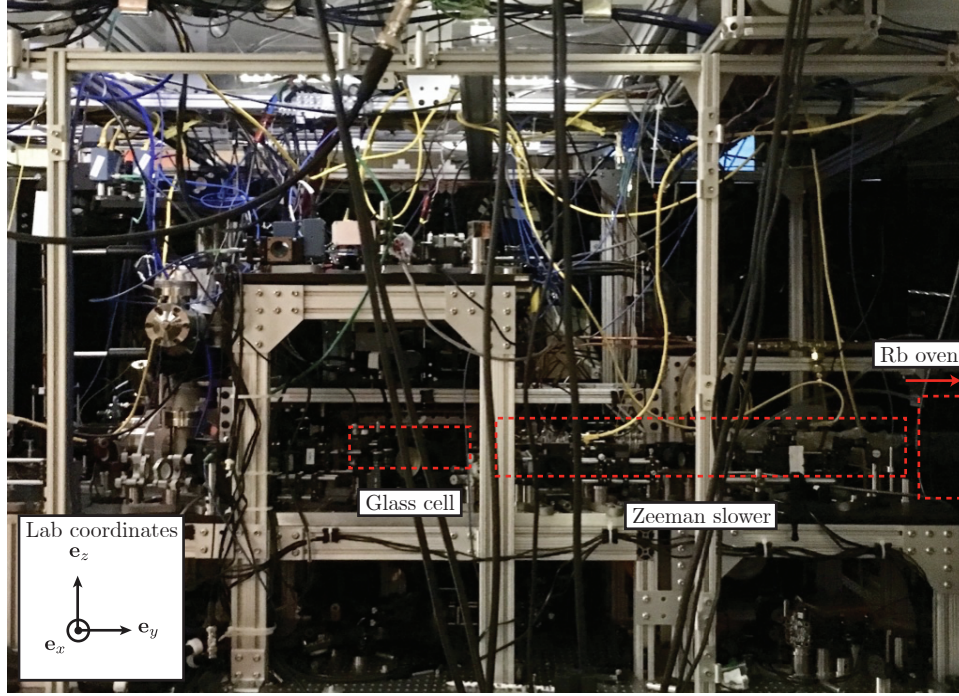


Figure 1: The vacuum system of the RbLi apparatus is divided into an oven, a Zeeman slower and an UHV region containing a glass-cell where all experiments are performed.

light used for imaging the atoms along two different directions. The frequencies of each laser with respect to the master laser frequency with respect to the transitions in  $^{87}\text{Rb}$  can be visualized in Figure 2.

We have two additional lasers that are used to generate potentials for the atoms. The first one is a 30 W 1064nm IPG Photonics laser located at the main experiment table and that we use to make a cross dipole trap for the atoms. The two dipole beams come from the zeroth and first order of an acusto-optic modulator (AOM) and the beams propagate along the  $\mathbf{e}_x + \mathbf{e}_y$  and  $\mathbf{e}_x - \mathbf{e}_y$  direction (the lab coordinate system is shown in Figure 1). The other laser system is a Ti:Sapphire laser used to generate Raman transitions and will be described in more detail in

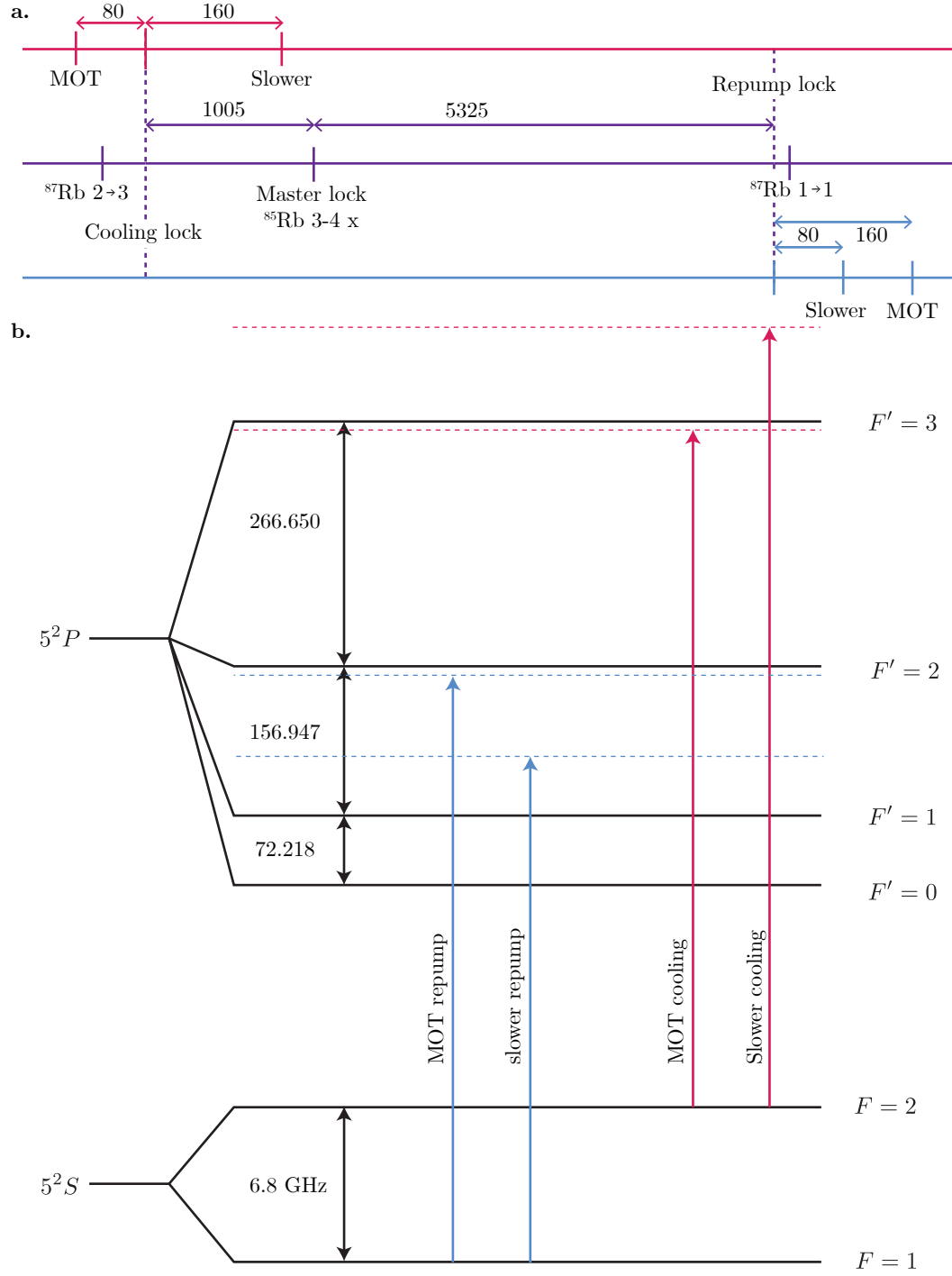
Section 4.3.2.

We can detect atoms using two different imaging systems. The first one is used primarily for diagnostics and it images the  $yz$  plane of the atoms from the  $+\mathbf{e}_x$  side of the glass cell. The second system looks at the  $xy$  plane from below the glass cell and is the main system used for data acquisition. Figure 3 shows a simplified diagram with a top and a side view of the apparatus including all the lasers that are used for cooling, trapping, Raman coupling and imaging.

#### 4.1.2 Magnetic field control

The precise control of magnetic fields is essential during the multiple stages in our experimental sequence. The RbLi apparatus has multiple coils as is illustrated in Figure 4. Three pairs of coils in the vicinity of the glass cell generate bias magnetic fields  $\mathbf{B} = (B_x, B_y, B_z)$  along  $\mathbf{e}_x$ ,  $\mathbf{e}_y$  and  $\mathbf{e}_z$ . Once BECs are produced we typically use bias fields along  $\mathbf{e}_z$  to change the Zeeman energy of the different  $|m_F\rangle$  states. One pair of anti-Helmholtz coils generates a strong quadrupole magnetic field along  $\mathbf{e}_z$  that is used in the MOT, for magnetic trapping, and to separate the different  $|m_F\rangle$  states before imaging via the Stern-Gerlach effect. An additional set of coils arranged in a ‘cloverleaf’ pattern generates small gradients along  $\mathbf{e}_z$ ,  $\mathbf{e}_x + \mathbf{e}_y$  and  $\mathbf{e}_z - \mathbf{e}_y$  which allow us to cancel stray magnetic gradients in  $B_z$  near the atoms.

The experiment also has the capability of producing oscillatory magnetic fields. A set of coils on a printed circuit board (PCB) and placed right above the glass cell produce linearly polarized radio-frequency (RF) magnetic fields either in the  $\mathbf{e}_y$  or  $\mathbf{e}_z$



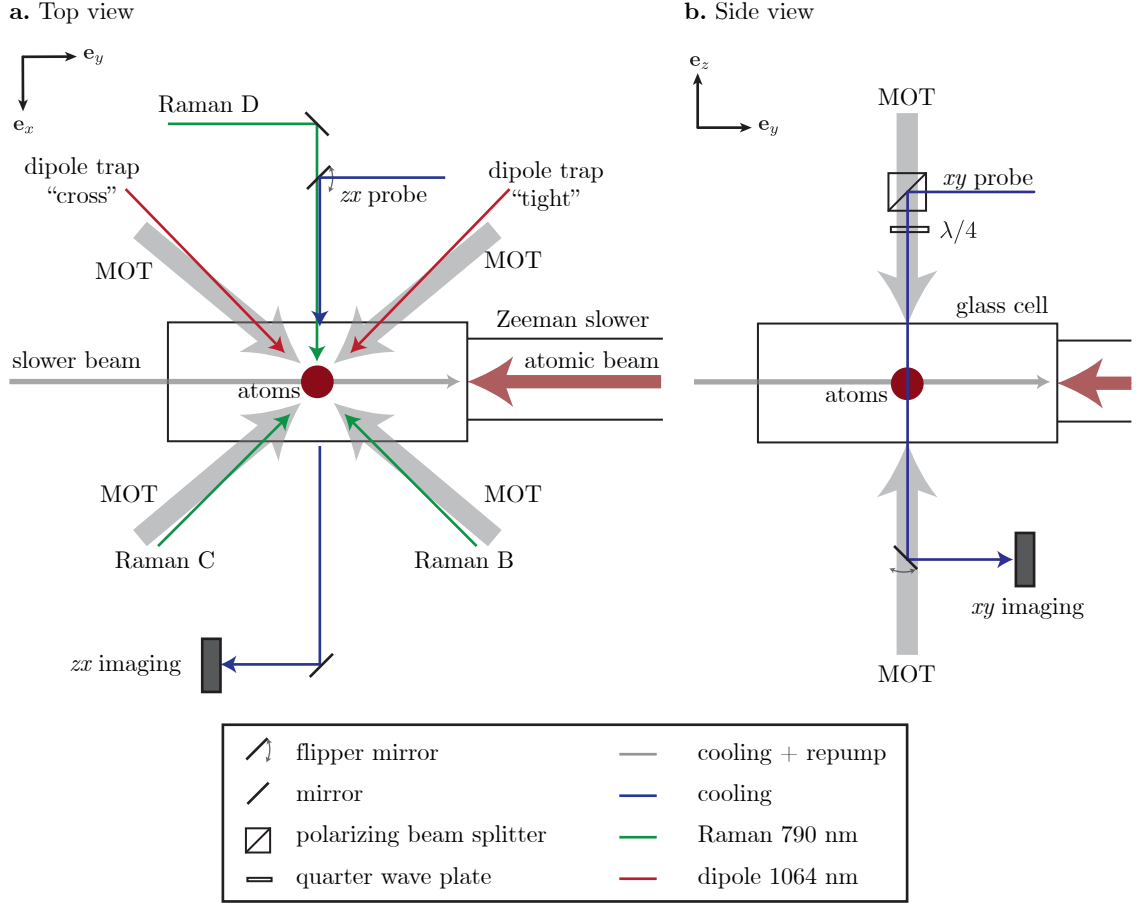


Figure 3: Diagram of the RbLi apparatus as seen from **a.** the top and **b.** the side.

direction that are used for RF induced evaporation and to drive transitions between  $|m_F\rangle$  states. There is an additional setup for producing high-power RF fields which will be described in more detail in Section 4.3.3.

## 4.2 Experimental sequence to make BECs

The production of BECs starts in an oven where Rb atoms are heated to 120 C to produce an atomic beam. The Rb atoms come from the same 5 g ampule that was installed when the apparatus was first built almost 10 years ago. The atoms then

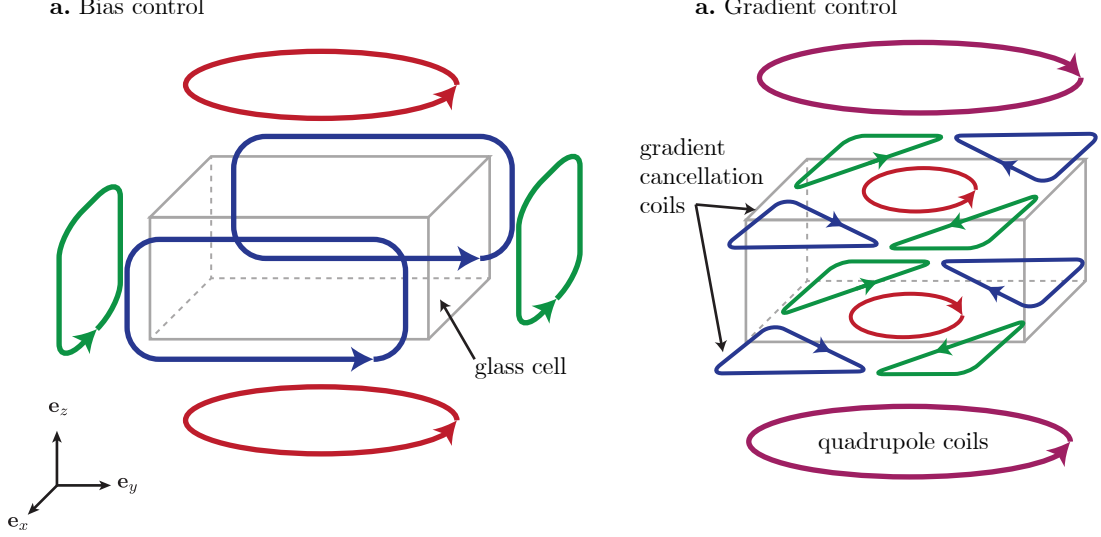


Figure 4: Magnetic coil geometry in the RbLi apparatus **a.** We use three pairs of Helmholtz coils to produce bias fields along  $\mathbf{e}_x$ ,  $\mathbf{e}_y$  and  $\mathbf{e}_z$ . **b.** We have a pair of coils that produce strong quadrupole magnetic fields for the MOT and magnetic trapping. Additionally we have a pair of ‘cloverleaf’ coils to control the gradients in  $B_z$ .

travel down a Zeeman slower [64] where they are laser cooled and then captured in a MOT. For the MOT we apply a 5.5 A current to the quadrupole coils corresponding to a 15.62 G/cm gradient. The cooling light is blue detuned by 18 MHz  $\sim -3\Gamma$  from the  $F = 2 \rightarrow F' = 3$  cycling transition and the repump light is 16 MHz below the  $F = 1 \rightarrow F' = 2$  transition. We typically load the MOT for a total time of 1 – 5 s before we turn off the Zeeman slower currents. In preparation for the molasses stage we do a 30 ms decompression stage where we ramp down the gradient to 10 G/cm and reduce the repump power.

The atoms are cooled further down in an optical molasses stage due to the polarization gradient from interfering counter-propagating lasers [65]. For this stage we completely switch off the quadrupole coils and adjust the bias fields in all three di-

rections so that they are canceled at the atoms. During this stage, the repump power is kept low and the frequency of the cooling light is decreased to 140 MHz below the MOT frequency for 10 ms. We then completely turn off the MOT repump light allowing atoms to decay into the  $F = 1$  manifold and use a small amount of repump light from the Zeeman slower to optically pump atoms into the  $|F = 1, m_F = -1\rangle$  magnetically trappable state for a total of 1.5 ms.

Once the atoms are successfully pumped into  $|F = 1, m_F = -1\rangle$  we capture them in a magnetic trap with a gradient of 62 G/cm and compress them by increasing the current in the coils until we reach a gradient of 160 G/cm in 300 ms to enhance the elastic collisional rates and promote faster thermalization during evaporation. We perform RF-induced evaporation in the magnetic trap by turning on an RF field polarized along  $\mathbf{e}_y$  with a frequency of 24 MHz, which transfers the hotter atoms at the edges of the trap into the  $m_F = 0$  state which is not magnetically trappable. We then perform an exponential ramp from the initial frequency to a final frequency of 4.5 MHz in 1 s, and as the frequency is swept the most energetic atoms are selectively transferred into untrappable states.

For the final stage of evaporative cooling, we transfer the atoms from the magnetic trap into an optical dipole trap. We start by turning on only the ‘tight’ arm of the trap at full power (about 11 W) and slowly decompressing the quadrupole trap to 45 G/cm in 1.5 s. We then turn on the second ‘cross’ beam in 1 s, splitting the power so that 70 – 30% is distributed between the tight and cross beams. As the cross dipole beam is being turned on, we ramp the quadrupole field further down to 14 G/cm, slightly above the value necessary for the trap to suspend atoms against

gravity while simultaneously shifting the bias field along  $\mathbf{e}_z$  to align the center of the quadrupole trap to the dipole trap.

We evaporate atoms in the dipole trap in two stages. First we exponentially ramp down the power to about 20% of its initial power in 1.5 s (0.5 s  $1/e$  time). Before the final evaporation stage we completely turn off the quadrupole trap in 1 s. Finally, we perform a second exponential ramp where the power is dropped to about 30% of the intermediate power in 2 s (1 s  $1/e$  time). The slow ramps ensure that there is enough time for the atoms to thermalize as they evaporate. During the second evaporation stage the atoms reach the critical temperature for Bose-Einstein condensation and we are able to produce BECs with about  $4 \times 10^4$  atoms in the  $|F = 1, m_F = -1\rangle$  state. To transfer atoms into different  $|F, m_F\rangle$  states within the ground  $5^2S_{1/2}$  manifold we use the ARP protocol described in Section 3.5.2.

Our experiments are very sensitive to changes in the environmental magnetic field. In the past we used flux gate sensors (Stefan-Mayer model FL1-100 f) in the apparatus to feedback and stabilize the magnetic field (see [63]). These sensors are a useful tool, however due to space constraints we were not able to measure the fields close to the atoms and additionally the range of magnetic fields that they operate at is small (only 1 G, we typically operate at  $B > 10$  G). We built a 6.8 GHz microwave system (see Section 4.3.4) so that we could use the atoms themselves as sensors of magnetic field. Once BEC is achieved and before performing any experiment, we typically use a protocol involving two microwave assisted partial transfer absorption images described in Section 3.5.3 to monitor and stabilize the magnetic field at the atoms.

To keep the bias magnetic fields as stable as possible we additionally synchronize the timing of our experiments to the 60 Hz line; this step is performed in different stages of the experiment but its biggest impact is right before performing any magnetically sensitive experiment.

## 4.3 Upgrades to the RbLi Machine

### 4.3.1 Master laser system

Previously we used a **New Focus Vortex II TLB-6900** extended cavity diode laser as our master laser locked using a homemade saturation spectroscopy setup using a Rb glass cell and an FPGA based PID circuit (see [62, 63]). The frequency of this laser was not very stable and the laser would constantly get out of lock. We replaced the old master laser with a **Vescent photonics DBR Laser Module System** which uses a distributed Bragg reflector laser diode with no external cavity and is therefore very mechanically stable. The frequency of the laser is stabilized and controlled using the D2-210 spectroscopy module and D2-125 laser servo. The master laser system is considerably simplified as can be seen in Figure 5 and the lab no longer suffers from an unstable master laser.

### 4.3.2 Raman laser system

The RbLi apparatus has a laser system with wavelength close to 790 nm that is used to generate Raman induced transitions and spin-dependent potentials (see Section 3.3.2). The original Raman laser system consisted of a **Toptica DL Pro**



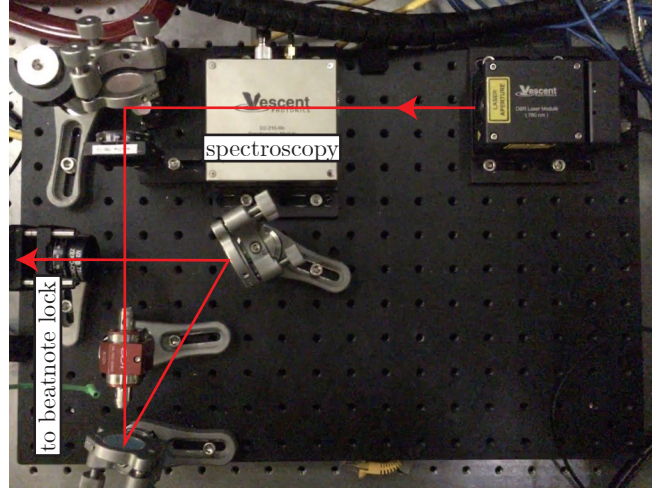


Figure 5: Master laser system. We replaced the old Vortex II laser with a Vescent photonics DBR Laser Module System that is considerably more stable.

laser seeding a tapered amplifier chip mounted on a homemade copper holder. This laser system was replaced by an M squared Ti:Sapphire laser (SolisTiS-400-SRX-F) that is pumped by a 532 nm IPG GLR30 laser. We typically operate the pump laser at 14.5 W. A fraction of this light is redirected into the path of a 1D optical lattice and the remaining power is used to pump the Ti:Sapphire laser. Figure 6 shows the original (at setup) dependence of the Ti:Sapph output power as a function of pump power. We switched to using a Ti:Sapph laser because of its wide range of tunable wavelengths in the near infrared (725 – 875 nm) and its high power output. In addition to this, the TA system had considerable amplified spontaneous emission (ASE) close to 780 nm which caused increased scattering of photons and reduced lifetimes due to our inability to completely filter it.

The output of the laser is split into 3 different Raman beams. The frequency and power of each beam is independently controlled using IntraAction ATM-801A2 AOMs centered at 80 MHz. We drive the AOMs using homemade drivers made from

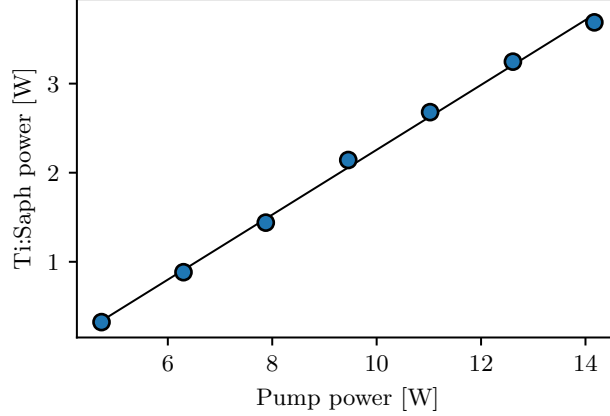


Figure 6: Ti:Sapphire laser output as a function of pump power. This data was taken when the laser was being setup for the first time. The alignment of the laser cavity was optimized to produce maximum power at 15 W pump power. The output power is proportional to the pump power  $P_{\text{out}} \approx 0.364P_{\text{pump}}$ .

the **Minicircuits** components listed in Table 4.1 and **Novatech Model 409B** direct digital synthesizer (DDS) to generate an RF signal at the desired frequency. Figure 8 shows the arrangement of the components: we control the amplitude of the RF signal using a mixer connected to a DC signal and the switch can turn off the signal in less than  $1 \mu\text{s}$  using a TTL signal. We fiber-couple the light using single-mode optical fibers (non-polarization maintaining) with angle-cut FC/APC type connectors at the input (laser side) and flat-cut FC/PC at the output (experiment side). We made this choice so to implement a phase lock that would cancel phase noise added by the fibers. The idea behind this method is that a small fraction of the fiber-coupled light is reflected at the flat-cut edge of the optical fiber and coupled back where it can be heterodyne probed with the input light (see Figure 7). Our implementation of the fiber phase lock is based on Section 3.6.3 of [66]. Because the fibers are not polarization maintaining we control the output polarization of the light using paddles

(Thorlabs FPC030 and Thorlabs FPC560) that produce a controllable amount of stress in the fibers and change their birefringence. This method makes injecting light into the fiber very straightforward and painless because, unlike with polarization-maintaining fibers, the polarization of the incoming light can be arbitrary and does not need to be aligned to any fiber axis. None of the experiments presented in this thesis used the phase lock but the experiments described in Chapter 8 were performed using the new Raman laser system. Figure 7 shows a diagram of the Raman optics as well as the 532 nm optical lattice optics which are shared on the same breadboard.

Table 4.1: List of AOM driver components

Part number	Description
ZHL-1-2W	2 W amplifier
ZAD-3+	Mixer
ZYSWA-2-50DR	Digital switch

### 4.3.3 High power RF system

The experiments described in Chapters 6 and 8 required the use of high power RF magnetic fields to achieve coupling strengths at the atoms  $\Omega \sim 100 - 200$  kHz. After multiple attempts to build a resonant coil either on a PCB (similar to the coil used for RF induced evaporation, see [62, 63]) or winding some wires with the right dimensions we found that the product that worked best for our needs was a wireless power charging receiver coil (Würth Elektronik Digikey part number 732-5646-ND) shown in the bottom panel of Figure 4.3.3a. The coil has a self-

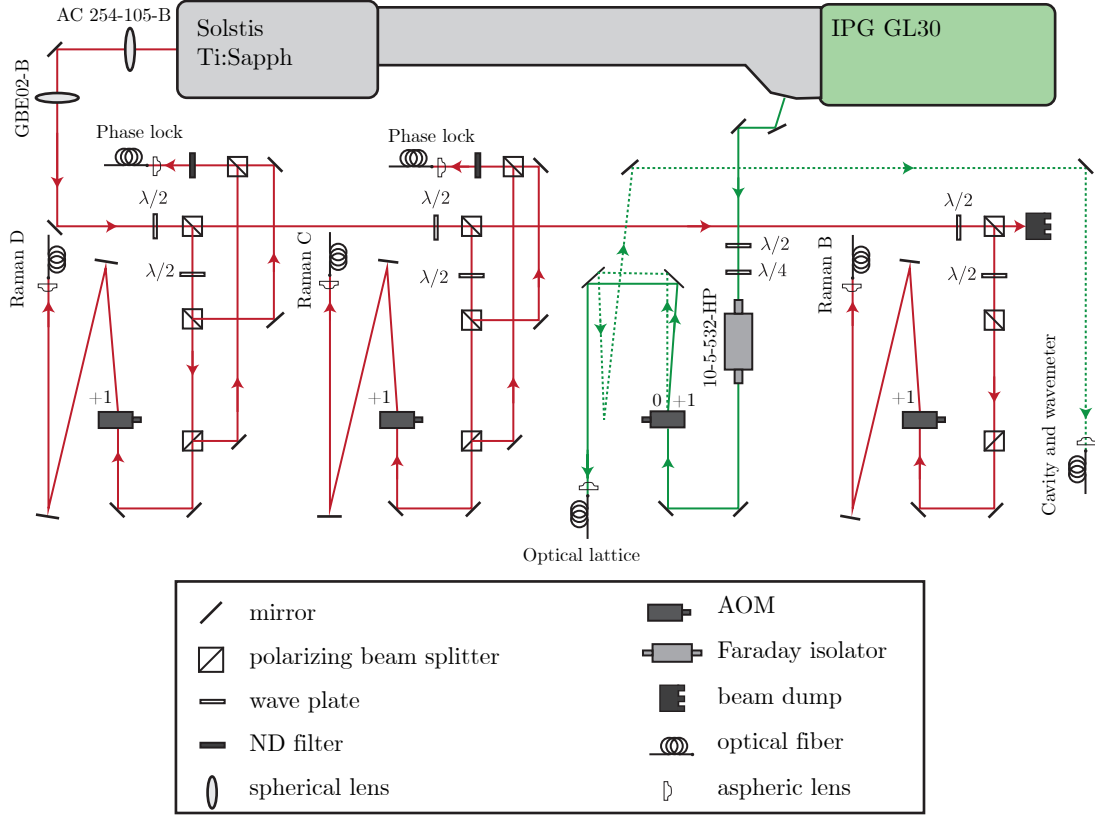


Figure 7: Optical layout of Raman and optical lattice lasers. The Ti:Sapphire laser provides tunable infrared light that we use for three different Raman beams. The beams labeled as ‘Raman C’ and ‘Raman D’ can be phase stabilized. A small fraction of the green pump laser is used to produce a blue-detuned 1D optical lattice.

resonant frequency at 22 MHz and a Q-Factor of 45. It has an inner diameter of 1.62 cm and an outer diameter of 2.8 cm, just the right size for us to place it snugly next to the glass cell (on the  $-\mathbf{e}_x$  side) with minimal perturbations to the laser beams in its vicinity (it only slightly clips one MOT beam).

The loop is mounted on the PCB shown in Figure 9. The board has two connections: the top one in Figure 9 has a small loop used as a pickup antenna that we attach to a power detector Minicircuits ZX47-40-S+ to monitor the power and the bottom lines have pads that can be used to make an impedance matching

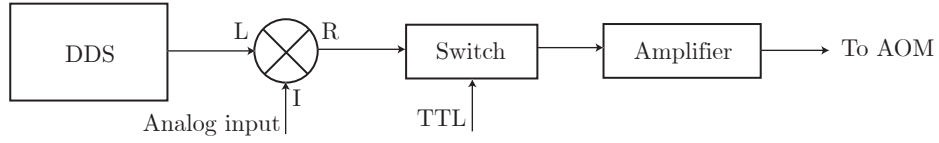


Figure 8: Setup used to drive the AOMs controlling the power and frequency of Raman beams. A similar setup is used to drive a coil used to generate high-power RF fields at the atoms.

network.

We used a vector network analyzer (VNA) to perform the impedance matching. The VNA sends a small amplitude RF signal into the circuit and measures the amplitude and phase of the reflected power as a function of frequency from which the impedance can be inferred. Figure 10a shows the reflected power as a function of frequency for a test circuit and Figure 10b shows the complex valued impedance as a function of frequency displayed on a Smith chart. The Smith chart is a helpful way to visualize the impedance of a circuit: the black circles correspond to constant resistance, with the rightmost point corresponding to an open circuit (infinite resistance) and the largest circle corresponding to a short circuit (zero resistance). The arcs correspond to constant reactance; the horizontal axis corresponds to zero reactance ( $\text{Im}(Z) = 0$ ), the top arcs correspond to  $\text{Im}(Z) > 0$ , and the lower arcs to  $\text{Im}(Z) < 0$ . The circuit is impedance matched when  $Z = 50 \text{ Ohm}$  (the center of the Smith chart), the standard value of RF transmission lines. We tested different components on the pads until we found a peak in reduced reflected power at the desired frequency. It is also important to note that it was essential that the circuit was installed in its final location in the experiment when measuring the impedance

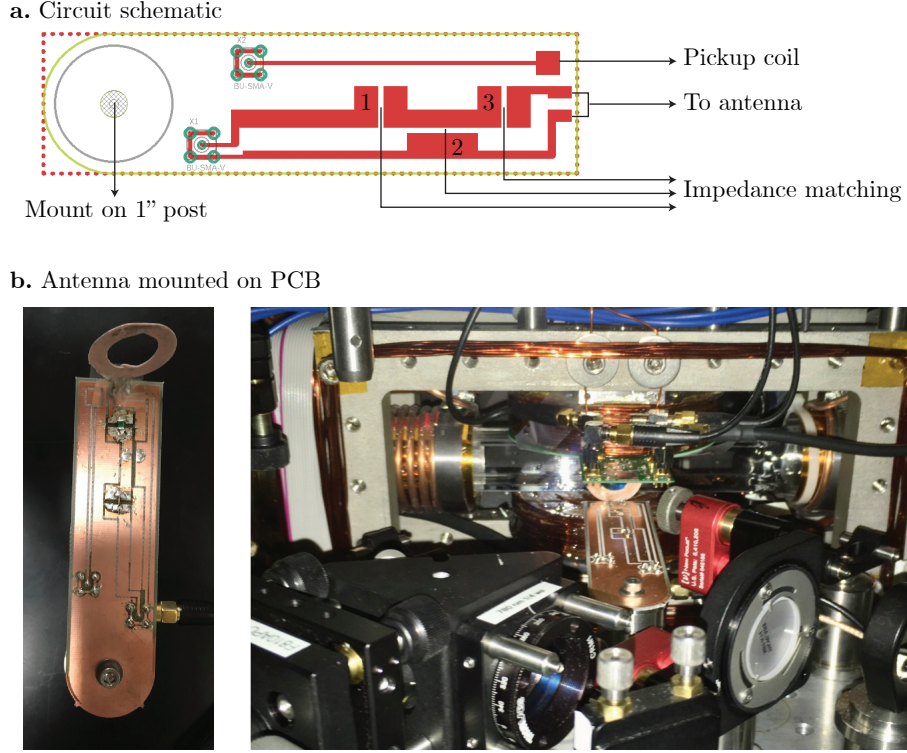


Figure 9: We use a commercial resonator with an impedance matching network to produce high power RF fields. **a.** Diagram of the impedance matching network. **b.** A picture of the resonator mounted on a PCB. We place this device as close to the atoms as possible next to the glass cell.

as the other parts in the vicinity of the antenna can shift the resonant frequency.

The driving electronics are very similar to the AOM drivers described in Section 4.3.2. The only difference is we use a 30 W amplifier (Minicircuits LZY-22+) instead of the smaller amplifiers needed to drive the AOMs.

#### 4.3.4 6.8 GHz microwave system

We built a 6.8 GHz microwave system to couple atoms between the  $F = 1$  and  $F = 2$  ground hyperfine manifolds of  $^{87}\text{Rb}$ . We mostly use this system to stabilize the bias magnetic field along  $\mathbf{e}_z$  at the atoms using microwave assisted

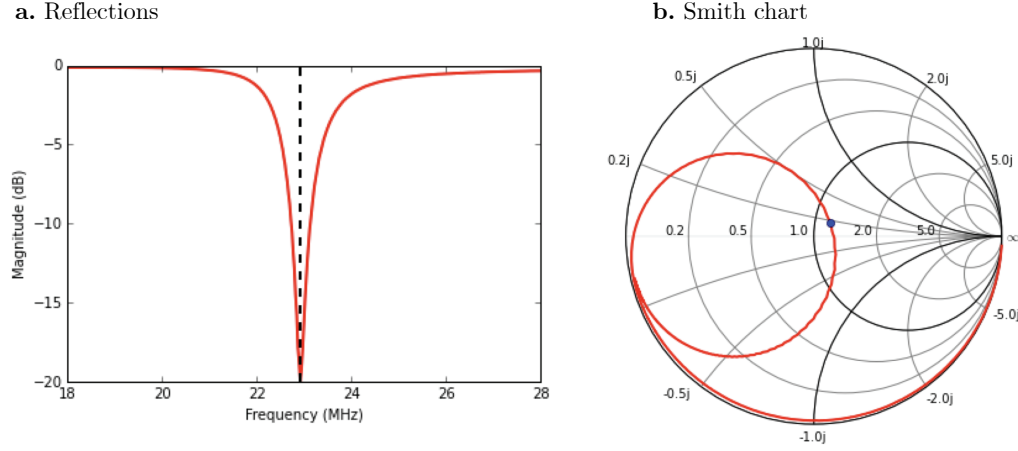


Figure 10: Impedance matching of high power RF antenna. **a.** Reflected power as a function of frequency. **b.** Impedance as a function of frequency, visualized on a Smith chart.

PTAI (Section 3.5.3). Table 4.2 presents a list of components used in the setup is presented and Figure 11 shows a diagram of the connections.

Table 4.2: 6.8 GHz microwave system components

Part number	Description
SRS SG384	Signal generator
Narda 4014C-30	Directional coupler
Marki IRW0618	Mixer
Minicircuits VBFZ-6260-S+	Bandpass filter 6 – 8 GHz
Herley D1956	Voltage controlled attenuator
MSI MSH-5727901	46 dB gain amplifier
Narda 4014C-30	Circulator
Minicircuits ZX47-40-S+	Power detector
Maury microwave 1819C	Stub tuner
ZYSWA-2-50DR	Digital switch

The SRS generator serves as a source of a fixed frequency and amplitude signal. We control the frequency by mixing a programmable  $\sim 100$  MHz signal from a Novatech into a double balanced mixer; the RF signal can be turned on

or off using a TTL switch. The amplitude is controlled by commanding a  $0 - 6$  V signal from the control computer into an attenuator. The signal is amplified by +43 dB using an amplifier mounted on a water cooled plate. The microwave signal is broadcast to the atoms using a horn antenna. In order to get a coupling microwave coupling strength of the order of 10 kHz or larger it was important to place the horn as close to the atoms as possible and impedance matching the transmission line to maximize the radiated power. We additionally use a circulator that prevents any reflected power to go back into the amplifier and couplers at different locations to monitor the performance of the system. The last coupler and the circulator of the system are connected to a power detector that outputs a DC signal proportional to the microwave power. The impedance is tuned with a stub tuner by changing the length of the stubs until the reflected power measured at the exit port of the circulator is minimized at the desired frequency of operation.

#### 4.4 Computer control and data acquisition

There have been two main changes in our computer control and data acquisition system. We have transitioned from using a LabVIEW based control system to a Python based control system, The `labscript suite` [67]. With the previous control software the lab devices were programmed using a graphic interface. `Labscript` instead uses a hybrid approach in which the experimental sequences are text based scripts. The use of scripted programming has given us more flexibility and modularity for programming experiments and additionally, it is now very easy



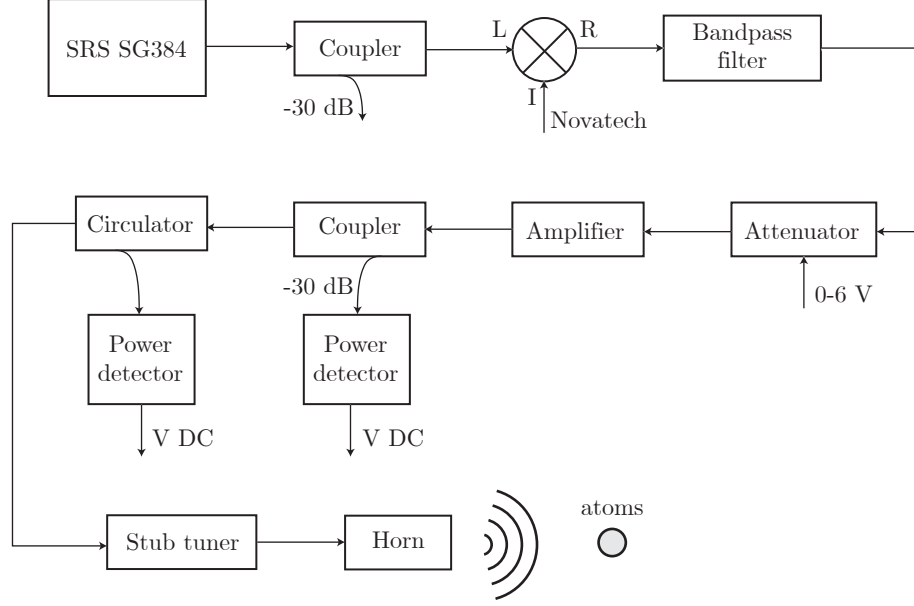


Figure 11: Schematic of 6.8 GHz microwave system.

to do multi-dimensional parameter scans. Each experimental shot is saved in a Hierarchical Data Format version 5 file (HDF5) that includes images from cameras, oscilloscope traces, and analog inputs as well as copy of the script used in the experiment and the values of all of the parameters used. This has been a great upgrade as we no longer rely on the person running an experimental sequence pushing the ‘save’ button and thoroughly documenting the experiment in question<sup>1</sup>.

The other upgrade worth mentioning is replacing our old Flea3 (FL3-FW-03S1M-C from FLIR, formerly Point Grey) CCD camera with a Mako G-030 camera from Allied Vision. With this new camera the time between two consecutive shots can be as short as  $96 \mu\text{s}$  (we used to wait  $\sim 30 \text{ ms}$  with the Flea3 camera), greatly reducing the effect of mechanical vibrations in the experiment that produce fringes in the

---

<sup>1</sup>As I have been digging into old data, I greatly wish we had this feature sooner.

absorption images. Table 4.3 summarizes relevant specifications of both cameras. In our experimental sequence the probe and atoms images are separated by  $150\ \mu\text{s}$ , which is not enough time for the atoms in the first absorption image to be cleared out. We therefore had to change the order in the absorption imaging protocol: first we take a picture of the probe which is 6.8 GHz detuned and minimally disturbs the atoms, we then apply repump light during the  $150\ \mu\text{s}$  interval in between the images to transfer atoms into  $F = 2$  and then take the absorption image of the atoms. Figure 12 shows the OD computed using both cameras with no atoms present. When there is a long interval in between the two images the probe captured on the camera changes, leading to the fringes shown in Figure 12a. In contrast for probe images captured within a short interval the main noise contribution is shot noise as can be seen in Figure 12b. The addition of this camera was essential to get a better signal to noise ratio in the experiments reported in Chapter 8.

Table 4.3: Relevant specifications of Flea3 and Mako cameras.

	Flea3	Mako
Sensor type	CCD	CMOS
Quantum efficiency at 780 nm	$\sim 40\%$	$\sim 30\%$
Dark noise	19.94 e/s	12.9 e/s
Frame rate at max resolution	120 fps	309 fps
Pixel size	$5.6 \times 5.6\ \mu\text{m}$	$7.4 \times 7.4\ \mu\text{m}$
Resolution	$648 \times 488$	$644 \times 484$
Dynamic range	65 dB	56.7 dB
Analog to digital converter	12 Bit	12 Bit

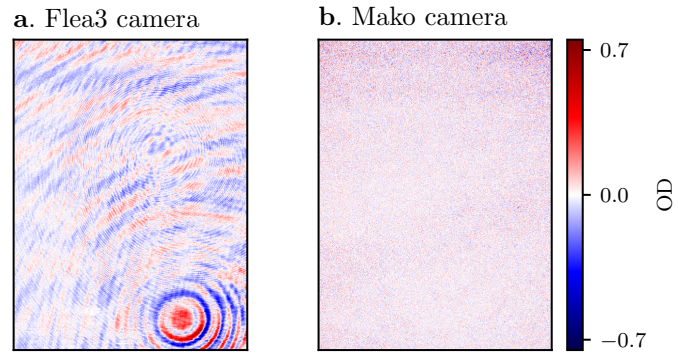


Figure 12: OD computed using two consecutive probe images without atoms. **a.** With the Flea3 camera images are spaced by  $\sim 30$  ms. Changes in the probe result in fingers in the OD. **b.** With the Mako camera images are spaced by  $\sim 150 \mu\text{s}$ .

## Chapter 5: Fourier Transform Spectroscopy

The high level of control in ultracold atomic systems makes them an ideal platform for analog simulation of materials and other complex systems. The properties of these engineered ‘atomic’ materials depend on the underlying single particle energies and it is important to characterize them. We worked on Fourier transform spectroscopy for this purpose.

Many spectroscopy techniques in atomic physics rely on using a source of coherent electromagnetic radiation with a well known frequency that probes the internal structure of a system (atom). For example, in absorption spectroscopy [68] coherent light is sent through an atomic medium and if the frequency of the light is resonant with an atomic transition it is absorbed and a reduced transmission is measured. Other variants of spectroscopy (e.g. Rabi spectroscopy [55], spin-injection spectroscopy [69]) work under a similar principle: atoms absorb and emit photons with frequencies equal to the transition energies between internal states.

Fourier transform spectroscopy instead employs the connection between the energy spectrum of a system and its dynamics. This connection has been exploited to study the spectrum of both condensed matter [70] and cold atom systems [71, 72] alike. As opposed to other techniques, Fourier spectroscopy relies only on follow-

ing the unitary evolution of an initial state suddenly subjected to a Hamiltonian of interest and measuring probabilities in a basis that does not diagonalize that Hamiltonian.

The frequency resolution of Fourier transform spectroscopy is limited only by the coherent evolution timescale of the system under study and can otherwise be applied to any system. Other applications of this technique implemented in our laboratory that are not included in this Chapter include measuring the dispersion relation of a Rashba spin-orbit coupled gas (see Chapter 8) and the band structure of a fractional period adiabatic superlattice [27].

This Chapter is organized as follows: First, I give a general description of the Fourier transform spectroscopy technique in Section 5.1. In Section 5.2 I describe a set of experiments where we engineered a tunable spin-orbit coupled system and applied Fourier transform spectroscopy. This work was published in [25].

## 5.1 Operating principle of Fourier spectroscopy

We focus on a system where we can measure the occupation probabilities of a set of orthonormal states  $\{|\psi_i\rangle\}$  that fully span the accessible Hilbert space of the system. We then consider the time evolution of an arbitrary initial state  $|\Psi_0\rangle = \sum_i a_i |\psi_i\rangle$  as governed by a Hamiltonian  $\hat{H}'(\{\Omega_i\})$  and observe the occupation probabilities of the  $\{|\psi_i\rangle\}$  states of the measurement basis as a function of time. When  $\hat{H}'$  is applied, the evolution of the initial state is  $|\Psi(t)\rangle = \sum_{i,j} a_i c_{i,j} e^{-iE'_j t/\hbar} |\psi'_j\rangle$ , where  $E'_j$  and  $|\psi'_j\rangle$  are the eigenenergies and eigenstates of  $\hat{H}'$ , and  $c_{i,j}(t) = \langle\psi_i|\psi'_j\rangle$ .

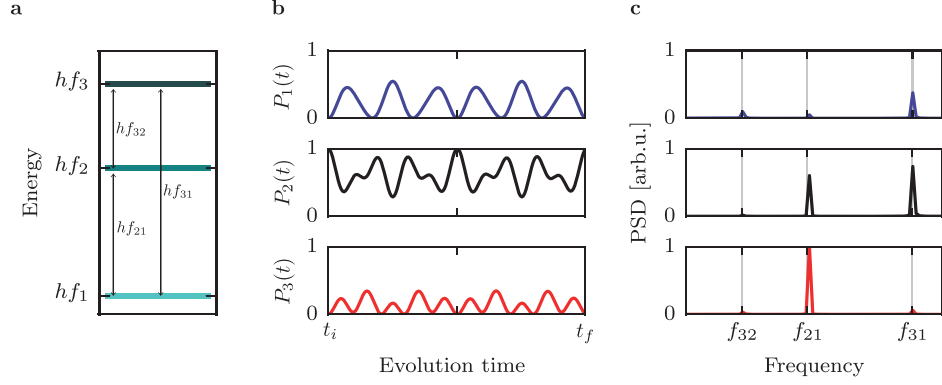


Figure 1: **a.** Eigenenergies of a three-level system described by  $\hat{H}'(\Omega_1, \Omega_2, \Omega_3)$ . **b.** The system is prepared in  $|\psi_2\rangle$  and subjected to  $\hat{H}'$  at time  $t_i$ . The three panels show the occupation probabilities of the states  $|\psi_1\rangle$  (blue),  $|\psi_2\rangle$  (black), and  $|\psi_3\rangle$  (red) in the measurement basis, for evolution times up to  $t_f$ . **c.** Power spectral density of the occupation probabilities from panel b. The three peaks in the Fourier spectra correspond to the energy differences present in panel a.

The probability

$$P_k(t) = |\langle \psi_k | \Psi(t) \rangle|^2 = \left| \sum_{i,j} a_i c_{i,j} c_{j,k}^* e^{-iE'_j t / \hbar} \right|^2 \quad (5.1)$$

of finding the system in a state  $|\psi_k\rangle$  of the measurement basis can be expressed as a sum of oscillatory components, with amplitude given by the magnitude of the overlap integrals between the initial state and the eigenvalues of  $\hat{H}'$

$$P_k(t) = 1 + \sum_{i,j \neq l} 2|a_i^2 c_{i,j} c_{j,k} c_{i,l} c_{l,k}| \cos(2\pi f_{j,l} t), \quad (5.2)$$

where  $f_{j,l} = (E'_j - E'_l)/h$  is the frequency associated with the energy difference of two eigenstates of the Hamiltonian. Fourier spectroscopy relies on measuring the populations on each state of the measurement basis as a function of time and extracting the different frequency components  $f_{j,l}$  directly by computing the discrete

Fourier transform. The bandwidth and frequency resolution of the measurement are determined by the total sampling time and the number of samples. For  $N$  samples separated by a time interval  $\Delta t$ , the highest resolved frequency will be  $f_{\text{bw}} = 1/2\Delta t$ , with resolution  $\Delta f = 1/\Delta t N$ . This resolution can be decreased if the Fourier transform is calculated using certain types of windowing functions that enhance signal to noise ratio. Any higher frequency  $f > f_{\text{bw}}$  will be aliased and measured in the Fourier spectrum as  $f_{\text{alias}} = |f - m/\Delta t|$ , where  $m$  is an integer. If interactions are present in the system, the dynamics get modified in a time scale given by the magnitude of the interactions, giving an additional constraint to the smallest frequency components of a single particle Hamiltonian that can be resolved with our technique.

Figure 1 illustrates the principle of Fourier spectroscopy for a three level system, initially prepared in the state  $|\Psi_0\rangle = |\psi_2\rangle$ , subject to the Hamiltonian

$$\hat{H}' = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} + \hbar \begin{pmatrix} 0 & \Omega_1 & \Omega_2 \\ \Omega_1^* & 0 & \Omega_3 \\ \Omega_2^* & \Omega_3^* & 0 \end{pmatrix}, \quad (5.3)$$

where we measure the occupation probability as a function of time for each of the  $\{|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle\}$  states. The three eigenenergies  $E'_i = \hbar f_i$  that result from diagonalizing  $\hat{H}'$  are displayed in Figure 1a. The three energy differences  $\hbar f_{jj'}$  between the levels determine the oscillation frequencies of the occupation probabilities, as can be seen in Figure 1b. Finally, Figure 1c shows a plot of the power spectral

densities (PSD) with three peaks corresponding to the three relative energies of  $\hat{H}'$ .

## 5.2 Measuring the SOC dispersion with Fourier transform spectroscopy

We applied the Fourier transform spectroscopy technique to measure the dispersion relation of spin-1 BECs with an equal superposition of Rashba and Dresselhaus SOC, and tunable SOC strength.

### 5.2.1 System

All of our experiments started with BECs containing about  $4 \times 10^4$  atoms in the  $|f = 1, m_F = -1\rangle$  hyperfine state. The experiments described in Section 5.2.3 were performed in an optical dipole trap with frequencies  $(\omega_x, \omega_y, \omega_z)/2\pi = (42(3), 34(2), 133(3))$  Hz. We later modified the trapping frequencies in the  $xy$  plane to try to make them more symmetric for the experiments described in Section 5.2.4. We broke the degeneracy of the three  $m_F$  magnetic sub-levels by applying a 1.9893(3) mT bias field along  $\mathbf{e}_z$  that produced a  $\omega_Z/2\pi = 14.000(2)$  MHz Zeeman splitting, and a quadratic Zeeman shift  $\epsilon$  that shifted the energy of  $|f = 1, m_F = 0\rangle$  by  $-h \times (28.45 \text{ kHz})$ . We transferred atoms into the  $|f = 1, m_F = 0\rangle$  state using ARP (see Section 3.5.2) and then we monitored and stabilized the magnetic field using partial transfer absorption imaging (Section 3.5.3) by applying a pair of 250  $\mu\text{s}$  microwave pulses, each of them detuned by  $\pm 2 \text{ kHz}$  from the  $|f = 1, m_F = 0\rangle \leftrightarrow |f = 2, m_F = 1\rangle$  transition.

We induced spin-orbit coupling using a pair of intersecting, cross polarized



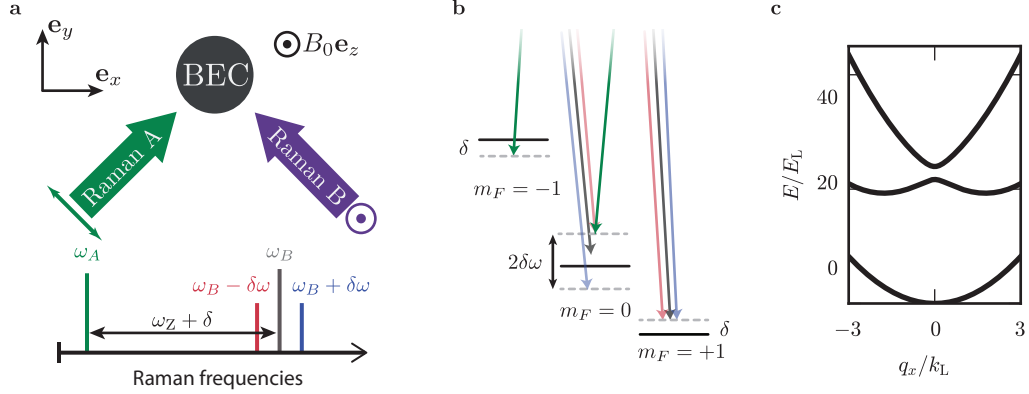


Figure 2: **a.** Setup. A bias magnetic field  $B_0 \mathbf{e}_z$ , with  $B_0 = 1.9893 \text{ mT}$  splits the hyperfine energy levels of the  $f = 1$  manifold of  $^{87}\text{Rb}$  by  $\omega_Z/2\pi = 14 \text{ MHz}$ . A pair of cross polarized Raman beams propagating along  $\mathbf{e}_x + \mathbf{e}_y$  and  $\mathbf{e}_x - \mathbf{e}_y$  couple the atoms' momentum and spin states. **b.** The Raman frequencies are set to  $\omega_A = \omega_L + \delta$  and  $\omega_B = \omega_L + \omega_Z$ . We add frequency sidebands to  $\omega_B$ , separated by  $\pm \delta\omega$ . The amplitude modulation from the interference between the multiple frequency components results in tunable SOC. **c.** SOC dispersion for Raman coupling strength  $\Omega_0 = 12E_L$  and  $\Omega = 0$ , on four photon resonance.

Raman laser beams propagating along  $\mathbf{e}_x + \mathbf{e}_y$  and  $\mathbf{e}_x - \mathbf{e}_y$ , as shown in Figure 2a and b. The beams had angular frequency  $\omega_A = \omega_L + \delta$  and  $\omega_B = \omega_L + \omega_Z$ , where  $2\delta$  is the, experimentally controllable, detuning from four photon resonance between  $m_F = -1$  and  $m_F = +1$ .

Our system was well described by the Hamiltonian including atom-light interaction along with the kinetic contribution

$$\hat{H}_{\text{SOC}} = \frac{\hbar^2 q_x^2}{2m} + \alpha q_x \hat{F}_z + 4E_L \hat{1} + \hbar \Omega_R \hat{F}_x + (4E_L - \epsilon)(\hat{F}_z^2 - \hat{1}) + \hbar \delta \hat{F}_z, \quad (5.4)$$

where  $q$  is the quasimomentum,  $\hat{F}_{x,y,z}$  are the spin-1 angular momentum matrices,  $\alpha = \hbar^2 k_L/m$  is the SOC strength, and  $\Omega_R$  is the Raman coupling strength, propor-

tional to the Raman laser intensity. The Raman field coupled  $|m_F = 0, q = q_x\rangle$  to  $|m_F = \pm 1, q = q_x \mp 2k_L\rangle$ , generating a spin change of  $\Delta m_F = \pm 1$  and imparting a  $\mp 2k_L$  momentum. The eigenstates of  $\hat{H}_{\text{SOC}}$  were linear combinations of these states and  $|m_F = 0, q = q_x\rangle$ , and the set  $\{|m_F, q\rangle\}$  constituted the measurement basis for Fourier transform spectroscopy.

Figure 2c shows a typical band structure of our spin-1 SOC system as a function of quasimomentum for a large and negative quadratic Zeeman shift  $-\epsilon > 4E_L$ . In this parameter regime, the ground state band had a nearly harmonic dispersion with an effective mass  $m^* = \hbar^2[d^2E(k_x)/d^2x]^{-1}$ , only slightly different from that of a free atom.

### 5.2.2 Tunable SOC

We engineered a highly tunable dispersion relation in which we could independently control the size of the gap at  $q_x = 0$  as well as the SOC strength  $\alpha$  by adding frequency sidebands to one of the Raman beams. The state of the system could change from  $|m_F = -1, q = q_x + 2k_L\rangle$  to  $|m_F = 1, q = q_x - 2k_L\rangle$  by absorbing a red detuned photon first followed by a blue detuned photon and vice versa, in a similar way to the Mølmer-Sørensen entangling gate in trapped ion systems [73]. When we set the angular frequencies of the sidebands to  $\omega = \omega_A + \omega_Z \pm \delta\omega$ , the Hamiltonian (Equation 5.4) acquired a time-dependent coupling  $\Omega_R(t) = \Omega_0 + \Omega \cos(\delta\omega t)$ . This periodically driven system was well described by Floquet theory [74] (see Section 3.5.5). Figure 3 shows the spectrum of Floquet quasi-energies for a system

described by Equation 5.4 where  $\Omega_R$  oscillated with angular frequency  $\delta\omega$ .

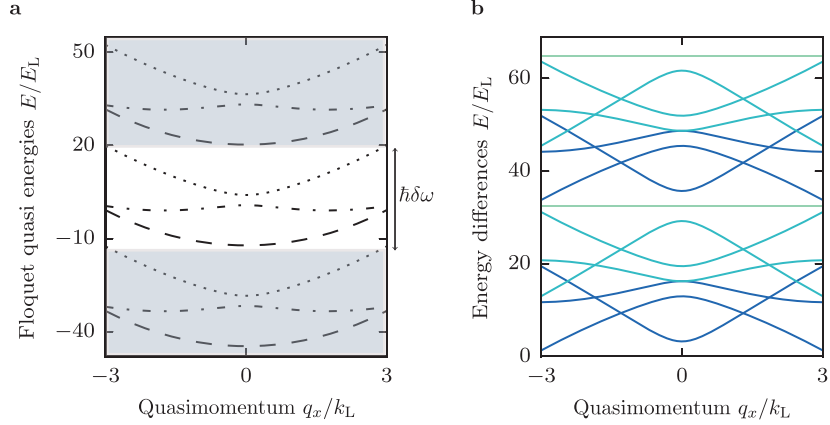


Figure 3: **a.** Floquet quasi-energies of a three level Hamiltonian with SOC and time periodic coupling strength. The quasi-energies are grouped into manifolds consisting of three levels that get repeated with a periodicity equal to  $\hbar\delta\omega$ . **b.** Energy differences of the Floquet quasi-energies. Each color represents the energy difference, separated by a fixed number of neighboring levels. When the number of neighboring levels is a multiple of 3, the energy differences are straight lines, a result of the periodic structure of the Floquet manifolds.

We found an effective time-independent Hamiltonian using a unitary transformation  $\hat{U}(t)$  and applying a RWA. The time evolution of the transformed wave function  $|\psi'\rangle = \hat{U}^\dagger |\psi\rangle$  was given by the time dependent Schrödinger equation for the Hamiltonian  $\hat{H}' = \hat{U}^\dagger \hat{H} \hat{U} - i\hbar \hat{U}^\dagger \partial_t \hat{U}$ . We used

$$\hat{U}(t) = \exp\left[-i \frac{\Omega}{\delta\omega} \sin(\delta\omega t) \hat{F}_x\right] \quad (5.5)$$

so that  $i\hbar \hat{U}^\dagger \partial_t \hat{U} = \hbar\Omega_R(t) \hat{F}_x$ . The transformed Hamiltonian  $\hat{H}'(t)$  had terms proportional to  $\sin(\Omega/\delta\omega \sin(\delta\omega t))$ ,  $\sin^2(\Omega/\delta\omega \sin(\delta\omega t))$ ,  $\cos(\Omega/\delta\omega \sin(\delta\omega t))$  and

$\cos^2(\Omega/\delta\omega \sin(\delta\omega t))$  which we simplified using the Jacobi-Anger expansion

$$\begin{aligned}\cos(z \sin \theta) &= J_0(z) + 2 \sum_{n=1}^{\infty} J_{2n}(z) \cos(2n\theta) \approx J_0(z) \\ \sin(z \sin \theta) &= 2 \sum_{n=0}^{\infty} J_{2n+1}(z) \sin((2n+1)\theta) \approx 0,\end{aligned}$$

where  $J_n$  is the the  $n$ th order Bessel function of the first kind and we neglected the ‘fast’ terms proportional to  $\cos(2n\theta)$  and  $\sin((2n+1)\theta)$ , essentially performing a RWA.

This approximation is valid for  $\hbar\delta\omega > |\epsilon| + 12E_L$  and  $|q_x| \leq 2k_L$  so that quasi-energy manifolds are well separated as in Figure 3a. The resulting Hamiltonian retained the form of Equation 5.4 with renormalized coefficients and an additional coupling term

$$\hat{H}_{Fl} = \hat{H}_{SOC}(q, \Omega_0, \tilde{\alpha}, \tilde{\delta}, \tilde{\epsilon}) + \tilde{\Omega} \hat{F}_{xz}, \quad (5.6)$$

where  $\tilde{\alpha} = J_0(\Omega/\delta\omega)\alpha$ ,  $\tilde{\Omega} = 1/4(\epsilon + 4E_L)[J_0(2\Omega/\delta\omega) - 1]$ ,  $\tilde{\delta} = J_0(\Omega/\delta\omega)\delta$ , and  $\tilde{\epsilon} = 1/4(4E_L - \epsilon) - 1/4(4E_L + 3\epsilon)J_0(2\Omega/\delta\omega)$ .  $\hat{F}_{xz}$  is the  $\hat{\lambda}_4$  Gell-Mann matrix that directly couples  $|m_F = -1, q = q_x + 2k_L\rangle$  and  $|m_F = +1, q = q_x - 2k_L\rangle$  states. The experimentally tunable parameters  $\delta\omega$ ,  $\Omega$  and  $\Omega_0$  can be used to tune the SOC dispersion.

### 5.2.3 Application of Fourier spectroscopy

We used Fourier transform spectroscopy to measure the spectrum of the SOC Hamiltonian (Equation 5.6) for three coupling regimes: (i)  $\Omega_0 \neq 0$  and  $\Omega = 0$ , (ii)  $\Omega_0 = 0$  and  $\Omega \neq 0$  and (iii)  $\Omega_0 \neq 0$  and  $\Omega \neq 0$ . We turned on the Raman laser non-adiabatically, in approximately  $1 \mu\text{s}$ . We let the system evolve subject to  $\hat{H}_{\text{SOC}}$  for up to  $900 \mu\text{s}$ , and then turned off the laser while releasing the atoms from the optical dipole trap. We resolved the spin and momentum distribution using a Stern-Gerlach gradient and a 21 ms TOF which allowed us to measure the fraction of atoms in each state of the measurement basis  $\{|m_F, q\rangle\}$ . We chose the density of sampling points and the maximum evolution time so that the bandwidth of the Fourier transform was comparable to, or larger than, the highest frequency in the evolution of the system while maximizing resolution. Experimental decoherence resulting in loss of contrast of the oscillations due to magnetic field noise and small magnetic field gradients present in our apparatus, was an additional constraint that became significant around 1 ms.

Working with a BEC with  $k = 0$  gave us access to only a single point in the dispersion relation. In order to map the full spin and momentum dependent dispersion relation of  $\hat{H}_{\text{SOC}}$ , we measured the time dependent occupation probabilities at a fixed Raman coupling strength and different values of Raman detuning  $\delta$  for the same initial state. The detuning corresponded to the Doppler shift experienced by atoms moving relative to a light source with quasimomentum  $q_x/k_L = \hbar\delta/4E_L$ . We controlled the frequency and the detuning of the Raman beams using two AOMs,

one of which was driven by up to three phase coherent frequencies (the carrier frequency plus two sidebands). For each of the three coupling cases that we measured, we applied the Raman beams at detuning values within the interval  $\pm 12E_L$  which corresponds to quasimomentum values  $\pm 3k_L$ .

This approach of changing detuning rather than using atoms with non-zero quasimomentum had the advantage that the state preparation was very reliable (making BECs at rest is easy<sup>1</sup>!) and we got very good signal to noise ratios due to the relatively high densities of the BECs. The downside was that if one is interested in looking at a large range of different momenta it can take a long time to repeat each experiment for different detuning values. In subsequent implementations of Fourier transform spectroscopy (Chapter 8 and [27]) we sacrificed some signal to noise ratio for speed and used the momentum distribution of non-condensed atoms to parallelize our measurements.

We mapped the band structure of SOC atoms for three different coupling regimes. Figure 4a shows representative traces of the measured occupation probabilities for short evolution times along with fits to the unitary evolution given by  $\hat{H}_{\text{SOC}}$  with  $\delta$ ,  $\Omega_0$ , and  $\Omega$  as free parameters. The fit parameters agree well with independent microwave and Raman power calibrations. In the lower two panels, where the Raman coupling strength was periodically modulated, the occupation probabilities oscillated with more than three frequencies since the full description of the system was given by a Floquet quasi-energy spectrum. Figure 4b,c shows the occupation probabilities for the parameter regime (iii) for longer evolution times

---

<sup>1</sup>Well, nothing in the lab is really ‘easy’...

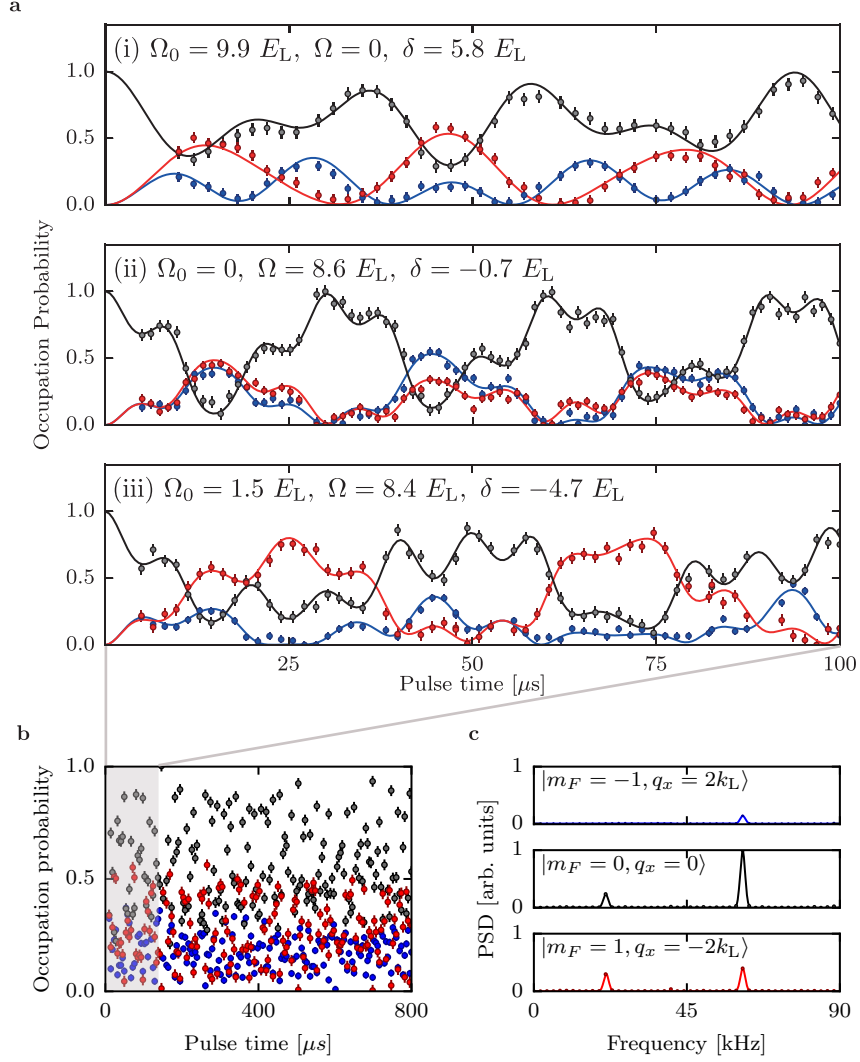


Figure 4: **a.** Occupation probability for the three states in the measurement basis  $|m_F = -1, q = q_x + 2k_L\rangle$  (blue),  $|m_F = 0, q = q_x\rangle$  (black), and  $|m_F = +1, q = q_x - 2k_L\rangle$  (red), following unitary evolution under  $\hat{H}_{\text{SOC}}$  for times up to  $100 \mu\text{s}$  at different spin-orbit coupling regimes: (i)  $\Omega_0 = 9.9 E_L, \Omega = 0, \delta = 5.8 E_L$ , (ii)  $\Omega_0 = 0, \Omega = 8.6 E_L, \delta = -0.7 E_L, \delta\omega = \epsilon + 12 E_L$ , and (iii)  $\Omega_0 = 1.5 E_L, \Omega = 8.4 E_L, \delta = -4.7 E_L, \delta\omega = \epsilon + 17 E_L$ . **b.** Occupation probability for long pulsing up to  $800 \mu\text{s}$  for parameters as in (iii). **c.** Power spectral density of the occupation probability. We subtract the mean value of each probability before taking the Fourier transform to remove peaks at  $f = 0$ . The peaks in the PSD then correspond to the relative eigenenergies of  $\hat{H}_{\text{SOC}}$ .

along with the PSD of the occupation probability of each spin state.

We used a non-uniform fast Fourier transform algorithm (NUFFT) on a square

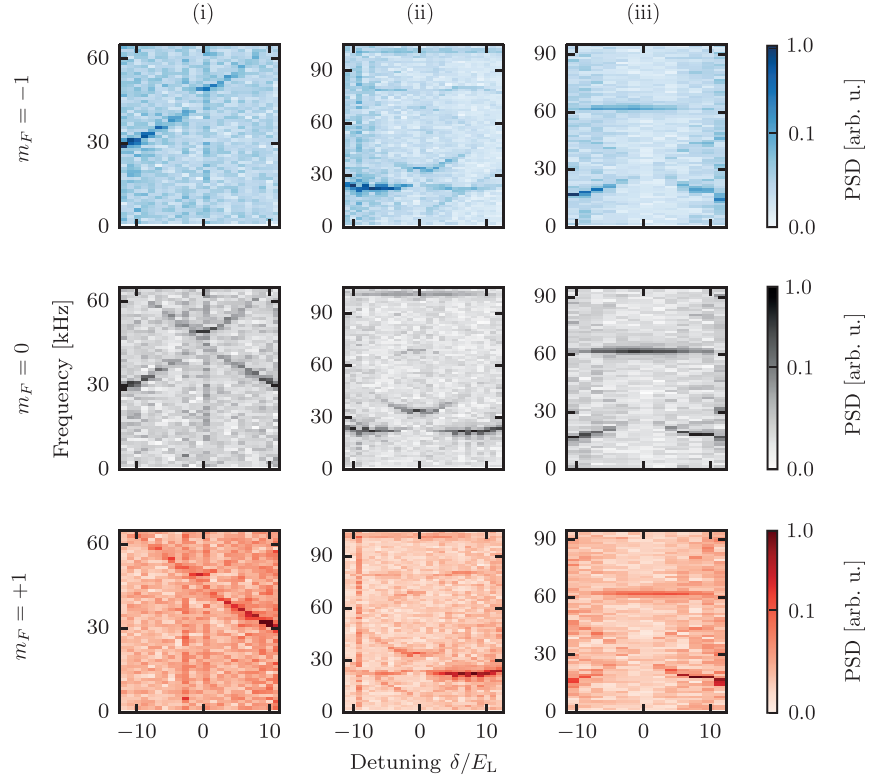


Figure 5: Power spectral density of the time dependent occupation probability for each state in the measurement basis for three coupling regimes: (Left)  $\Omega_0 = 9.9E_L$ ,  $\Omega = 0$ , (Center)  $\Omega_0 = 0$ ,  $\Omega = 8.6E_L$ ,  $\delta\omega = \epsilon + 12E_L$ , and (Right)  $\Omega_0 = 4.9E_L$ ,  $\Omega = 8.4E_L$ ,  $\delta\omega = \epsilon + 17E_L$ . Each panel is normalized to peak amplitude to highlight small amplitude features in the PSD of the periodically driven SOC, and the highest value on the frequency axis corresponds to the FFT bandwidth.

window to obtain the power spectral density of the occupation probabilities since our data points were not always evenly spaced because of imperfect imaging shots. The heights of the peaks in the PSD are related to the magnitude of the overlap integrals between the initial state and the Raman dressed states. Figure 4c shows the raw PSD of the time evolution of the system under  $\hat{H}_{\text{SOC}}$  for a given Raman coupling strength and detuning. We put together all the PSDs for the three coupling regimes in the spectra shown in Figure 5. Each column corresponds to a different



coupling regime and the colors represent the different spin states of the measurement basis. The spectra show that some overlap integrals vanish near  $\delta = 0$ , which is manifested as missing peaks in the PSD. The periodic structure of the Floquet quasi-energy spectrum gave rise to peaks at constant frequencies of  $\delta\omega$  and  $2\delta\omega$  independently of the Raman detuning, and a structure that is symmetric about the frequencies  $2\pi f_1 = \delta\omega/2$  and  $2\pi f_2 = \delta\omega$ . A reader is interested in seeing another nice experiment where the Floquet quasienergy spectrum becomes can be visualized is advised to see [75].

#### 5.2.4 Effective mass

Fourier transform spectroscopy only gives access to the relative energies of a Hamiltonian. If we want to recover the absolute energies we need to have an additional energy reference. The particular Hamiltonian  $\hat{H}_{SOC}$  had a ground state with a nearly quadratic dispersion. We measured its effective mass to obtain the ground state energy which we used as a reference to recover the absolute energies.

We measured the effective mass of the Raman dressed atoms by adiabatically preparing the BEC in the lowest eigenstate and inducing dipole oscillations [31]. The effective mass of the dressed atoms was related to the bare mass  $m$  and the bare and dressed trapping frequencies  $\omega$  and  $\omega^*$  along the Raman recoil direction by the ratio  $m^*/m = (\omega/\omega^*)^2$ . We measured this ratio following [19]. To induce the oscillations we started in  $|m_F = 0, k_x = 0\rangle$  state and adiabatically turned on the Raman laser in 10 ms while simultaneously ramping the detuning to  $\delta \approx 0.5 E_L$  which shifted the

minima in the ground state energy away from zero quasi-momentum. We suddenly brought the field back to resonance causing an abrupt change in the dispersion relation that excited the dipole mode of the BEC. To obtain the bare state  $\omega$  we only used the Raman beams to induce the oscillations and then subsequently turned off the beams while the BEC oscillated in the dipole trap and to obtain  $\omega^*$  the Raman beams were kept on the whole time. For this set of measurements, we adjusted our optical dipole trap to give new trapping frequencies  $(\omega_x, \omega_y, \omega_z)/2\pi = (35.6(4), 32.2(3), 133(3))$  Hz, nominally symmetric in the plane defined by  $\mathbf{e}_x$  and  $\mathbf{e}_y$ .

The Raman beams were co-propagating with the optical dipole trap beams; therefore, the primary axes of the dipole trap frequencies were at a  $45^\circ$  angle with respect to the direction of  $\mathbf{k}_L$ . The kinetic and potential terms in the Hamiltonian including the contribution of the Raman and optical dipole trap were

$$\begin{aligned}\hat{H}_\perp &= \frac{\hbar^2 q_x^2}{2m^*} + \frac{\hbar^2 q_y^2}{2m} + \frac{m}{2}[\omega_{x'}^2 x'^2 + \omega_{y'}^2 y'^2] \\ &= \frac{\hbar^2}{2m^*} k_x^2 + \frac{1}{2m} k_y^2 + \frac{m}{4}[(\omega_{x'}^2 + \omega_{y'}^2)(x^2 + y^2) + 2xy(\omega_{x'}^2 - \omega_{y'}^2)],\end{aligned}\quad (5.7)$$

where  $x' = (x + y)/\sqrt{2}$  and  $y' = (x - y)/\sqrt{2}$  are position coordinates rotated by  $45^\circ$ . For an axially symmetric trap with  $\omega_{x'} = \omega_{y'}$ , the frequency of oscillation along the Raman recoil direction is

$$\omega_x^2 = \frac{m}{2m^*}(\omega_{x'}^2 + \omega_{y'}^2). \quad (5.8)$$

Our trap had a small 3.4 Hz asymmetry and there was some coupling of the motion along the axis perpendicular to  $\mathbf{k}_L$  which becomes more significant at larger values of the effective mass. The sampling times for the measurements were small compared to the trap asymmetry and we can locally approximate the motion of the atoms by a simple harmonic function with a frequency along  $\mathbf{e}_x$  given by Equation 5.8.

Figure 6 shows the dipole oscillations along the  $\mathbf{e}_x$  and  $\mathbf{e}_y$  directions for the three different coupling regimes we explored, as well as the bare state motion. The resulting mass ratios for the three coupling regimes are  $m/m^* =$  (i) 1.04(8), (ii) 0.71(7), and (iii) 0.62(4).

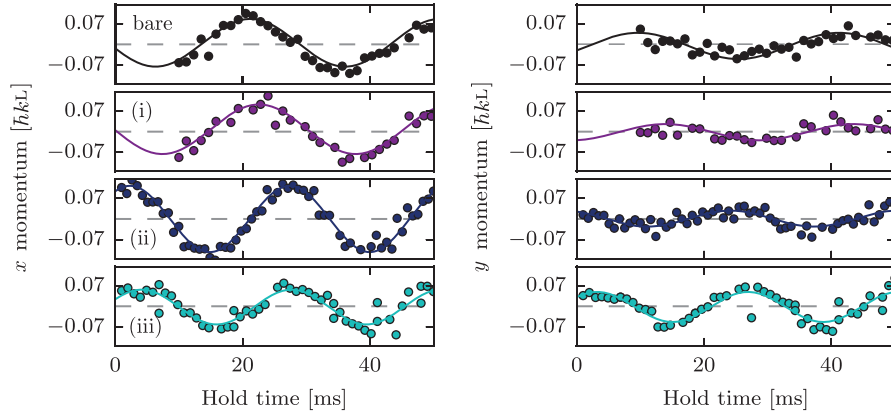


Figure 6: Oscillation of the BEC in the dipole trap along the recoil directions  $\mathbf{e}_x$  and  $\mathbf{e}_y$  for (top) bare atoms, and the three parameter regimes that we explored (i), (ii), and (iii). We believe that the observed low amplitude oscillations along  $\mathbf{e}_y$  are due to the initial detuning ramp not being fully adiabatic.

### 5.2.5 Measured dispersions

Figure 7 explains in detail the interpretation of the multiple peaks in the PSD and the steps that were taken to recover the dispersion relation using the effective mass and the Fourier spectra. The red line in Figure 7a represents a level within a Floquet manifold that has the largest overlap integral with the initial  $|m_F = 0, q = 0\rangle$  state. The peaks in the PSD correspond to energy differences between the marked level and the levels in neighboring Floquet manifolds pointed by the colored arrows. We show the theoretically computed energy differences on top of the measured PSD in panel b. The lowest frequency dominant peaks of the PSD correspond to energy differences with the adjacent lower Floquet manifold. To properly recover the SOC dispersion we shifted the PSD by a negative quadratic term  $-\hbar^2 q_x^2 / 2m^*$  as we show on panel c. We finally invert the frequency axis and shift it by  $\delta\omega$ .

We obtained the characteristic dispersion of a SOC system after adding a quadratic term to the PSD, proportional to the measured effective mass and rescaling the detuning into recoil momentum units. We combined the PSD of the time evolution of the three  $|m_F\rangle$  states to look at the spin dependence of the spectra. Figure 8 shows the measured dispersion relations as well as the Floquet quasi-energies calculated for the Hamiltonian parameters obtained from our calibrations. The spectral lines that can be resolved with our technique depend on the overlap integrals of the initial state with the target Hamiltonian eigenstates. Additional energies can be measured by repeating the experiment with different initial states. The spectral

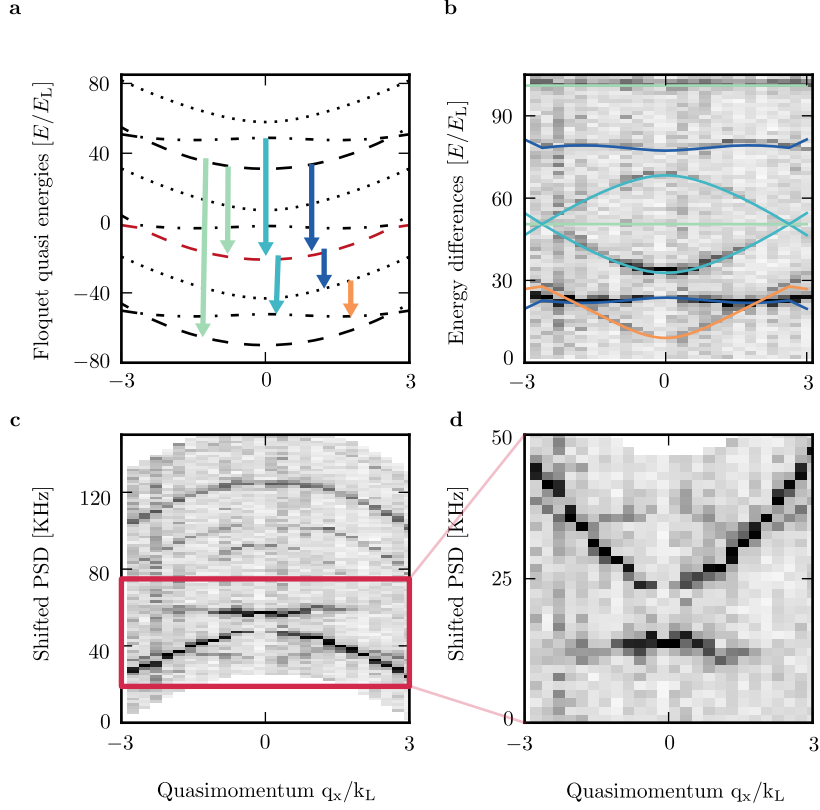


Figure 7: **a** Floquet quasi-energy spectrum of a SOC Hamiltonian with periodic coupling strength. The red line represents the eigenstate that has the largest overlap with the initial  $|m_F = 0\rangle$  state. The arrows indicate the energies of the states that have non-zero overlap with the initial state and can be measured with Fourier transform spectroscopy. **b** PSD of the occupation probability and numerically calculated energy differences between the levels indicated by the arrows on panel a. **c** PSD shifted by a quadratic term  $-\hbar^2 q_x^2 / 2m^*$ . The red box indicates the region of interest where we can recover the SOC spectrum. **d** We invert the frequency axis and shift it by  $\delta\omega$ .

lines we were able to resolve are in good agreement with the calculated energies of the Hamiltonian.

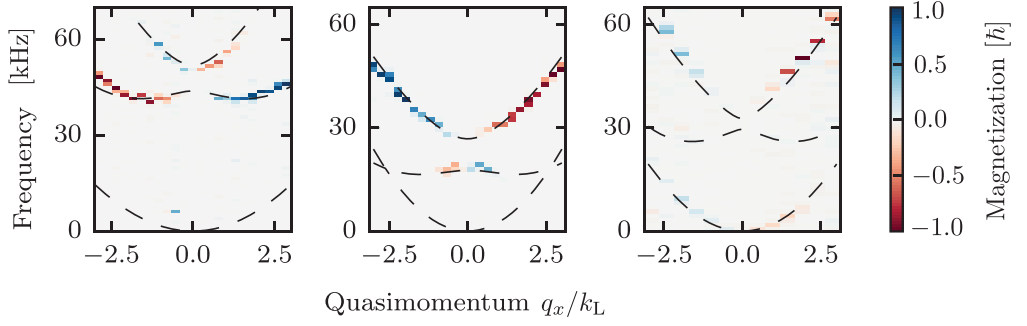


Figure 8: Spin-dependent SOC dispersion for three different coupling regimes. We combine the PSD of the occupation probability of the states  $|m_F = \pm 1, q_x = \mp 2k_L\rangle$ , and shift each frequency by an amount proportional to the squared quasimomentum and the effective mass. The dashed lines are the calculated Floquet energies for the Hamiltonian using our calibration parameters.

## Conclusion

This chapter introduced the basic principles of the Fourier transform spectroscopy technique and used it to measure the spin and momentum dependent dispersion relation of a spin-1 spin-orbit coupled BEC. We additionally studied a periodically driven SOC system and found a rich Floquet quasi-energy spectrum. Our method can be applied generically to any system with long enough coherent evolution to resolve the energy scales of interest and could prove particularly useful to study systems where it is harder to predict or compute the exact energies, such as cold atom realizations of disordered or highly correlated systems [76]. In our lab, this technique has been used to study the spectrum of a Rashba spin-orbit coupled system [26] and of a fractional period adiabatic superlattice [27].

Our main initial interest was to create tunable spin-orbit coupling and Fourier spectroscopy was conceived as a tool to characterize it. We realized that the use of

Raman transitions from multiple frequency beams was equivalent to another experiment that achieved tunable SOC using amplitude modulated Raman coupling [\[77\]](#).

## Chapter 6: Synthetic clock transitions through continuous dynamical decoupling

Most of the experimental techniques described in this thesis so far have used the hyperfine  $|m_F\rangle$  states as effective spins dressed with an RF or Raman field. However, due to the linear dependence of their energies with respect to magnetic field, and our lack of control of environmental changes we always had to take special care to stabilize the magnetic field in the laboratory (see Section 3.5.3). An alternative to doing active magnetic field stabilization is to use clock transitions which are first-order insensitive to changes in magnetic field but unfortunately they are not present in all systems or for arbitrary system parameters. However, under almost all circumstances, clock transitions can be synthesized using dynamical decoupling protocols. These protocols involve driving the system with an external oscillatory field, resulting in a dynamically protected ‘dressed’ system.

The idea of implementing continuous dynamical decoupling (CDD) came from a theoretical proposal to engineer Rashba type SOC (see Chapter 8) using Raman beams and a strong RF field [78], the second being a necessary ingredient for CDD. We initially worked in implementing CDD protocols to create ‘synthetic clock states’ as an intermediate step towards our final goal of engineering Rashba SOC. Just like



with Fourier spectroscopy, CDD became a workhorse of the laboratory both for the stability it provides against environmental fluctuations and because it has given us access to non-zero matrix coupling elements that we otherwise would not have when working with the bare  $|m_F\rangle$  states. We have continued to use CDD not only for engineering Rashba SOC (Chapter 8) but also to engineer subwavelength optical lattices [27] and Hofstadter [79] cylinders (work in preparation). On the theory side, we developed a proposal that uses them as a platform for emulating  $\mathcal{PT}$  symmetric Hamiltonians [80].

This Chapter discusses the implementation of CDD in the ground  $F = 1$  hyperfine manifold of ultracold  $^{87}\text{Rb}$ . First I give a general overview of dynamical decoupling and continuous dynamical decoupling. Then I describe the technical details and characterization of our CDD protocol which produces a protected three-level system of dressed-states whose Hamiltonian is fully controllable. Finally, I discuss an implementation of concatenated CDD that renders the system first-order insensitive to both magnetic field noise and noise in the control field. This work was published in [81] and was done in parallel with [82].

## 6.1 Basic principles of CDD

Dynamical decoupling (DD) protocols consist in applying an external control Hamiltonian, generally implemented by a series of pulses, which has the effect of canceling out the dynamics that arise from a quantum system coupling to the environment. DD was first introduced in the context of nuclear magnetic resonance

(NMR) with the discovery of spin-echoes [83], where a single ‘refocusing’ pulse was applied to eliminate dephasing of spins resulting from variations in magnetic field. These ideas were later generalized in [84] to protect a system from decoherence induced by interactions with a quantum environment. Continuous dynamical decoupling (CDD) relies on the application of time-periodic continuous control fields, rather than a series of pulses.

A number of dynamical decoupling protocols, pulsed or continuous, have been shown to isolate quantum systems from low-frequency environmental noise [85–93]. Thus far, CDD has inoculated multi-level systems in nitrogen vacancy centers in diamond, nuclear magnetic resonance experiments, and trapped atomic ions [94–101], from spatiotemporal magnetic field fluctuations.

## 6.2 CDD of a spin-1 system

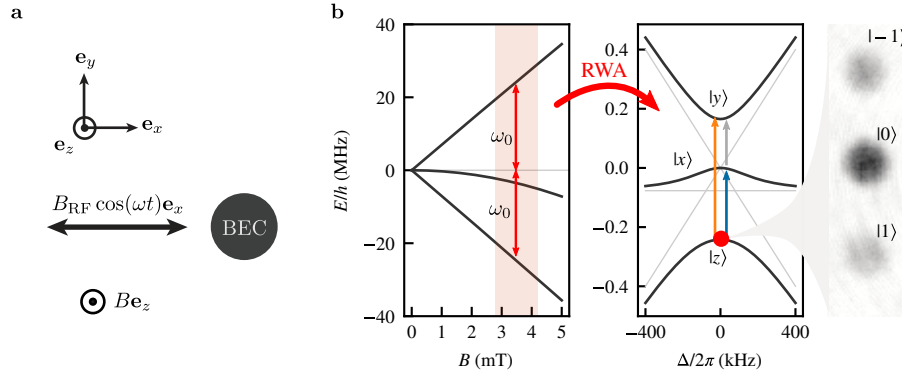


Figure 1: **a.** Setup for implementing CDD using a strong RF magnetic field. **b.** Left: dependence of the  $5^2S_{1/2}$ ,  $F = 1$  ground state of  $^{87}\text{Rb}$  on magnetic field, where the quadratic dependence of the  $|m_F = 0\rangle$  state’s Zeeman shift has been exaggerated so it is visible on the same scale. Center: energies of the  $|xyz\rangle$  eigenstates, for  $\Omega/2\pi = 200$  kHz (black curves) and  $\Omega = 0$  (grey curves). Right: TOF absorption image of  $|z\rangle$  at  $\Delta = 0$ , showing the constituent  $|m_F\rangle$  states.

We implemented CDD using a strong RF magnetic field with strength  $\Omega$  which linked the three  $|m_F\rangle$  states comprising the  $F = 1$  electronic ground state manifold of  $^{87}\text{Rb}$ . The RF field was linearly polarized along  $\mathbf{e}_x$ , and had angular frequency  $\omega$  close to the Larmor frequency  $\omega_0 = g_F\mu_B B_0$  from a magnetic field  $B_0\mathbf{e}_z$ ;  $g_F$  is the Lande  $g$ -factor and  $\mu_B$  is the Bohr magneton. Using the rotating wave approximation for the frame rotating at  $\omega$  (which is valid for  $\omega \gg \Omega$ ), the system was described by

$$\hat{H} = \hbar\Delta\hat{F}_z + \hbar\epsilon(\hat{F}_z^2 - \hat{\mathbb{1}}) + \hbar\Omega\hat{F}_x, \quad (6.1)$$

with detuning  $\Delta = \omega - \omega_0$ , quadratic Zeeman shift  $\epsilon$ , spin-1 angular momentum operators  $\hat{F}_{x,y,z}$ , and the identity operator  $\hat{\mathbb{1}}$ .

### 6.3 The $|xyz\rangle$ states

The eigenstates of Equation 6.1 correspond to the CDD basis, and we denote them the  $|x\rangle$ ,  $|y\rangle$  and  $|z\rangle$  states.

### 6.3.1 State decomposition

The CDD states are linear combinations of the  $|m_F\rangle$  basis states, and for  $\Delta = 0$  the (non-normalized) eigenvectors are:

$$\begin{aligned} |x\rangle &= |-1\rangle - |1\rangle, \\ |y\rangle &= |-1\rangle - \frac{\epsilon + \tilde{\Omega}}{\sqrt{2}\Omega} |0\rangle + |1\rangle, \\ |z\rangle &= |-1\rangle - \frac{\epsilon - \tilde{\Omega}}{\sqrt{2}\Omega} |0\rangle + |1\rangle. \end{aligned} \quad (6.2)$$

Figure 2 shows the normalized full state decomposition as a function of  $\Delta$ . The  $|xyz\rangle$  states adiabatically map to the  $|m_F\rangle$  states for  $|\Delta| \gg \Omega$ : for positive (negative) detuning  $|z\rangle$  maps to  $|1\rangle$  ( $|-1\rangle$ );  $|y\rangle$  maps in the exact opposite way to  $|z\rangle$ ; and  $|x\rangle$  always maps to  $|0\rangle$ .

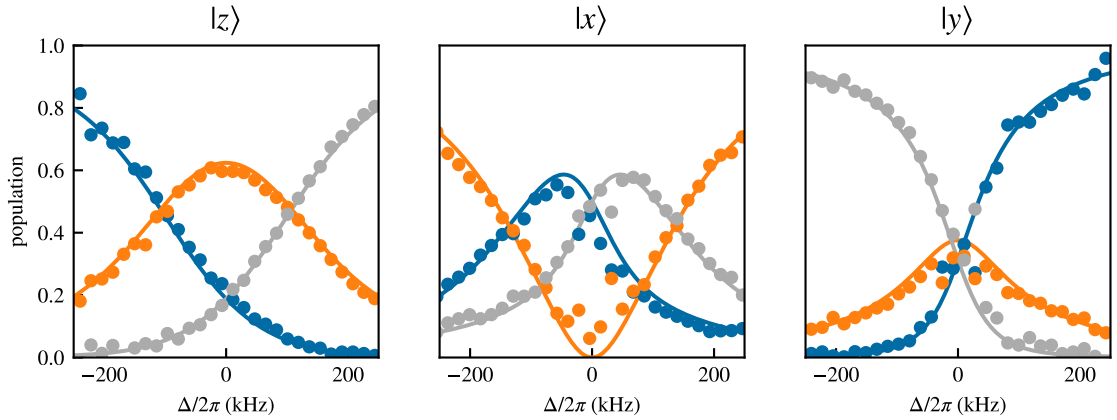


Figure 2: Decomposition of the  $|xyz\rangle$  states on the  $|m_F\rangle$  basis for  $\Omega/2\pi = 145(1)$  kHz. We adiabatically prepared the system in any of the  $|xyz\rangle$  states and then suddenly turned off the RF dressing field. The  $|m_F = -1, 0, 1\rangle$  states correspond to blue, orange, gray respectively.

We labeled our dressed states  $|xyz\rangle$  since for  $\Omega \rightarrow 0^+$  and  $\Delta = 0$ , they continuously approach the  $|XYZ\rangle$  states familiar from quantum chemistry [21]:

$$\begin{aligned} |X\rangle &= \frac{|1\rangle - |-1\rangle}{\sqrt{2}}, \\ |Y\rangle &= i\frac{|1\rangle + |-1\rangle}{\sqrt{2}}, \\ |Z\rangle &= |0\rangle. \end{aligned} \tag{6.3}$$

which transform under the application of the spin-1 operators as  $\epsilon_{jkl}\hat{F}_j|k\rangle = i\hbar|l\rangle$ , so that a resonant probe field can induce transitions between at least one pair of states, irrespectively of its polarization.

Finally, when  $\Omega \rightarrow \infty$  they are independent of the driving field amplitude and continuously approach the eigenstates of the  $\hat{F}_x$  operator

$$\begin{aligned} |x\rangle &= |1\rangle - |-1\rangle, \\ |y\rangle &= |1\rangle + \sqrt{2}|0\rangle + |-1\rangle, \\ |z\rangle &= |1\rangle - \sqrt{2}|0\rangle + |-1\rangle. \end{aligned} \tag{6.4}$$

### 6.3.2 Energies

The clock-like nature of these states is determined by their eigenvalues which are even functions with respect to  $\Delta$  as can be seen by the leading order expansion

of the eigenenergies  $E_i = \hbar\omega_i$  for  $\Delta \rightarrow 0$

$$\begin{aligned}\omega_x &= -\frac{\epsilon}{\Omega^2}\Delta^2 + \mathcal{O}(\Delta^4), \\ \omega_y &= \frac{1}{2}(-\epsilon + \tilde{\Omega}) - \frac{(\epsilon + \tilde{\Omega})}{-\epsilon^2 - 4\Omega^2 + \epsilon\tilde{\Omega}}\Delta^2 + \mathcal{O}(\Delta^4), \\ \omega_z &= \frac{1}{2}(-\epsilon - \tilde{\Omega}) + \frac{(\epsilon - \tilde{\Omega})}{\epsilon^2 + 4\Omega^2 + \epsilon\tilde{\Omega}}\Delta^2,\end{aligned}\tag{6.5}$$

where we have defined  $\tilde{\Omega} = \sqrt{4\Omega^2 + \epsilon^2}$ . The energy differences  $\hbar\omega_{xy}$ ,  $\hbar\omega_{zy}$  and  $\hbar\omega_{zx}$  are only quadratically sensitive to  $\Delta$  for  $\Delta \ll \Omega$ <sup>1</sup> so that detuning fluctuations  $\delta\Delta$  are suppressed to first order, making these a trio of synthetic clock states. The curvatures of  $\omega_x$  and  $\omega_z$  have the same sign and in principle there is a critical value of  $\Omega$  where the quadratic term in transition energy can be made arbitrarily small, making it quartic in  $\Delta$ . However, this cancellation does not take place when we consider the dependence of  $\epsilon$  on  $\Delta$  using the Breit-Rabi expression. It is still possible to find an optimal  $\Omega$  for which  $\omega_{zx}$  depends quartically on  $\Delta$ , but it does not occur at  $\Delta = 0$  as is predicted by Equation 6.5 for constant  $\epsilon$ .

### 6.3.3 Transition matrix elements

Unlike the  $|m_F\rangle$  basis, an oscillatory magnetic field with the right polarization can drive transitions between all pairs of the  $|xyz\rangle$  states with non-zero transition matrix elements. The transition matrix elements between the  $|xyz\rangle$  have a dependence on both  $\Omega$  and  $\Delta$ . For the  $\Delta = 0$  case they can be read from the representation of the spin-1 matrices in the  $|xyz\rangle$  basis

---

<sup>1</sup>The energies are quadratic in  $\Delta$  for  $\Delta \ll \Omega$ , and linear for  $\Delta \gg \Omega$  with a slope of 7 MHz/mT.

$$\begin{aligned}
\hat{F}_x &\rightarrow \begin{pmatrix} \frac{2\Omega}{\Omega} & 0 & -\frac{\epsilon}{\Omega} \\ 0 & 0 & 0 \\ -\frac{\epsilon}{\Omega} & 0 & -\frac{2\Omega}{\Omega} \end{pmatrix} \\
\hat{F}_y &\rightarrow \begin{pmatrix} 0 & -\frac{i(\tilde{\Omega}-\epsilon)}{\Omega\sqrt{\frac{(\epsilon-\tilde{\Omega})^2}{\Omega^2}+4}} & 0 \\ \frac{i(\tilde{\Omega}-\epsilon)}{\Omega\sqrt{\frac{(\epsilon-\tilde{\Omega})^2}{\Omega^2}+4}} & 0 & -\frac{i(\tilde{\Omega}+\epsilon)}{\Omega\sqrt{\frac{(\tilde{\Omega}+\epsilon)^2}{\Omega^2}+4}} \\ 0 & \frac{i(\tilde{\Omega}+\epsilon)}{\Omega\sqrt{\frac{(\tilde{\Omega}+\epsilon)^2}{\Omega^2}+4}} & 0 \end{pmatrix} \\
\hat{F}_z &\rightarrow \begin{pmatrix} 0 & -\frac{\sqrt{\frac{\epsilon}{\Omega}+1}}{\sqrt{2}} & 0 \\ -\frac{\sqrt{\frac{\epsilon}{\Omega}+1}}{\sqrt{2}} & 0 & -\frac{2}{\sqrt{\frac{(\tilde{\Omega}+\epsilon)^2}{\Omega^2}+4}} \\ 0 & -\frac{2}{\sqrt{\frac{(\tilde{\Omega}+\epsilon)^2}{\Omega^2}+4}} & 0 \end{pmatrix}, \tag{6.6}
\end{aligned}$$

where the states have been ordered by decreasing energy ( $|y\rangle$ ,  $|x\rangle$ ,  $|z\rangle$ ). We see that a term in a Hamiltonian that is proportional to  $\hat{F}_x$  can only drive transitions between  $|z\rangle$  and  $|y\rangle$  and that a coupling term proportional to  $\hat{F}_y$  and  $\hat{F}_z$  can drive both drive transitions between  $|z\rangle$  and  $|x\rangle$  or  $|x\rangle$  and  $|y\rangle$  with different strengths. The matrices in Equation 6.6 show that when  $\Omega$  and  $\epsilon$  are comparable in magnitude there exists at least one non-zero transition matrix element for each pair of dressed states and they can all be coupled cyclically.

## 6.4 $|xyz\rangle$ state preparation

We implemented CDD to BECs with  $N \approx 5 \times 10^4$  atoms in the  $F = 1$  ground hyperfine manifold of  $^{87}\text{Rb}$ . For all of the experiments described in this Chapter the dipole trap had trapping frequencies of  $(f_x, f_y, f_z) = (42(3), 34(2), 133(3))$  Hz. We applied a  $B_0 \approx 3.27$  mT bias field that lifted the ground state degeneracy, giving an  $\omega_0/2\pi = 22.9$  MHz Larmor frequency, with a quadratic shift  $\epsilon/2\pi = 76.4$  kHz. We determined that the ambient magnetic field fluctuations were dominated by contributions from line noise giving an RMS uncertainty  $\delta\Delta/2\pi = g_F\mu_B\delta B/h = 0.67(3)$  kHz.

The state preparation consisted of two stages of ARP. On the first stage we followed the usual protocol described in Section 3.5.2 to prepare the BEC in any of the  $|m_F = 0, -1, 1\rangle$  states. On the second stage, we adiabatically transformed the  $|m_F\rangle$  states into the  $|xyz\rangle$  states. We started with the bias field far from resonance ( $\Delta(t = 0)/2\pi \approx -450$  kHz) and with all coupling fields off. Then we ramped on  $\Omega$  in a two-step process. We first ramped from  $\Omega = 0$  to an intermediate value  $\Omega_{\text{mid}}$ , approximately half its final value in 1 ms. We then ramped  $\Delta$  to zero in 3 ms by increasing the magnetic field  $B_0$ . After allowing  $B_0$  to stabilize for 30 ms, we ramped the RF dressing field to its final value  $\Omega$  in 1 ms, yielding the dynamically decoupled  $|xyz\rangle$  states. It was important that we waited for the field to stabilize at an intermediate  $\Omega_{\text{mid}}$  as we found several times that the capacitors on the impedance matching network of the antenna used to generate the RF field would burn if we kept the power on for too long.



After performing any experiment with the  $|xyz\rangle$  states we measured their populations by adiabatically deloading them back into the  $|m_F\rangle$  basis. We first ramped  $B_0$  so that  $\Delta$  approached its initial detuned value in 2 ms, and then ramped off the dressing RF field in 1 ms. A typical experimental sequence for  $\Delta$  and  $\Omega$  can be visualized in Figure 3. As usual, we obtained the spin-resolved momentum distribution using absorption imaging after TOF, with a Stern-Gerlach field to spatially separate the spin components. The right panel of Figure 1b shows a TOF image of the  $|m_F\rangle$  state decomposition of the  $|z\rangle$  state. For this image as well as for the measurement of the dressed state decomposition shown in Figure 2 we suddenly (not-adiabatically) turned the RF coupling off, thereby projecting the  $|xyz\rangle$  states back into the  $|m_F\rangle$  basis.

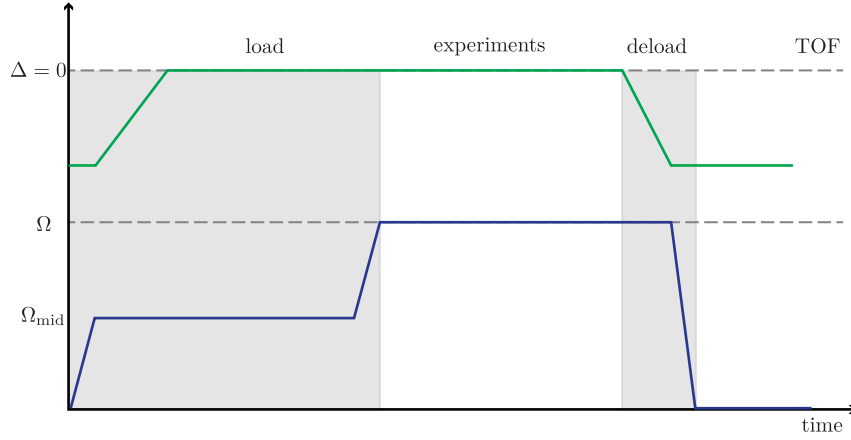


Figure 3: Detuning and RF coupling strengths ramps (not to scale) performed to adiabatically prepare the  $|xyz\rangle$  states starting in the  $|m_F\rangle$  states and vice versa.

## 6.5 Initial characterization of $\Omega$

Producing RF fields with large coupling strength was not a trivial task and when testing different antenna designs it was important to have an easy and quick way of characterizing them. We mostly relied on two different techniques to get an initial estimate of  $\Omega$ : first, we prepared atoms in  $|m_F = -1\rangle$  and pulse on the RF to drive transitions between the three  $|m_F\rangle$  states. We fit the populations in the three states as a function of pulsing time to the time evolution given the time dependent Schödinger equation for the RF Hamiltonian (Equation 6.1) using a least squares minimizer with  $\Omega$  and  $\Delta$  as free parameters.

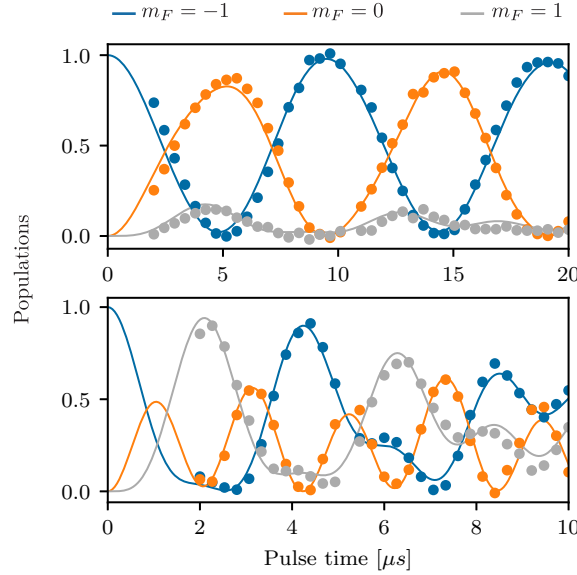


Figure 4: We prepared the system in the  $|m_F = -1\rangle$  state and pulsed  $\Omega$  for variable times. We used a least squares minimizer to fit the populations in the  $|m_F\rangle$  states as a function of pulsing time to get an initial estimate of  $\Omega$ . The top panel shows the time evolution of  $\Omega/2\pi \approx 76$  kHz and the bottom panel shows the evolution for  $\Omega/2\pi \approx 238$  kHz

Alternatively, we followed the loading procedure described in Section 6.4 but

suddenly turned  $\Omega$  off for different values of  $\Delta$  to get the decomposition of the  $|xyz\rangle$  states in terms of the  $|m_F\rangle$  states. We then fit the populations to the eigenstates of the Equation 6.1 with  $\Omega$  and  $\Delta$  as free parameters. Figure 2 is an example of such type of calibration.

For an antenna with a quality factor such as ours ( $q \sim 20$ ) we could not ‘suddenly’ turn  $\Omega$  on or off as it takes some time for power to build up and to die out when the RF fields are turned on or off<sup>2</sup>. If we did not include this into the model used to calibrate  $\Omega$  we could get some results that were slightly off. We only used these measurements as initial estimates and once we found an antenna design that could produce a large enough  $\Omega$  we used the spectroscopy techniques described in the next section to fully characterize the system.

## 6.6 Spectroscopy

We confirmed our control and measurement techniques spectroscopically by measuring the energy differences between the  $|xyz\rangle$  states with an additional probing field with angular frequency  $\omega + \omega_p$ , coupling strength  $\Omega_p$  and polarized along  $\mathbf{e}_y$ . In the frame rotating with angular frequency  $\omega$  and after using a RWA the system was described by the Hamiltonian

$$\begin{aligned} \hat{H} = & \Delta \hat{F}_z + \hbar \epsilon (\hat{F}_z^2 / \hbar^2 - \hat{\mathbb{I}}) + \Omega \hat{F}_x \\ & + \Omega_p \left( \sin(\omega_p t) \hat{F}_x + \cos(\omega_p t) \hat{F}_y \right). \end{aligned} \quad (6.7)$$

---

<sup>2</sup>It is possible to get the desired response by using a different function for the turn on/off that overshoots (not just a step).

In this rotating frame the probe field initially polarized along  $\mathbf{e}_y$  had components along  $\mathbf{e}_x$  and  $\mathbf{e}_y$ , resulting in at least one non-zero transition matrix element for all transitions between pairs of dressed states. If the probing field was polarized along  $\mathbf{e}_z$  we would not be able to drive the  $zy$  transition as can be seen from the matrix elements in Equation 6.6.

To probe the dependence of the  $|xyz\rangle$  state energies on detuning, we performed Rabi spectroscopy (Section 3.5.1) by pulsing  $\Omega_p$  on for a constant time and scanned  $\omega_p$  for different values of  $\Delta$ . Figure 1b shows the spectroscopically resolved values of  $\omega_{xy}/2\pi$ ,  $\omega_{yz}/2\pi$ , and  $\omega_{zx}/2\pi$  for  $\Omega/2\pi = 194.5(1)$  kHz and the side panel shows a sample spectra measured with coupling strength  $\Omega_p/2\pi \approx 1$  kHz and  $\Delta/2\pi \approx 9$  kHz. The dashed curves were computed by diagonalizing Equation 6.1, and they clearly depart from our measurements for the  $zx$  transition. This departure results from neglecting the weak dependence of the quadratic shift  $\epsilon$  on bias field  $B_0$ . In near-perfect agreement with experiment, the solid curves from the full Breit-Rabi expression account for this dependency.

## 6.7 Robustness

We focused on the robustness of the  $zx$  transition which can be made virtually independent of magnetic field variations due to the similar curvature of  $\omega_z(\Delta)$  and  $\omega_x(\Delta)$  (see the middle panel of Figure 1b). We quantified the sensitivity of this transition to field variations with three methods corresponding to the different markers in Figure 6b: (1) Triangles denote data using Rabi spectroscopy as in Figure 6a.

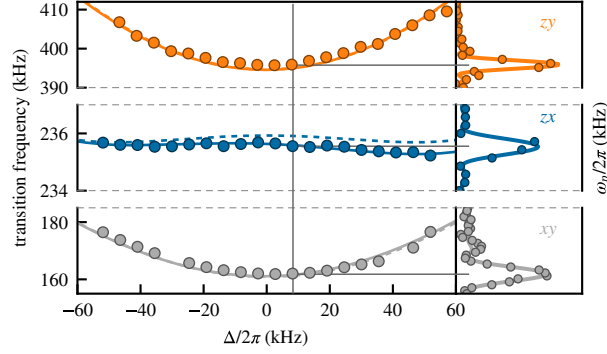


Figure 5: Spectroscopic data showing transitions between the  $|xyz\rangle$  states for  $\Omega/2\pi = 194.5(1)$  kHz. The vertical scale of the center panel ( $zx$  transition) has only 10% the range of the other panels. The dashed lines correspond to the Hamiltonian of Equation 6.1 while the solid lines include the dependence of the quadratic shift on  $\Delta$ . Right: representative spectra.

(2) Squares denote data in which a detuned  $\pi$ -pulse of the probe field transferred approximately half of the atoms from  $|z\rangle$  to  $|x\rangle$ . This ‘side-of-peak’ technique overcomes the limitation of Rabi spectroscopy being first-order insensitive to changes in  $\omega_{zx}$ . (3) Circles describe data using a double dressing technique that will be described in Section 6.9. In each case we measured the energy shift from resonance as a function of detuning (magnetic field) and then used a fourth-order polynomial fit to extract the RMS residuals  $\delta\omega_{zx}$  due to the known detuning noise<sup>3</sup>. The results are not consistent with the simple theory from Equation 6.7 (dashed) and instead require the Breit-Rabi expression (solid) to obtain full agreement<sup>4</sup>.

Even at our smallest coupling  $\Omega/2\pi = 69(1)$  kHz the typical magnetic field noise was attenuated by two orders of magnitude, rendering it essentially undetectable. Ideally, the radius of curvature of  $\omega_{zx}(\Delta)$  changes sign at about

<sup>3</sup>Our procedure also quantifies the small fluctuations that survive for spectra that are flat beyond second order, as in Equation 6.1.

<sup>4</sup>The fluctuations can be even smaller for a given  $\Omega$  if we allow for  $\Delta \neq 0$ .

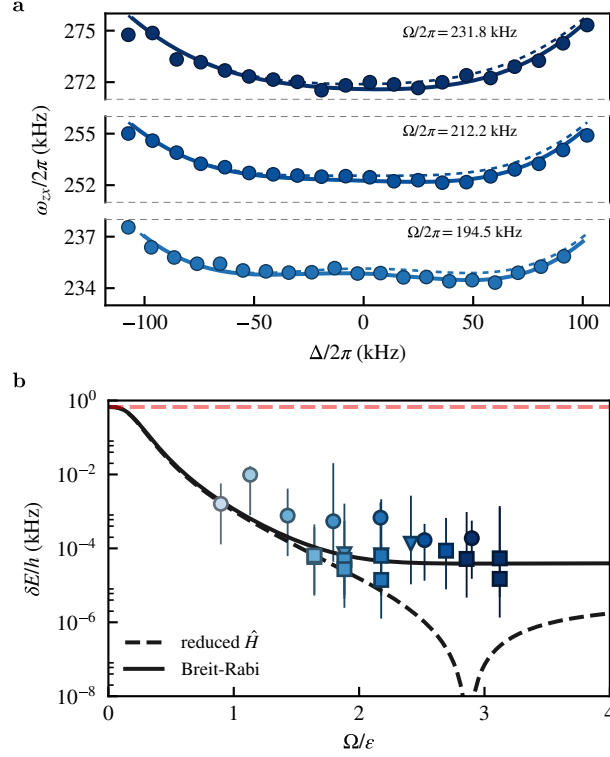


Figure 6: **a.** Transition frequency  $\omega_{zx}/2\pi$  for three values of  $\Omega/2\pi$ . The dashed curves correspond to Equation 6.7, while the solid curves use the Breit-Rabi expression. **b.** The change in energy from our experimental detuning fluctuations as measured in the  $|m_F\rangle$  basis is  $\delta\Delta/2\pi = 0.67$  kHz (red dashed line). Triangles correspond to  $|xyz\rangle$  spectroscopy data, squares to side-of-peak  $\pi$ -pulse data, and circles to double-dressed data. The black dashed (solid) curve was calculated using Equation 6.7 (the Breit-Rabi expression). The shading of the data points corresponds to the Rabi frequencies in Figure 8.

$\Omega/2\pi = 220$  kHz, leaving only a  $\Delta^4$  contribution, however, in practice the small dependence of  $\epsilon$  on  $B$  prevents this perfect cancellation. Still it is possible to see the changing curvature of  $\omega_{zx}(\Delta)$  near  $\Delta = 0$  for different values of  $\Omega$  in Figure 6a.

### 6.7.1 Optimal response to noise

The sensitivity of the  $zx$  transition to detuning fluctuations can be optimized further by working at  $\Delta \neq 0$  as shown in Figure 7.

For small values of  $\Omega$ , the optimum value of  $\Delta$  corresponds to one of the concave features of the  $zx$  transition energy that arises due to the asymmetry introduced by the quadratic shift. As  $\Omega$  gets larger, these features merge into a single one and the optimum value is  $\Delta \approx 0$ . The deviation from  $\Delta = 0$  is due to an overall tilt of the transition energy coming from the dependence of the quadratic shift on  $\Delta$ . At the optimum point  $\Omega/\epsilon \approx 3$  the sensitivity of the synthetic clock transition is  $1.9 \times 10^{-7}$  kHz, c.f, the  $^{87}\text{Rb}$  clock transition which scales as  $57.5 \text{ kHz/mT}^2$  and gives  $5.8 \times 10^{-7}$  kHz.

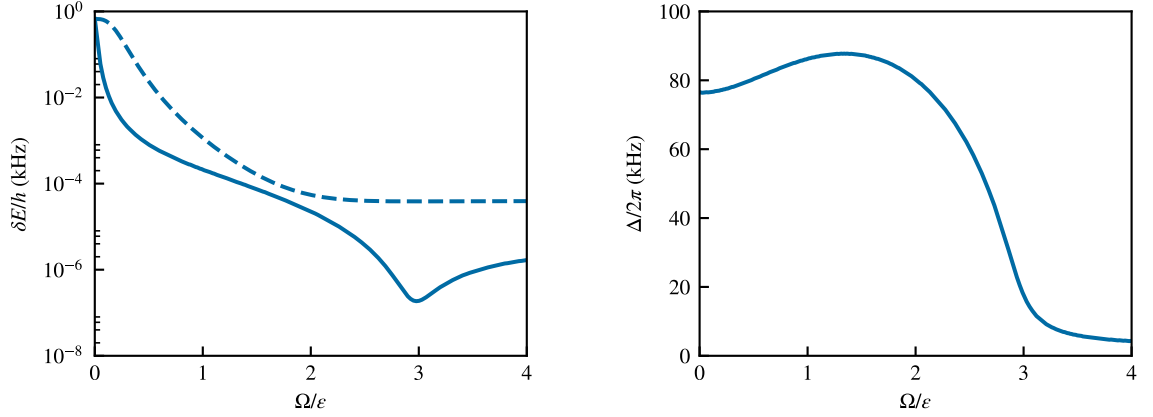


Figure 7: Left: Optimum response (solid) of the  $zx$  transition to detuning fluctuations allowing for finite  $\Delta$  compared to  $\Delta = 0$  (dashed) for the full Breit-Rabi model (note the logarithmic vertical scale). Right: The values of  $\Delta$  that correspond to the minimum derivative of  $\omega_{zx}$ .

## 6.8 Driving dressed state transitions

We explored the strength of the probe-driven transitions between these states by observing coherent Rabi oscillations (Figure 8a) where our BEC was prepared in  $|z\rangle$  and the probe field had strength  $\Omega_p/2\pi \approx 1 \text{ kHz}$ . The top panel shows

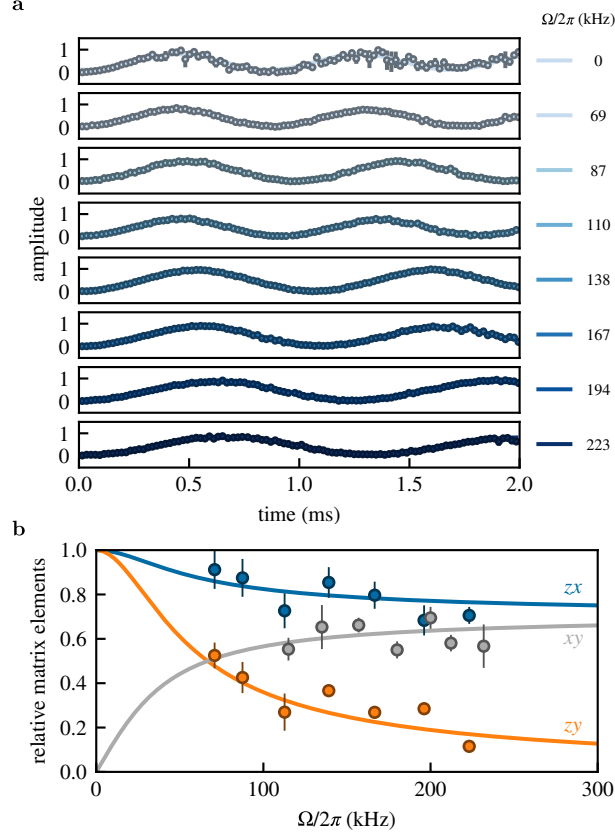


Figure 8: **a.** Rabi oscillations. Phase coherence is maintained throughout the oscillations in the dressed basis, while it is quickly lost in the  $|m_F\rangle$  basis. The marker size reflects the typical uncertainties on the dressed basis oscillations. **b.** Relative transition matrix elements measured by driving Rabi oscillations between all the  $|xyz\rangle$  states at constant probe power for different values of  $\Omega$ . The  $zx$  (blue) and  $zy$  (orange) transitions decrease monotonically with increasing  $\Omega$  for  $\Delta = 0$ , while they increase for  $xy$ .

Rabi oscillations between  $|m_F = 0\rangle$  and  $|m_F = -1\rangle$  states for reference, and the remaining panels show oscillations between  $|z\rangle$  and  $|x\rangle$ . The observed Rabi frequency between dressed states decreased with increasing  $\Omega$  indicating a dependence of the  $zx$  transition matrix elements on  $\Omega$ . We repeated this experiment driving all possible pairs of dressed state transitions at fixed  $\Omega_p$  for, and Figure 8b shows the dependence of these matrix elements on  $\Omega$  for  $\Delta = 0$ .

The coherence of the Rabi oscillations for longer times was limited by gradients



in  $\Omega$  that lead to phase separation of the dressed states, and therefore loss of contrast in the oscillations. This effect was faster for smaller frequency Rabi oscillations. For example for  $\Omega_p/2\pi = 5$  kHz we observed coherent Rabi oscillations with almost full contrast for more than 10 ms while for the  $\Omega_p/2\pi = 870$  Hz oscillation shown in Figure 9 the contrast was significantly reduced after 5 ms. The loss of contrast was even worse when we tried performing a Ramsey sequence where the time evolution was most sensitive to the environment. One solution to this problem would be to change the experimental setup to a double loop antenna to generate a more spatially uniform magnetic field.

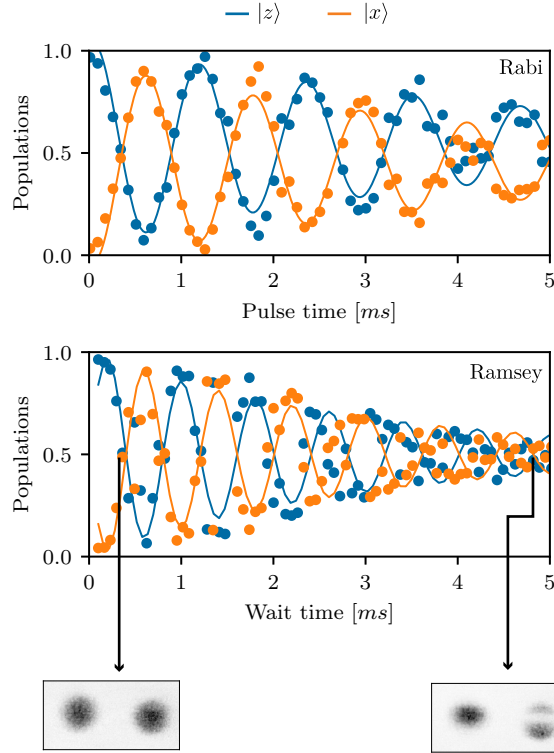


Figure 9: Loss of contrast in coherent oscillations. A Rabi oscillation (top) between the  $|z\rangle$  and  $|x\rangle$  states with  $\Omega_p/2\pi = 870$  Hz decays by  $1/e$  in 4.6 ms and a Ramsey oscillation (middle) with about 1 kHz frequency decays in about 3 ms. The gradients in  $\Omega$  lead to phase separation of dressed states and loss of contrast for longer pulse/wait times.

In comparison, we found that for both Rabi and Ramsey oscillations between the  $|m_F\rangle$  states the phase started deteriorating after a few hundreds of  $\mu s$ , this is not surprising due to bias magnetic field temporal noise. We canceled gradient magnetic fields so that no phase separation of the bare states was observed for  $> 10$  sec. As a result, the system can in principle undergo coherent evolution without loss of contrast for a long time but because of field fluctuations between shots what we observed instead was full contrast noise.

## 6.9 Concatenated CDD

The driving field  $\Omega$  coupled together the  $|m_F\rangle$  states, giving us the  $|xyz\rangle$  synthetic clock states that were first-order insensitive to magnetic field fluctuations. However, the spectrum of these states is still first-order sensitive to fluctuations of the driving field  $\delta\Omega$ . Reference [89] showed that an additional field coupling together with these  $|xyz\rangle$  states can produce doubly-dressed states that are insensitive to both  $\delta\Omega$  and  $\delta\Delta$ : a process called concatenated CDD (CCDD). In our experiment, the probe field provided the concatenating coupling field. Because  $\Omega_p \ll \Omega$ , we focused on a near-resonant two-level system formed by a single pair of dressed states, here  $|z\rangle$  and  $|x\rangle$ , which we considered as pseudospins  $|\uparrow\rangle$  and  $|\downarrow\rangle$ . These states were described by the effective two-level Hamiltonian

$$\hat{H}_p = \frac{\hbar\Delta'}{2}\hat{\sigma}_3 + \hbar\Omega'\cos(\omega_pt)\hat{\sigma}_1, \quad (6.8)$$

with energy gap  $\Delta' \approx \omega_{z,x}$  (shifted by off-resonant coupling to the  $zy$  and  $xy$  transitions) and coupling strength  $\Omega' \propto \Omega_p$ , as set by the matrix elements displayed in Figure 8b. Here  $\hat{\sigma}_{1,2,3}$  are the three Pauli operators.

We performed a second transformation into a frame rotating with angular frequency  $\omega_p$  and use a RWA to compute the eigenenergies of Equation 6.8. For large values of  $\Omega'$  the energies take the values  $E_{\uparrow,\downarrow} \approx \pm\Omega'/2 + (\Delta')^2/2\Omega'$ . Even though  $E_{\uparrow,\downarrow}$  are still first order sensitive to  $\Omega$  because  $\Delta' \approx \omega_{z,x} \propto \Omega$ , its effect is suppressed by a factor of  $1/\Omega'$ . Thus, the concatenated CDD field protects from the fluctuations  $\delta\Delta'$  of the first dressing field in a similar way that CDD provided protection from detuning noise  $\delta\Delta$ . Table 6.1 summarizes the dependence of the  $|xyz\rangle$  and  $|\uparrow\downarrow\rangle$  energies on  $\Delta$ ,  $\Omega$  and  $\Omega'$ .

Table 6.1: Energies of the CDD and CCDD states as a function of  $\Delta$ ,  $\Omega$  and  $\Omega'$ . The dependence on parameters not relevant to the expansion is given by the functions  $f_1$ ,  $f_2$ ,  $g_1$  and  $g_2$ .

	CDD	concatenated CDD
$\Delta$ dependence	$f_1(\epsilon, \Omega)\Delta^2$	$f_2(\Omega, \epsilon)\frac{\Delta^2}{\Omega'}$
$\Omega, \Omega'$ dependence	$\Omega + g_1(\Delta, \epsilon)\frac{1}{\Omega}$	$\left[\Omega^2 + \epsilon\Omega + g_2(\Delta, \epsilon)\frac{1}{\Omega}\right]\frac{1}{\Omega'}$

We produced doubly-dressed states by doing (one more!) ARP sequence. We initialized the system in the  $|\downarrow\rangle$  state with RF coupling strength  $\Omega_i$ . We set the probe frequency to be  $\sim 20$  kHz off resonant with respect to the  $|\downarrow\rangle \rightarrow |\uparrow\rangle$  transition and ramped it on in 10 ms. We then ramped  $\Omega_i \rightarrow \Omega_f$  in 30 ms, changing the detuning of the probe with respect to the  $|\uparrow\downarrow\rangle$ . The experimental sequence can be visualized in Figure 10. We chose the value of  $\Omega_f$  such that it would bring  $\omega_p$  to resonance

at  $\Delta = 0$ , creating double dressed states that were equal superposition of  $|\downarrow\rangle$  and  $|\uparrow\rangle$ . We quantified the sensitivity of this transition to large changes in the detuning  $\Delta$  in terms of the fractional population imbalance  $\langle\hat{\sigma}_3\rangle = P_\downarrow(\Delta) - P_\uparrow(\Delta)$ , shown in Figure 11a for  $\Omega_f/2\pi = 138.2(1)$  kHz<sup>5</sup>. This signal is first-order sensitive to  $\omega_{\downarrow,\uparrow}$ , and provided our third measurement of sensitivity to detuning in Figure 6b denoted by circles.

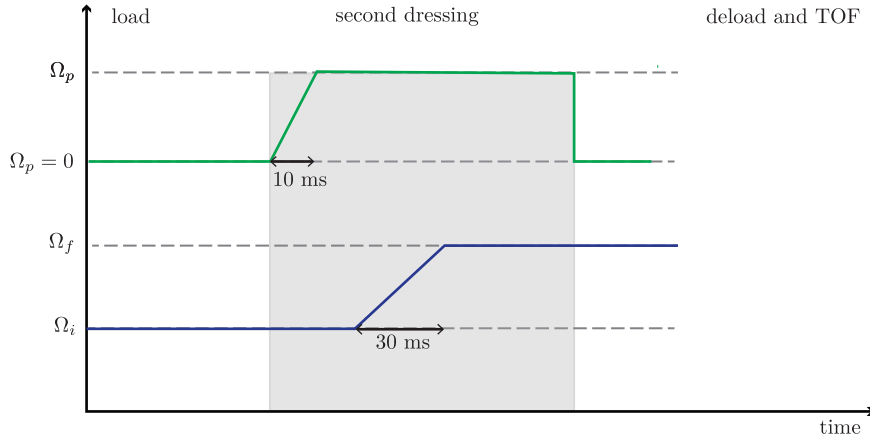


Figure 10: Experimental protocol for implementing concatenated CDD. We started an initial RF coupling strength  $\Omega_i$  and ramped on the probe field  $\Omega_p$  in a few ms with  $\omega_p = \omega_{z,x}(\Omega_f)$  so that it was initially slightly off resonant with the  $zx$  transition. We then ramped the RF field to  $\Omega_f$ , brining  $\omega_p$  to resonance.

We compared the fidelity of preparing a superposition of the  $|\downarrow\rangle$  and  $|\uparrow\rangle$  states to adiabatically preparing a similar superposition of the the  $|m_F = 0\rangle$  and  $|m_F = -1\rangle$  states using a single ARP (no dressed states involved), both with a probe field strength of  $\approx 1$  kHz. Figure 11b shows the RMS deviation of the population imbalance measured over a few hundred repetitions of the experiment. The RMS deviation for the dressed basis is 0.024(1) and is an order of magnitude smaller than

---

<sup>5</sup>We chose the maximum value of  $\Delta$  such that the population of  $|y\rangle$ , was negligible after de-loading.

for the  $|m_F\rangle$  basis 0.29(1), where it is practically impossible to prepare a balanced superposition for the parameters used here <sup>6</sup>.

Figure 11c shows the response of the  $|\downarrow\rangle \rightarrow |\uparrow\rangle$  transition to small changes in the RF coupling strength  $\delta\Omega$  for different values of  $\Omega_p$ . We prepared an equal superposition of  $|\downarrow\rangle$  and  $|\uparrow\rangle$  following the same procedure as before for  $\Omega_f/2\pi = 138.2(1)$  kHz. We then measured how the population imbalance changes for small variations of  $\Omega$  — an effective detuning in the ‘twice-rotated frame’ — for different probe amplitudes  $\Omega_p$ . We defined a sensitivity parameter  $d\langle\hat{\sigma}_3\rangle/d\Omega$ , obtained from the linear regime of the population imbalance measurements (see inset in Figure 11c). The robustness of the doubly-dressed states against  $\delta\Omega$  fluctuations increased with  $\Omega_p$ , thus verifying the concatenating effect of CDD in the  $|xyz\rangle$  basis.

However promising the application of multiple concatenating fields might seem, this procedure has a fundamental limitation. Each time a new coupling field is applied the energies of the dressed states are reduced to something on the order of magnitude of the coupling strength from the applied concatenating field. For example, in the experiments we described here we started with  $|m_F\rangle$  states with transition frequencies on the order of MHz. The transition frequencies of the  $|xyz\rangle$  states were reduced to hundreds of kHz (or in general the magnitude of  $\Omega$ ). After applying the second concatenating RF field the transition frequencies of the  $|\downarrow\uparrow\rangle$  were reduced to the order of  $\Omega_p \sim 10$  kHz which needed to be smaller than  $\Omega$  for the second RWA to be valid. Therefore we see that after applying multiple concatenating

---

<sup>6</sup>In Figure 11b, the noise in the  $|m_F\rangle$  basis does not follow a Gaussian distribution. This feature is typical of line noise in these experiments.

fields we are at the risk of having some very robust states that are also very closely spaced in energy which might not be desirable for some applications.

## 6.10 Conclusions

We realized a three-level system that is dynamically decoupled from low-frequency noise in magnetic fields, measured now-allowed transitions between all three states, and demonstrated control techniques for creating arbitrary Hamiltonians. These techniques add no heating or loss mechanisms, yet within the protected subspace retain the full complement of cold-atom coherent control tools such as optical lattices and Raman laser coupling, and permit new first-order transitions that are absent in the unprotected subspace. These transitions enable experiments requiring a fully connected geometry as for engineering exotic states, e.g., in cold-atom topological insulators, and two-dimensional Rashba spin-orbit coupling in ultracold atomic systems [78, 102].

The synthetic clock states form a decoherence-free subspace that can be used in quantum information tasks where conventional clock states might be absent, or incompatible with other technical requirements [103]. Moreover, their energy differences are proportional to the amplitude of the dressing field, and hence tunable, so they can be brought to resonance with a separate quantum system. The effective quantization axis can be arbitrarily rotated so that the two systems can be strongly coupled, pointing to applications in hybrid quantum systems [104, 105]. Introducing a second coupling field shields the system from fluctuations of the first, a process

that can be concatenated as needed. More broadly, synthetic clock states should prove generally useful in any situation where fluctuations of the coupling field can be made smaller than those of the environment.

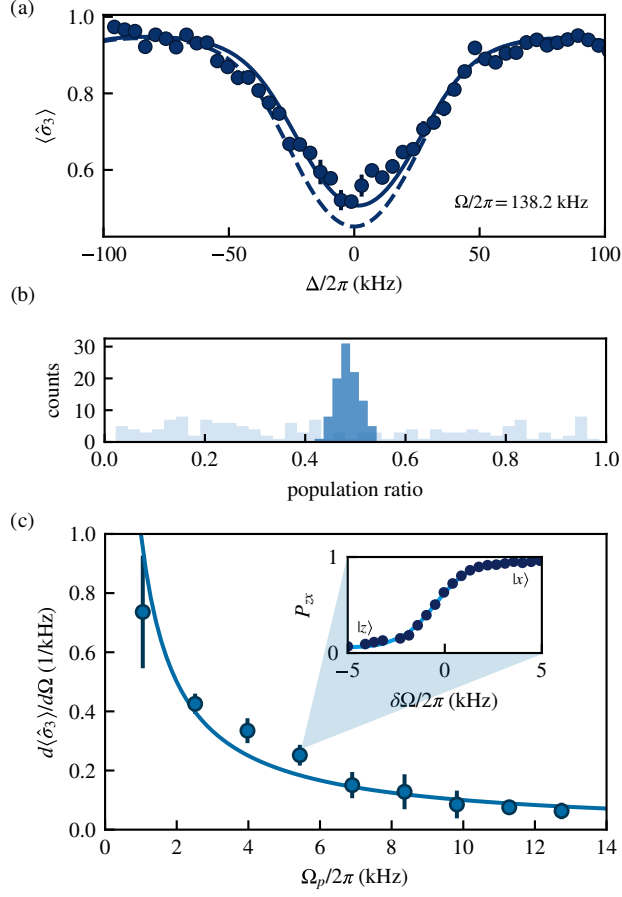


Figure 11: (a) The fractional population imbalance of the  $\downarrow\uparrow$  transition for  $\Omega/2\pi = 138.2(1)$  kHz over detuning  $\Delta$ . The dashed curve is calculated using Equation 6.7 and the solid one using the full Breit-Rabi expression. (b) The fidelity of preparing a balanced superposition of  $|\downarrow\rangle$  and  $|\uparrow\rangle$  (dark blue) states compared to  $|m_F = 0\rangle$  and  $|m_F = -1\rangle$  states (light blue). (c) The robustness of  $\downarrow, \uparrow$  transition against fluctuations  $\delta\Omega$  for different probe field coupling strengths. The points represent the slope of the fitted curves to the fractional population imbalance (inset).



## Chapter 7: Topological order in quantum systems

Topological order can be found in a wide range of physical systems, from crystalline solids [23], photonic meta-materials [106] and even atmospheric waves [107] to optomechanic [108], acoustic [109] and atomic systems [110]. Topological systems are a robust foundation for creating quantized channels for transporting electrical current, light, and atmospheric disturbances. These topological effects can be quantified in terms of integer-valued invariants such as the Chern number, applicable to the quantum Hall effect [111, 112], or the  $\mathbb{Z}_2$  invariant suitable for topological insulators [113].

The topology of Bloch bands defines integers that serve to both classify crystalline materials and precisely specify properties, such as conductivity, that are independent of small changes to lattice parameters [23]. Topologically non-trivial materials first found application in metrology with the definition of the von Klitzing constant as a standard of resistance, which is now applied in the realization of the kilogram [114]. Today, topological systems have found applications in the engineering of low loss optical waveguides [106] and present a promising path to quantum computation [115].

We got interested in topology when working on a system with engineered

Rashba [24] type spin-orbit coupling. Our system had non-trivial topology but it broke from the usual mold of topological materials as it didn't have an underlying crystalline structure that conventionally yields to integer Chern numbers.

Before describing our experiments to both engineer and characterize the unconventional topology of a Rashba spin-orbit coupled gas, in this Chapter I take a step back to describe some basic concepts of topology and its applications to the band theory of solids. The ideas of topology and how exactly one can connect donuts with band structures might feel a bit obscure and complicated for non-experts in the field. The concepts introduced in this Chapter will be necessary for understanding the results presented in Chapter 8.

## 7.1 Topology in mathematics

Topology is a branch of mathematics that studies continuity [116]. The most familiar example might be that of objects being continuously deformed into one another. For example, a donut can be continuously deformed into a coffee mug but if we want to deform it into a pretzel we need to poke more holes in it. This gives us some intuition that the donut and the mug must share the same topology, which is different from that of the pretzel. Topology also studies more abstract objects but I will limit the discussion to closed two-dimensional surfaces (manifolds) in three dimensions, which will be enough to provide some intuition when we define topological invariants for band structures in the following sections.

The topology of 2D surfaces can be classified by the Euler characteristic, and it

is related to the local Gaussian curvature of a surface by the Gauss-Bonnet theorem. The Gaussian curvature can be interpreted in the following way: at any point  $\mathbf{x}_0$  on a surface we can find a normal vector  $\hat{n}$  that is orthogonal to the tangent plane of the surface (red arrow in Figure 1). We can then define a family of planes containing  $\hat{n}$  and the intersection between any of these planes and the surface defines a curve. If we perform a Taylor expansion of the curve around  $\mathbf{x}_0$  the quadratic term defines what is known as the normal curvature  $\kappa$ . The orange and blue lines in Figure 1 are two examples of such curves and their curvatures correspond to the minimum and maximum value of the surface; they are what is known as the principal curvatures  $\kappa_{min}$  and  $\kappa_{max}$ . The Gaussian curvature at any point of a surface is defined in terms of the principal curvatures as  $K = \kappa_{min}\kappa_{max}$  [116].

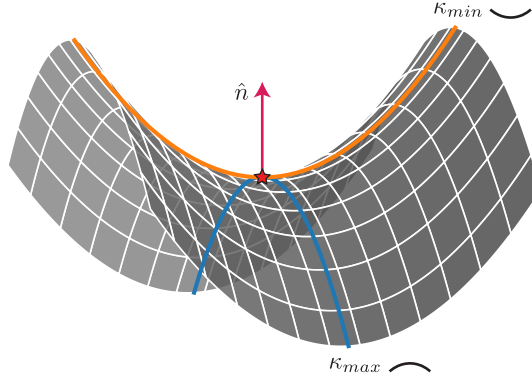


Figure 1: The intersection of planes containing the normal vector  $\hat{n}$  at  $\mathbf{x}_0$  (red star) and the gray surface define a family of curves. The minimum and maximum curvatures, corresponding to the orange and blue lines respectively, correspond to the principal curvatures of the surface at  $\mathbf{x}_0$ .

The Gauss-Bonnet theorem states that the integral of the local Gaussian cur-

vature over a closed surface is equal to the integer valued Euler characteristic

$$\chi = \frac{1}{2\pi} \int_S K dA, \quad (7.1)$$

which is related to the genus  $g$  (number of holes or handles in the surface) by  $\chi = 2(1 - g)$ . The Gauss-Bonnet theorem is a very powerful result as it relates the local properties of a surface, the Gaussian curvature, with a global topological invariant, the Euler characteristic.

In the following sections I will introduce topological invariants in the context of condensed matter physics, which even though might seem a bit more abstract, their interpretation can be closely related to the concepts just defined in this section.

## 7.2 Topological order in condensed matter

Just like topology classifies properties of geometric objects, one important task of condensed matter physics has been to classify phases of matter. Many of these phases, for example, magnetic or conducting phases, can be described in terms of order parameters related to spontaneously broken symmetries [117]. However, in the past few decades, an increasing number of systems have been found where it is only possible to understand their phases and properties in terms of the underlying topology of their quantum states. This new paradigm of physics has been so important that in 2016 the Nobel prize in physics was awarded to David J. Thouless, F. Duncan M. Haldane and J. Michael Kosterlitz for the theoretical discoveries of topological phase transitions and topological phases of matter

The effects of topology in condensed matter systems were first observed when von Klitzing and colleagues [118] measured the quantized Hall resistance in two-dimensional electron gases subjected to a strong perpendicular magnetic field. The effect can be understood semi-classically by thinking of the electrons' quantized cyclotron orbits<sup>1</sup> that give rise to Landau levels. If the Landau levels are filled then there is an energy gap separating two consecutive levels and the material acts as an insulator but if an electric field is applied the orbits drift and the electrons will be 'skipping orbits' in the edge as can be seen in Figure 2, giving rise to what is known as edge states.

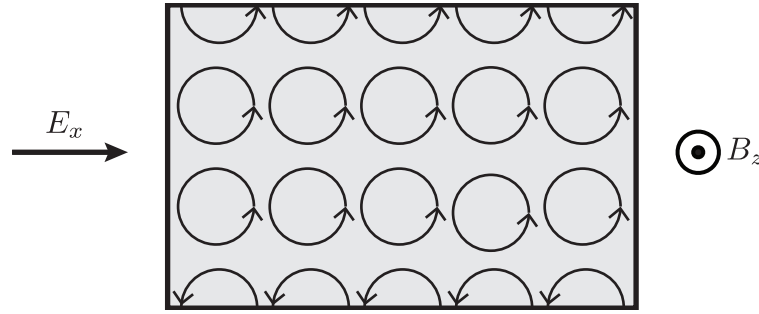


Figure 2: The quantum Hall effect. An electron gas is confined in a two-dimensional material and a strong magnetic field is applied perpendicular to the plane. The electrons on the bulk travel in cyclotron orbits while the electrons on the edge travel 'skipping orbits'.

In a seminal paper Thoules, Kohomoto, Nightingale, and den Nijs [111] explained that the quantization of the Hall conductivity is determined by the underlying topology of the band structure. Just like the Euler characteristic defined in

---

<sup>1</sup>This is an intuitive but not very complete explanation of the quantum Hall effect, see [119] if you want to learn more about this subject.

Equation 7.1 classifies 2D solids that can be continuously deformed without opening or closing holes, there is a topological invariant that classifies band structures that can be deformed into one another without opening or closing an energy gap. This invariant, initially known as the ‘TKNN invariant’, was later recognized by the mathematical physicist Bary Simon as the ‘first Chern class invariant from  $U(1)$  fiber bundles’<sup>2</sup> [121] and the TKNN invariant became what is known today as the Chern number or Chern invariant. Another very valuable contribution from Simon’s work was that he made the connection between the Chern number and the Berry’s geometrical phase [122] which will be defined in the following sections and will allow us to make a physical interpretation of this otherwise abstract seeming topological invariant.

### 7.3 Berry phase and Berry curvature

A Berry or geometric phase is used to describe the phase acquired by a state<sup>3</sup> as it moves through a closed trajectory in parameter space. It plays a key role in topological band theory and can help provide a physical interpretation of the Chern number.

Consider a Hamiltonian  $\hat{H}$  that depends on a set of parameters  $\mathbf{r} = (r_1, r_2, \dots)$ . If the parameters are slowly changed in time, the corresponding change in the system can be described by a path in parameter space  $\mathbf{r}(t)$ . The state  $|\psi(t)\rangle$  evolves

---

<sup>2</sup>See [120] if you want to dive into hardcore topology.

<sup>3</sup>Geometric phases are not unique to quantum systems and are also present, for example, in optics [123].

according to the time dependent Schödinger equation and at any given time  $t$  there is a basis that satisfies

$$\hat{H}(\mathbf{r}) |n(\mathbf{r})\rangle = E_n(\mathbf{r}) |n(\mathbf{r})\rangle \quad (7.2)$$

for  $\mathbf{r} = \mathbf{r}(t)$ . Suppose the system is initially in state  $|n(\mathbf{r}(t=0))\rangle$ , if the parameters are changed slowly such that the adiabatic theorem is valid, then at time  $t$  the state of the system can be written as

$$|\psi(t)\rangle = \exp \left\{ -\frac{i}{\hbar} \int_0^t dt' E_n(\mathbf{r}(t')) \right\} \exp(i\gamma_n(t)) |n(\mathbf{r}(t))\rangle, \quad (7.3)$$

where the first exponential term corresponds to a dynamical phase factor, and the second term is a geometric phase. By imposing that  $|\psi(t)\rangle$  satisfies the time-dependent Schrödinger equation one finds that

$$\gamma_n(t) = i\langle n(\mathbf{r}) | \nabla_{\mathbf{r}} n(\mathbf{r}) \rangle \cdot \dot{\mathbf{r}}(t), \quad (7.4)$$

where the term

$$\mathbf{A}_n(\mathbf{r}) = i\langle n(\mathbf{r}) | \nabla_{\mathbf{r}} n(\mathbf{r}) \rangle \quad (7.5)$$

is usually referred to as the Berry connection<sup>4</sup> or the Berry vector potential for reasons that will become apparent. Because eigenvectors can only be defined up to a global phase,  $\mathbf{A}$  is a gauge dependent quantity. If we make a gauge transformation such that  $|n(\mathbf{k})\rangle \rightarrow \mathbf{e}^{i\xi(\mathbf{k})} |n(\mathbf{k})\rangle$  then the Berry connection is also transformed as

---

<sup>4</sup>This is related to the connection defined in differential geometry that is used to describe things like parallel transport.

$\mathbf{A}_n(\mathbf{k}) \rightarrow \mathbf{A}_n(\mathbf{k}) - \nabla_{\mathbf{k}}\xi(\mathbf{k})$ . However if we integrate the Berry connection on a closed loop

$$\gamma_n(\mathcal{C}) = \oint_{\mathcal{C}} \mathbf{A}_n(\mathbf{r}) \cdot d\mathbf{l}, \quad (7.6)$$

we obtain the Berry phase which, unlike the Berry connection, is gauge independent (modulo  $2\pi$ ).

An alternative way to compute Berry's phase uses Stokes's theorem from vector calculus

$$\begin{aligned} \oint_{\mathcal{C}} \mathbf{A}_n \cdot d\mathbf{l} &= \int_{\mathcal{S}} \nabla \times \mathbf{A}_n \cdot d\mathbf{S} \\ &= \int_{\mathcal{S}} \boldsymbol{\Omega}_n \cdot d\mathbf{S}, \end{aligned} \quad (7.7)$$

where the vector field  $\boldsymbol{\Omega}_n = \nabla \times \mathbf{A}_n$  is known as the Berry curvature or Berry field. By rewriting the Berry phase in this way, its resemblance with the definition of the Euler characteristic from Equation 7.1 becomes apparent.

Using some vector calculus identities the Berry curvature can be rewritten as

$$\begin{aligned} \boldsymbol{\Omega}_n &= i[\nabla_{\mathbf{r}} \langle n | \times \nabla_{\mathbf{r}} | n \rangle] \\ &= \sum_{j \neq n} i[\langle n | \nabla_{\mathbf{r}} | j \rangle \times \langle j | \nabla_{\mathbf{r}} | n \rangle] \\ &= i \sum_{j \neq n} \frac{\langle n | \nabla_{\mathbf{r}} \hat{H} | j \rangle \times \langle j | \nabla_{\mathbf{r}} \hat{H} | n \rangle}{(E_j - E_n)^2}, \end{aligned} \quad (7.8)$$

where  $\langle n | \nabla_{\mathbf{r}} | j \rangle$  was replaced with  $\langle n | \nabla_{\mathbf{r}} \hat{H} | j \rangle / (E_j - E_n)$  by differentiating Equa-



tion 7.2. This expression shows that  $\mathbf{\Omega}_n$  is a gauge independent quantity as it does not depend on the derivatives of a particular gauge choice for  $|n\rangle$  but rather on  $\nabla_{\mathbf{r}}\hat{H}$  which is gauge independent. Also we can see that  $\mathbf{\Omega}_n$  becomes singular when there are degeneracies present in the Hamiltonian, and these degeneracies act as ‘sources’ for the Berry curvature. Finally, even though the system may remain in state  $|n\rangle$  during the adiabatic evolution, this expression for the Berry curvature makes it explicit that other eigenstates of the Hamiltonian have an influence in the Berry phase acquired.

### 7.3.1 Aharonov-Bohm phase as an example of a Berry’s phase

A familiar example of geometric phases is the Aharonov-Bohm phase [124] gained by an electron moving along closed trajectories around a solenoid. This phase was initially conceived as a way of showing that in quantum mechanics magnetic vector potentials, typically conceived only as mathematical objects, can have a physical effect on the wave function. They considered a coherent electron beam split into two paths around a solenoid that produces a magnetic field  $\mathbf{B}$  and later recombined as shown in Figure 3. Outside the solenoid the magnetic field  $\mathbf{B} = 0$ , but there can be a non-zero magnetic vector potential such that  $\mathbf{B} = \nabla \times \mathbf{A}$ . Even though the electron’s trajectories are not modified by the presence of the solenoid, when looking at the interference pattern one finds that the two paths acquired different phases, and their difference is remarkably equal to magnetic flux piercing the area enclosed by the path of the electrons  $\Delta\varphi = 2\pi\Phi_B/\Phi_0$ , where  $\Phi_0 = h/e$  is the

flux quantum. This Aharonov-Bohm phase can be interpreted as an example of a Berry phase in real space.

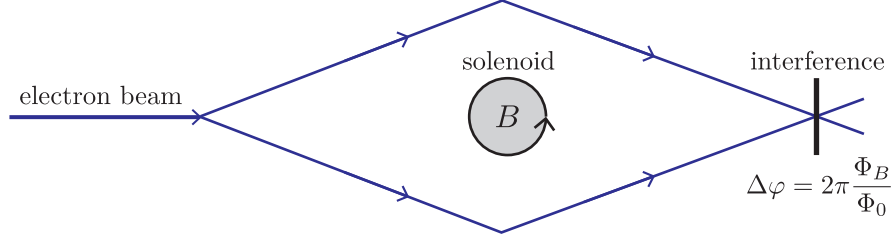


Figure 3: The Aharonov-Bohm experiment. A coherent electron beam is split into two paths surrounding a solenoid which produces a non-zero magnetic field  $\mathbf{B}$  inside the gray region and  $\mathbf{B} = 0$  outside. The two beams are later recombined and an interference pattern reveals a phase difference  $\Delta\varphi = 2\pi\Phi_B/\Phi_0$  equal to the magnetic flux enclosed by the electron's path.

For a charged particle in the presence of a vector potential, the momentum dependence of the free-particle Hamiltonian is modified  $\mathbf{p} \rightarrow \mathbf{p} - q\mathbf{A}$  so that the wave function will depend on the magnetic vector potential as well. Using Equations 7.6 and 7.7 it can be shown that the Berry phase associated with a closed path around the solenoid is exactly equal to the Aharonov-Bohm phase:

$$\begin{aligned}
 \gamma_n(\mathcal{C}) &= \frac{e}{\hbar} \oint_{\mathcal{C}} \mathbf{A}(\mathbf{r}) \cdot d\mathbf{r} \\
 &= \frac{e}{\hbar} \int_{\mathcal{S}} \nabla \times \mathbf{A} \cdot d\mathbf{S} \\
 &= \frac{e\Phi_B}{\hbar},
 \end{aligned} \tag{7.9}$$

In this particular example, the Berry connection is exactly equal to the magnetic vector potential and the Berry curvature is the magnetic field. This gives us a very physical intuition for interpreting the Berry phase in terms of the ‘magnetic

flux’ from abstract sources of ‘magnetic fields’ in parameter space.

### 7.3.2 Chern number

The Chern number is conventionally used to describe the topology of materials which have an underlying crystalline structure. According to Bloch’s theorem, the wave functions of a space periodic Hamiltonian can be written as  $|\psi(\mathbf{k})\rangle = e^{i\mathbf{k}\cdot\mathbf{r}}|u(\mathbf{k})\rangle$ <sup>5</sup>, where  $\mathbf{r}$  is the position and  $\mathbf{k}$  the crystal momentum. The wave functions  $|u(\mathbf{k})\rangle$  are periodic and therefore invariant under the displacement operator  $\hat{D}(nd)|u(\mathbf{k})\rangle = |u(\mathbf{k})\rangle$  when  $d$  is the unit cell size and  $n$  an integer. If we define the Bloch Hamiltonian

$$\hat{H}(\mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{r}}\hat{H}e^{-i\mathbf{k}\cdot\mathbf{r}}, \quad (7.10)$$

their eigenvectors are given by  $|u(\mathbf{k})\rangle$  and the eigenvalues define the band structure. Translational symmetry implies that  $\hat{H}(\mathbf{k} + \mathbf{a}) = \hat{H}(\mathbf{k})$  where  $\mathbf{a}$  is a reciprocal lattice vector. The crystal momentum or quasimomentum (in analogy to the Floquet quasienergy) is only defined within the periodic Brillouin zone and therefore can be mapped into a torus in  $d$  dimensions if we glue the edges together.

The Chern number of the  $n$ th band is defined as

$$C_n = \frac{1}{2\pi} \int_{BZ} \boldsymbol{\Omega}_n \cdot d\mathbf{k}, \quad (7.11)$$

---

<sup>5</sup>Very much like Floquet theory in momentum space

where the relevant parameter space is crystal momentum and the surface of integration corresponds to the BZ (a torus). The definition of the Chern number is closely related to the Berry phase from Equation 7.7. For our previous example of a quantum Hall system, the integer proportionality factor in the quantized conductance is exactly equal to the Chern number.

Just like two-dimensional surfaces are classified by the integral of their Gaussian curvature, the topology of Bloch bands and quantum systems, in general, is determined by the integral of the Berry curvature. Similarly, the integral connects local properties of a quantum system, the Berry connection, with a global topological invariant, the Chern number. One subtle difference is that the Euler characteristic is only determined by the surface (and its intrinsic Gaussian curvature) while the Chern number is defined both by a surface (the BZ) and an additional local curvature (the Berry curvature). By considering different lattice Hamiltonians one can obtain a different Berry curvature, but the geometry of the BZ and thereby the surface of integration is typically defined by a torus<sup>6</sup>. This difference will be important later on when we describe the experiments performed to study a system with Rashba spin-orbit coupling where the unit cell size is taken to infinity (i.e. we remove the lattice).

---

<sup>6</sup>In the next chapter we consider a case where this breaks down.

## 7.4 The bulk-edge correspondence principle

Earlier I mentioned that topological systems provide very robust channels for transporting things like electrical current and light. Transport phenomena typically arise when there is a spatial interface between two topologically distinct phases. The electrons skipping orbits at the interface of a (topological) quantum Hall material and (trivial) vacuum are one example of this. Notice that for this particular example the modes propagate along a given direction, they are chiral. In general, one can expect to have modes moving along two directions, and the difference between the number of these modes  $N_L - N_R$  is fixed and determined by the topology of the bulk states. The bulk-edge correspondence principle relates the difference in the number of these modes with the bulk topology of the materials at the interface:

$$\Delta C = N_R - N_L \tag{7.12}$$

where  $\Delta C$  is the difference of Chern number on the interface.

## 7.5 Example: two-level model

Many of the concepts introduced in the previous section can be readily applied and understood using a two-level model

$$\hat{H}(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \hat{\boldsymbol{\sigma}} \tag{7.13}$$

where  $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$  are the Pauli matrices and  $\mathbf{h}(\mathbf{k}) = (h_x(\mathbf{k}), h_y(\mathbf{k}), h_z(\mathbf{k}))$  are functions of  $\mathbf{k}$ . This model has been used to describe a number of physical systems like graphene [112] and spin-orbit coupled systems [24, 51]. Let us now consider the simple case  $h(\mathbf{k}) = \mathbf{k}$ , for which  $\nabla_{\mathbf{k}} \hat{H} = \boldsymbol{\sigma}$  and using Equation 7.8 it can be shown that

$$\boldsymbol{\Omega} = -\frac{\mathbf{h}}{2h^3} \quad (7.14)$$

which can be recognized as the field of a Dirac monopole [125] with charge  $-1/2$ . The degeneracy in the energies that gives rise to the monopole is known as a Dirac point as the energies in that vicinity resemble the dispersion of a massless Dirac particle. In 2D materials where  $\mathbf{k}$  is only defined within 2D plane  $h_z$  corresponds to the mass of a Dirac particle but its effect on the Berry curvature is equivalent to that of moving the Dirac monopole along a fictitious  $k_z$  dimension in a direction determined by the sign of  $h_z$ .

It follows from Equation 7.14 that the Berry phase gained by moving along a closed path  $\mathcal{C}$  is equal to the flux from the monopole in the surface enclosed by  $\mathcal{C}$  as is shown in Figure 4. This connects nicely with our intuition from the Aharonov-Bohm effect. For a closed surface enclosing the Dirac point, the Chern number is an integer equal to 1.

For a Hamiltonian with arbitrary  $\mathbf{h}(\mathbf{k})$  we can define a normalized vector  $\hat{h} = \mathbf{h}/|\mathbf{h}|$  and the Chern number takes the form

$$C = \frac{1}{4\pi} \int (\partial_{k_x} \hat{h} \times \partial_{k_y} \hat{h}) \cdot \hat{h} d^2k \quad (7.15)$$

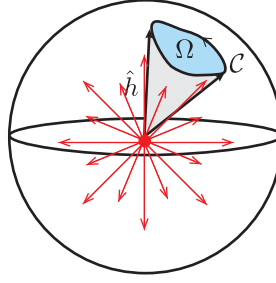


Figure 4: For a two-level system, the Berry curvature from a Dirac point can be viewed as a Dirac monopole in momentum (parameter) space. The Chern number can be interpreted as the flux from the monopole on the solid angle subtended by the vector  $\hat{h}(\mathbf{k})$  or alternatively as the number of times  $\hat{h}(\mathbf{k})$  wraps around a unit sphere.

which can be interpreted as the number of times that the vector  $\hat{h}(\mathbf{k})$  wraps around a unit sphere [126], a quantity that is known as the winding number. Depending on the sign of  $h_z$  the vector  $\hat{h}(\mathbf{k})$  will wrap around the north or south hemisphere, so to have integer valued Chern numbers, Dirac points must come in pairs. Luckily for lattice Hamiltonians this is guaranteed by the fermion doubling theorem [127]. In Chapter 8 I describe an engineered system that has a single Dirac point.

## 7.6 Monopoles and Dirac strings

We just gained some intuition about interpreting the Chern number as the flux from Dirac monopoles. But if we stick to our knowledge of electromagnetism we might remember that magnetic monopoles are forbidden since

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0. \quad (7.16)$$

So how is it possible to keep a vector potential and have  $\nabla \cdot \mathbf{B} \neq 0$ ? The solution to this problem was envisioned by Dirac [125] and is now called a Dirac string. If we consider a semi-infinitely long and infinitesimally thin solenoid, the magnetic field in the finite end will resemble that of a monopole as can be seen in Figure 5. This tiny solenoid corresponds to the Dirac string. A more mathematical interpretation

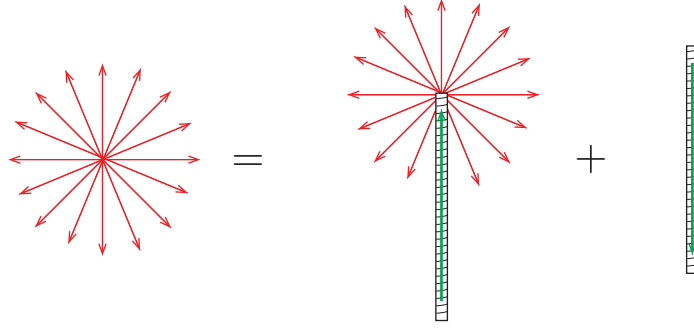


Figure 5: For a two-level system, the Berry curvature from a Dirac point can be viewed as a Dirac monopole in momentum (parameter) space. The Chern number can be interpreted as the flux from the monopole on the solid angle subtended by the vector  $\hat{h}(\mathbf{k})$  or alternatively as the number of times  $\hat{h}(\mathbf{k})$  wraps around a unit sphere.

of these strings comes from the fact that in order to have  $\nabla \cdot \mathbf{B} \neq 0$  the vector potential of a monopole must have ‘lines’ where it becomes singular. For example, we can write for a particular gauge

$$\mathbf{A}(\mathbf{r}) = g \frac{-y\mathbf{e}_x + x\mathbf{e}_y}{r(r+z)} \quad (7.17)$$

which is singular at the Dirac string located at the negative  $z$  axis where  $z = -r$ .

The orientation of the Dirac string is gauge dependent, something that should not surprise or bother us at this point. However, the physical effects of the Dirac string



should be gauge independent, or in other words, the Aharonov-Bohm phase gained by a charged particle moving in a path that encloses the string should be an integer multiple of  $2\pi$ . This argument gives rise to the Dirac charge quantization [125], and in the context of topology, it guarantees that when we calculate the Berry phase by integrating the Berry connection (vector potential) along a path that encloses a Dirac string, its effect will be indistinguishable.

## 7.7 Conclusions

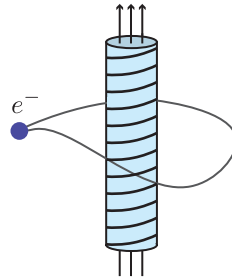
Topology plays a very important role both in mathematics and in physics. As a closing remark, Figure 6 summarizes the main concepts that were introduced and is a reminder that topological invariants are global properties defined in terms of integrals of local properties. Furthermore, we can use our intuition from electromagnetic theory to interpret topological invariants in quantum mechanics.

Euler characteristic



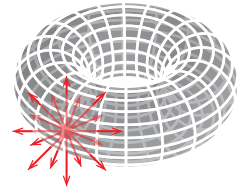
$$\chi = \frac{1}{2\pi} \int_{\mathcal{M}} K dS$$

Magnetic flux



$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

First Chern number



$$C_n = \frac{1}{2\pi} \int_{BZ} \boldsymbol{\Omega}_n \cdot d\mathbf{k}$$

Figure 6: The Euler characteristic and the Chern number are topological invariants defined by integrals of local curvatures. The Aharonov-Bohm phase gives us physical intuition to interpret the Chern number as the flux from a ‘Berry field’.

## Chapter 8: Unconventional topology in a quantum gas with Rashba SOC

Topological order is present in a wide range of physical systems and is quantified by integer valued topological invariants such as the Chern number. In this Chapter, I describe a system with Rashba-type spin-orbit coupling whose topological invariants can take half-integer values. If the concept of half-integer invariants does not sound odd, think of a donut with half a hole.

Ultracold atomic systems are an emerging platform for engineering topological lattices, from the Harper-Hofstadter model [20, 128], the Haldane model [129], to the Rice-Mele model [130, 131] as well as assembling spin-orbit coupled lattices without analogues in existing materials [132, 133]. However, experimental realizations of topological materials have mostly focused on engineering different topological bands (with different Berry curvatures) in lattice systems, where the BZ is always a torus. Here I show that by eliminating the lattice potential and changing the BZ from  $\mathbb{T}^2$  to  $\mathbb{R}^2$ , i.e. from a torus to a Cartesian plane, it is possible to create topological branches of the dispersion relation with half-integer Chern number. This work has been submitted for publication and is under review. A preprint of the manuscript is available in [26].

These experiments combine the techniques from Chapters 5 and 6, additionally I use key concepts of topology described in Chapter 7. This Chapter is organized in the following way: First I give a general overview of Rashba SOC and describe theoretical proposals for engineering this type of coupling in ultracold atom systems. Then I describe our experimental implementation of Rashba SOC in the lab using a trio of Raman coupled CDD states and validate our quantum engineering using Fourier transform spectroscopy. Finally, I describe a quantum state tomography procedure to measure the eigenstates of our system, from which we can directly obtain the Chern number.

To avoid confusion between dressed state  $xyz$  labels and Cartesian coordinates indicating the Raman geometry, in this Chapter I use the numbers 1, 2, 3 to label experimental coordinates and the letters  $x, y, z$  to label clock state parameters.

## 8.1 Rashba spin-orbit coupling

Rashba SOC [24] appears in condensed matter systems where electrons are confined in a 2D plane and experience an intrinsic out-of-plane electric field. If the electron's momentum is given by  $\hbar\mathbf{k} = \hbar(k_x\mathbf{e}_x + k_y\mathbf{e}_y)$  and the electric field is  $\mathbf{E} = E\mathbf{e}_z$ , in the electron's moving frame there will be a momentum dependent magnetic field  $\mathbf{B}_{\text{SOC}} = -\hbar\mathbf{k}/m \times \mathbf{E}/c^2 = \hbar E/mc^2(-k_y, k_x, 0)$ . The interaction between the electron's spin with this field through the magnetic Zeeman interaction

$-\boldsymbol{\mu} \cdot \mathbf{B}_{\text{SOC}}$  gives rise to the SOC Hamiltonian

$$\hat{H}_{\text{SOC}} = \frac{2\alpha}{m}(k_y\hat{\sigma}_x - k_x\hat{\sigma}_y) \quad (8.1)$$

where  $\alpha = g\mu_B E/c^2$ ,  $g$  is the electron gyromagnetic ratio,  $\mu_B$  is the Bohr magneton and  $\hat{\sigma}_i$  are the Pauli matrices.

Figure 1 shows the Rashba dispersion relation which is characterized by having a Dirac point located at  $\mathbf{k} = 0$  (see Section 7.5) and a degenerate ground state that is described by the ring  $k_x^2 + k_y^2 = \alpha^2$ . If we combine Equation 8.1 with the free particle Hamiltonian, the system is described by  $\hat{H} = (\hbar\mathbf{k} - \hat{\mathbf{A}})^2/2m$  where  $\hat{\mathbf{A}} = \alpha(\hat{\sigma}_y\mathbf{e}_x - \hat{\sigma}_x\mathbf{e}_y)$  can be interpreted as a (matrix-valued) non-Abelian gauge potential [134] whose elements do not commute. This non-Abelian gauge potential is closely related to the Berry connection discussed in Section 7.3 and in combination with the Dirac point hints at us that a system with Rashba SOC has non-trivial topology.

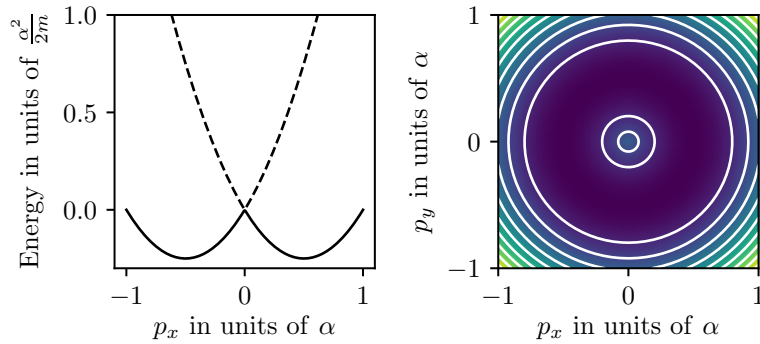


Figure 1: The Rashba dispersion relation has a Dirac point located at  $\mathbf{k} = 0$  and a degenerate ground state that is described by the ring  $k_x^2 + k_y^2 = \alpha^2$ .

## 8.2 Rashba SOC for neutral atoms

Proposals for engineering Rashba type SOC in neutral atoms consist of using lasers to link internal states of an atom with its linear momentum. In order to achieve non-trivial gauge potentials it is necessary to couple  $N \geq 3$  levels (see [16]). I will describe the proposal by [135] which considers a ‘ring coupling’ which is shown in Figure 2 for the case of  $N = 3$ . The states  $|j\rangle$  represent internal atomic states and they are linked to each other with complex valued matrix elements  $\frac{\Omega_j}{2}e^{i\mathbf{k}_j \cdot \mathbf{x}}$ , where  $\mathbf{k}_j$  is a momentum transfer associated with the  $|j\rangle \rightarrow |j+1\rangle$  transition and  $\Omega_i = e^{i\phi_i}|\Omega|$  represents the coupling strength. We require that  $\sum \mathbf{k}_i = 0$  so that no momentum is transferred when a closed loop  $|1\rangle \rightarrow |2\rangle \cdots \rightarrow |N\rangle \rightarrow |1\rangle$  is completed. For this case, the  $\mathbf{k}_i$  momenta vectors can be written as  $\mathbf{k}_j = \mathbf{K}_{j+1} - \mathbf{K}_j$  and we make  $\mathbf{K}_j = k_L \sin(2\pi j/N)\mathbf{e}_x + k_L \cos(2\pi j/N)\mathbf{e}_y$ , corresponding to the vertices of an  $N$  sided regular polygon. We can further make a gauge transformation such that we can replace the phases  $\phi_i$  associated to each coupling with  $\bar{\phi} = \sum_i \phi_i/N$ .

The Hamiltonian describing this coupling along with the kinetic term is

$$H_{j,j'} = \frac{\hbar^2 k^2}{2m} \delta_{j,j'} + \frac{\Omega}{2} (e^{i(\bar{\phi} + \mathbf{k}_j \cdot \mathbf{x})} \delta_{j,j'+1} + \text{h.c.}), \quad (8.2)$$

and after applying the unitary transformation  $U_{j,j'} = \exp[i\mathbf{K}_i \cdot \mathbf{x}] \delta_{j,j'}$ <sup>1</sup> it gets transformed to

$$H_{j,j'} = \frac{\hbar^2}{2m} |\mathbf{q} + \mathbf{K}_j|^2 \delta_{j,j'} + \frac{\Omega}{2} (e^{i\bar{\phi}} \delta_{j,j'+1} + \text{h.c.}), \quad (8.3)$$

---

<sup>1</sup>This transformation is equivalent to applying a state dependent momentum boost  $\mathbf{k} \rightarrow \mathbf{k} + \mathbf{K}_j$

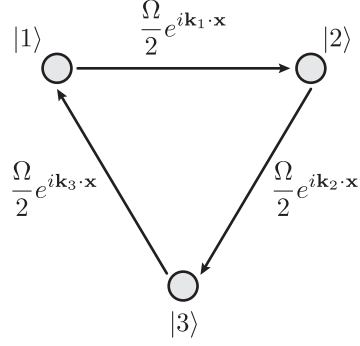


Figure 2: The Rashba ring coupling. To generate Rashba SOC in a system of cold atoms it is necessary to cyclically couple  $N \geq 3$  internal states such that the transition  $|j\rangle \rightarrow |j+1\rangle$  has a momentum transfer  $\mathbf{k}_j$  and  $\sum_j \mathbf{k}_j = 0$  such that there is no momentum transfer for a closed loop  $|1\rangle \rightarrow |2\rangle \dots |N\rangle \rightarrow |1\rangle$ . The ring coupling combined with the free particle Hamiltonian give rise to a 2-level subspace that can be described to first order by the Rashba Hamiltonian

where I replaced the momentum  $\mathbf{k}$  by the quasimomentum  $\mathbf{q}$ . The off diagonal terms of Equation 8.3 is related to a 1D periodic tight-binding Hamiltonian with hopping elements  $\Omega/2$  where the internal states  $|j\rangle$  represent lattice sites and completing one loop corresponds to gaining a ‘flux’ of  $N\bar{\phi}$ . To visualize how the Rashba Hamiltonian emerges from this coupling scheme, it is helpful to write the Hamiltonian in a basis that is conjugate to the index  $j$ <sup>2</sup>

$$|l\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i2\pi jl/N} |j\rangle \quad (8.4)$$

where the index  $l$  is analogous to the crystal momentum index for a Bloch Hamiltonian. In this new basis, terms in the diagonals are displaced to the off-diagonal and likewise, terms in the off diagonals are displaced to the diagonal. The Hamiltonian

---

<sup>2</sup>Just like position and momentum are conjugate variables related by Fourier transforms, the  $|j\rangle$  and  $|l\rangle$  basis are related by a discrete Fourier transform.

transformed to the Fourier basis starts looking very much Rashba-like

$$H_{l,l'} = \left[ \frac{\hbar^2}{2m}(q^2 + k_L^2) + E_l \right] \delta_{l,l'} + \frac{\hbar^2 k_L}{m} [(iq_x + q_y)\delta_{l-1,l'} + \text{h.c.}], \quad (8.5)$$

where  $E_L = 2\hbar\Omega \cos(2\pi l/3 + \bar{\phi})$  correspond to the eigenenergies when  $q = 0$ . The phase  $\bar{\phi}$  can be tuned such that a pair of states with consecutive  $l$  index become degenerate, indicating the presence of a Dirac point at  $q = 0$ . Figure 3 shows the energies  $E_l$  for  $N = 3$  and  $\bar{\phi} = 0$ .

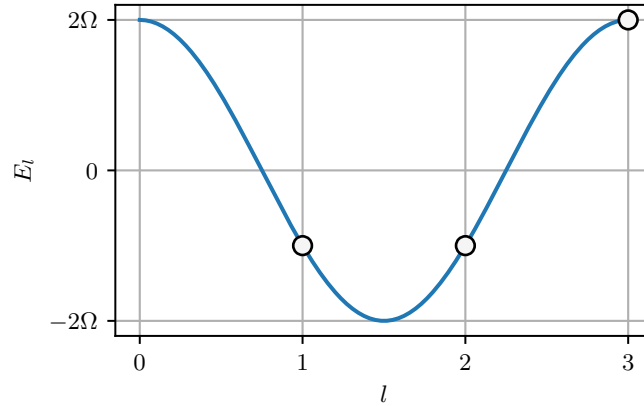


Figure 3: Eigenenergies of Equation 8.5 for  $q = 0$  for  $N = 3$  and  $\bar{\phi} = 0$ . For this particular choice of phase, the energies of the  $l = 1$  and  $l = 2$  states become degenerate

We consider the degenerate states as pseudospins which are described to zeroth order by the Rashba plus free particle Hamiltonian

$$\hat{H}^{(0)} = \frac{\hbar^2 q^2}{2m} + \frac{\hbar^2 k_L}{m} (\hat{\sigma}_x q_y - \hat{\sigma}_y q_x), \quad (8.6)$$

with spin-orbit coupling strength given by  $\alpha = \hbar^2 k_L/2$ . The zeroth-order Hamilto-

nian has continuous rotational symmetry while the proposed ring coupling only has discrete rotational symmetry. The symmetry of the Hamiltonian is recovered when higher-order corrections are added to the Hamiltonian. The complete expressions for the higher-order terms for  $N = 3$  and  $N = 4$  can be found in [135], and they are reminiscent of quadratic and cubic Dresselhaus SOC [136]. The largest leading order term for the  $N = 3$  case is inversely proportional to  $\Omega$  so that this ring-coupling scheme results in a more ‘Rashba-like’ Hamiltonian as one goes to higher coupling strengths. Figure 4 shows level curves of the ground state eigenenergies of Equation 8.5 for  $N = 3$  and  $\bar{\phi} = 0$  for increasing  $\Omega$ . At low  $\Omega$  the dispersion has discrete rotational symmetry and is characterized by three local minima<sup>3</sup>. As  $\Omega$  is increased the local minima start merging into each other and in the large  $\Omega$  limit we recover the characteristic Rashba ring-like dispersion.

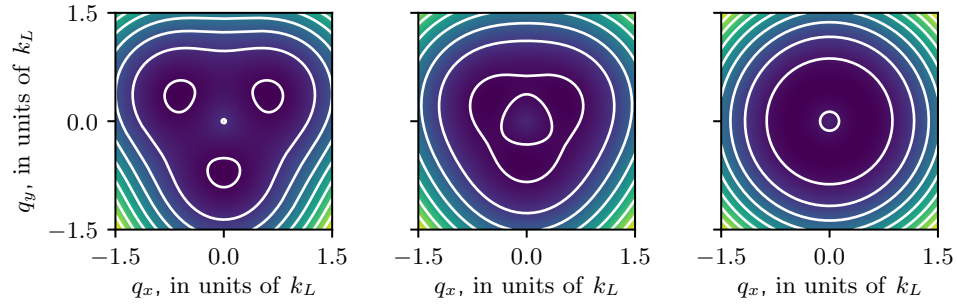


Figure 4: Ground state dispersion relation of Equation 8.5 for  $N = 3$  and  $\bar{\phi} = 0$  for  $\Omega = 1.75 E_L$  (left),  $\Omega = 3.5 E_L$  (middle) and  $\Omega = 175 E_L$  (right). Higher order corrections to  $\hat{H}^{(0)}$  decay as  $1/\Omega^2$  and in the large  $\Omega$  limit we recover the Rashba ring dispersion.

---

<sup>3</sup>or... an alien face?



### 8.3 Experimental implementation of Rashba SOC

We implemented the ring-coupling scheme and engineered Rashba SOC by resonantly coupling the  $|xyz\rangle$  states from Chapter 6 using two-photon Raman transitions [78] as depicted in Figure 5. The engineered system consisted of an effective spin-1/2 Rashba subspace, along with a topologically trivial high-energy branch. Our engineered Rashba system had a single Dirac cone near  $\mathbf{q} = 0$ , where the two lower dispersion branches become degenerate and the Berry curvature becomes singular. Each of these branches extend to infinite momentum, making the supporting manifold a plane rather than a torus. We characterized this system using both spectroscopy and quantum state tomography. This allowed us to measure the dispersion branches and directly observe the single Dirac point linking the lowest two branches as well as to reconstruct the Berry connection to derive the associated Chern numbers.

All of our experiments started with about  $N \approx 1 \times 10^6$   $^{87}\text{Rb}$  atoms in a crossed optical dipole trap [137], with frequencies  $(f_1, f_2, f_3) \approx (70, 85, 254)$  Hz, just above the transition temperature for Bose-Einstein condensation. We initially prepared the atoms in the  $|F = 1, m_F = -1\rangle$  state of the  $5S_{1/2}$  electronic ground state and transferred atoms to the  $m_F = 0$  and  $m_F = +1$  states as needed using ARP. A bias field  $B_0 \mathbf{e}_3$  gave a  $\omega_0/2\pi = 23.9$  MHz Larmor frequency along with a quadratic shift of  $\epsilon/2\pi = 83.24$  kHz. The RF field used to generate the  $|xyz\rangle$  states and implementing CDD had strength  $\Omega_{\text{RF}} = 1.41(2)\epsilon$ . We adiabatically prepared the  $|xyz\rangle$  states starting from the  $m_F$  states following the procedure described in

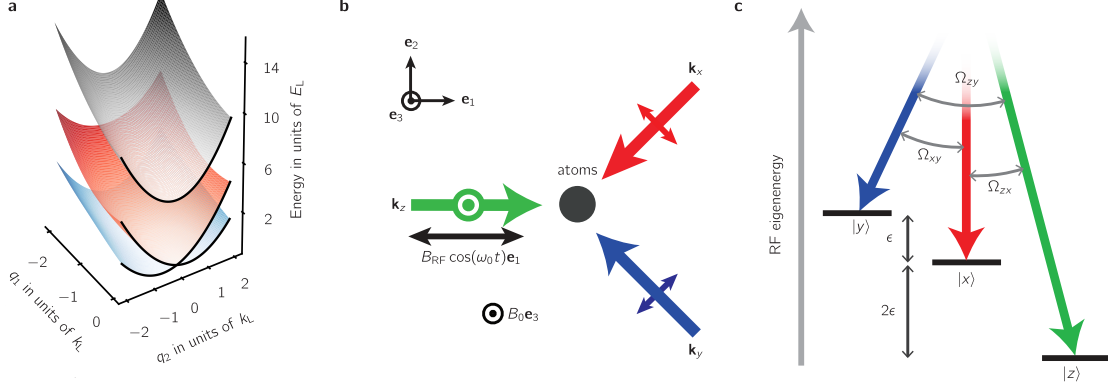


Figure 5: **a** Our engineered dispersion consisted of a two-level Rashba subspace (red and blue) with a single Dirac point linking the lowest two branches and a topologically trivial higher branch (gray). **b** We generated  $|xyz\rangle$  states by combining a bias magnetic field along  $\mathbf{e}_3$  with an RF magnetic field oscillating along  $\mathbf{e}_1$ . These states were coupled by three cross-polarized Raman laser beams propagating along  $\mathbf{e}_1$ ,  $\mathbf{e}_2 - \mathbf{e}_1$  and  $-\mathbf{e}_1 - \mathbf{e}_2$ . **c** Each pair of Raman lasers was in two-photon resonance with a single transition between the  $|xyz\rangle$  states which we coupled strengths  $(\Omega_{zx}, \Omega_{xy}, \Omega_{yz})/2\pi = (12.6(5), 8.7(8), 10(1))$  kHz.

Section 6.4.

### 8.3.1 Raman coupling the $|xyz\rangle$ states

We Raman-coupled atoms prepared in any of the  $|xyz\rangle$  states using the three cross-polarized Raman laser beams shown in Figure 5b, tuned to the ‘magic zero’ wavelength  $\lambda_L = 790$  nm where the scalar light shift vanishes (see Section 3.3.2.1). We arranged the Raman lasers into the tripod configuration shown in Figure 5c, bringing each pair into two-photon resonance with a single transition with strengths  $(\Omega_{zx}, \Omega_{xy}, \Omega_{yz})/2\pi = (12.6(5), 8.7(8), 10(1))$  kHz. The geometry of our experimental implementation differed from [78] where all Raman lasers are perpendicular. We had to go away from the ideal configuration because we needed all of the Raman recoil

vectors to lie on the imaging plane (spanned by  $\mathbf{e}_1$  and  $\mathbf{e}_2$ ) to image all the Raman induced  $\mathbf{k}$  dependent dynamics. As a result, the dispersion relation no longer has discrete rotational symmetry, however, the Dirac point is still present in our system. The only way to break the degeneracy of this system is to change the phase  $\bar{\phi}$  and for our laser configuration, we always have  $\bar{\phi} = 0$ .

The energies of the  $|xyz\rangle$  states were  $\omega_x = 0$  and  $\omega_{z,y} = -(\epsilon \pm \sqrt{4\Omega_{\text{RF}}^2 + \epsilon^2})/2$ . We set the frequencies of the Raman lasers to  $\omega_x = \omega_L + \omega_0 + \omega_{xy}$ ,  $\omega_y = \omega_L + \omega_0$  and  $\omega_z = \omega_L - \omega_{zx}$ , where  $\omega_L = 2\pi c/\lambda_L$  and  $(\omega_{zx}, \omega_{xy}, \omega_{zy})/2\pi = (166.47, 83.24, 249.71)$  kHz are the transition frequencies between pairs of dressed states and are integer multiples of  $\epsilon$  for our coupling strength  $\Omega = \sqrt{2}\epsilon$ .

The Raman coupled states can be described by the combined kinetic and light-matter Hamiltonian

$$\hat{H}_{\text{R}}(\mathbf{k}) = \sum_{j \in \{xyz\}} \left( \frac{\hbar^2 k^2}{2m} + \hbar\omega_i \right) |j\rangle \langle j| + \sum_{j' \neq j} \hbar\Omega_{j,j'} e^{i(\mathbf{k}_{j,j'} \cdot \mathbf{x} - i\omega_{j,j'} t)} |j\rangle \langle j'|, \quad (8.7)$$

where  $\mathbf{k}_{j,j'}$  is the recoil momentum from an  $|j\rangle \rightarrow |j'\rangle$  transition and  $\Omega_{ij}$  is the Raman coupling strength between a pair of RF dressed states. The Hamiltonian above only includes the matrix elements associated to resonant couplings. After applying the unitary transformation  $\hat{U}_{j,j'} = \exp(-i\mathbf{k}_j \cdot \mathbf{x} - \omega_j t) \delta_{j,j'} |j\rangle \langle j'|$  the Hamiltonian takes the familiar form of the ring coupling scheme

$$\hat{H}_{\text{R}} = \sum_{j \in \{xyz\}} \left( \frac{\hbar^2 (\mathbf{q} - \mathbf{k}_j)^2}{2m} + \hbar\delta_j \right) |j\rangle \langle j| + \sum_{j \neq j'} \hbar\Omega_{jj'} |j\rangle \langle j'|, \quad (8.8)$$

where  $\mathbf{k}_j$  are the Raman wave vectors and  $\delta_j$  is the detuning from Raman resonance.

This coupling scheme simultaneously overcomes three limitations of earlier experiments [138, 139] that performed the ring coupling using states in both  $5S_{1/2}$  hyperfine manifolds of  $^{40}\text{K}$  : (1) working in the same hyperfine manifold eliminates spin-relaxation collisions [140]; (2) unlike  $m_F$  states, the  $|xyz\rangle$  states can be tripod-coupled with lasers far detuned relative to the excited state hyperfine splitting greatly reducing spontaneous emission [21]; and (3) CDD renders the  $|xyz\rangle$  states nearly immune to magnetic field noise.

### 8.3.1.1 Floquet and off-resonant coupling effects

We operated in a regime where the transition energies between the  $|xyz\rangle$  states were integer multiples of  $\omega_{xy}$ :  $\omega_{zx} = 2\omega_{xy}$  and  $\omega_{zy} = 3\omega_{xy}$ , making the system suitable for a full description using Floquet theory [141]. We observed that the effective Raman coupling strengths for the driven three level system differed from our calibrations which were performed looking at Rabi oscillations from individual pairs of lasers because of the presence of nearby quasi-energy manifolds and off-resonant coupling terms. This effect could be mitigated for larger values of  $\omega_{xy}$  as the spacing between quasi-energy manifolds is increased and the off-resonant coupling terms less relevant. Figure 6 shows the dispersion relation for both the effective and the full Floquet Hamiltonians. The energies from the effective time independent model get closer to those of the full Floquet model when the spacing between Floquet manifolds is increased (left panel). Appendix C has a full derivation of the Raman

Hamiltonian starting from the  $|m_F\rangle$  basis in the laboratory frame including the full time dependence and off-resonant coupling terms.

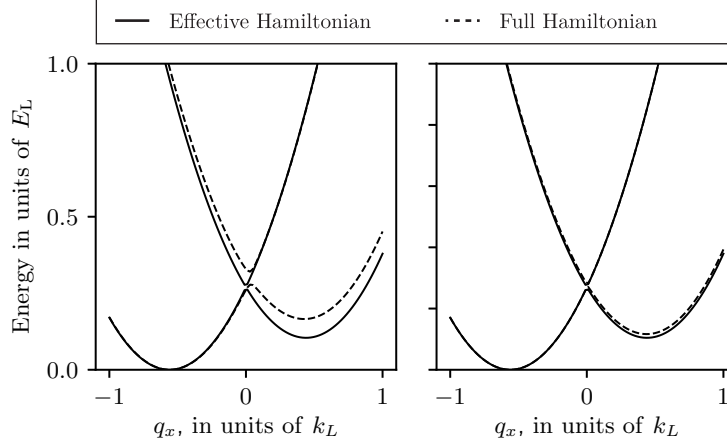


Figure 6: Solid lines: Dispersion relation from RWA Hamiltonian (Equation 8.8) as a function of  $q_x$  for  $\Omega_{i,j} = 1.5 E_L$  and  $\delta_i = 0$ . Dashed lines: Dispersion relation computed for the full Floquet Hamiltonian. We considered  $\omega_{zx} = 2\omega_{xy}$  and  $\omega_{zy} = 3\omega_{xy}$  in both cases so the separation between Floquet manifolds is  $\omega_{xy}$ . In the left panel  $\omega_{xy} = 83.24 \text{ kHz}$  as in our experiments and in the right panel  $\omega_{xy} = 416.2 \text{ kHz}$ . As the spacing between Floquet manifolds becomes we get a better agreement between the spectra of the RWA and Floquet Hamiltonians.

### 8.3.1.2 Lifetime

The limited lifetime due to spontaneous emission has always been a concern for Raman coupled systems. This was one of the reasons why we pursued the topology direction rather than trying to measure a fragile many-body phase. The measured spontaneous emission limited lifetime of our system was  $320(17) \text{ ms}$ , measured with the Raman lasers applied to the  $m_f$  states. However, it was reduced to  $40(2) \text{ ms}$  when we Raman coupled the  $|xyz\rangle$  states, which we attribute to technical noise in the relative phase between the RF dressing field and the Raman laser fields, which

has caused considerable consternation in ongoing experiments. All the experiments reported here were short compared to this timescale so this decreased lifetime was not an issue but it is a limitation that needs to be addressed in future experiments. Figure 7 shows measurements of the lifetimes of Raman dressed atoms in both  $|m_f\rangle$  and  $|xyz\rangle$  states. We obtained the lifetime by fitting decaying exponentials to the integrated OD from absorption images of thermal atoms where the Raman was turned on in 1 ms and held on for up to  $50\text{ }\mu\text{s}$ <sup>4</sup>.

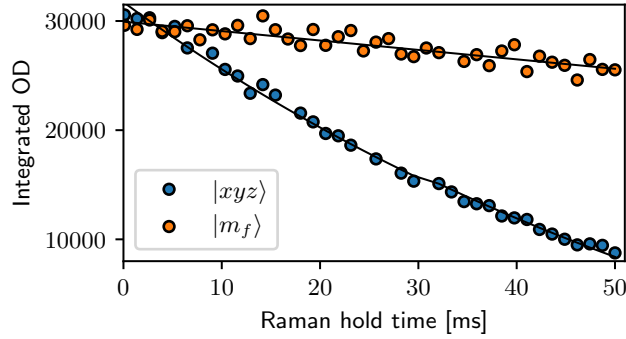


Figure 7: Lifetime of Raman dressed states. We Raman dressed atoms in the  $|m_f\rangle$  and  $|xyz\rangle$  states. The orange markers correspond to atoms initially prepared in  $|m_f = -1\rangle$  (no high power RF involved) and the blue markers correspond to atoms  $|xyz\rangle$  (three averaged traces). The lifetime of doubly dressed states is significantly reduced as compared to the lifetime of the Raman dressed  $|m_f\rangle$  states, indicating that of resonant scattering of photons is not our only loss mechanism.

### 8.3.2 Measuring quasimomentum distributions

Each pair of Raman lasers coupled states  $|i, \mathbf{k}\rangle \rightarrow |j, \mathbf{k} + \mathbf{k}_{i,j}\rangle$  where  $|i\rangle$  and  $|j\rangle$  denote the initial and final  $|xyz\rangle$  states,  $\mathbf{k}$  is the initial momentum and  $\mathbf{k}_{i,j} = \mathbf{k}_i - \mathbf{k}_j$  is the two-photon Raman recoil momentum. Dressed states with quasimomentum

---

<sup>4</sup>How long we could hold on the Raman was limited by the RF antenna heating up.

$\mathbf{q}$  are comprised of three bare states  $|j, \mathbf{k}\rangle$  with momentum  $\mathbf{k} = \mathbf{q} - \mathbf{k}_j$ . Figure 8 illustrates the mapping from bare momentum states into dressed quasimomentum states. The eigenstates of our Rashba SOC Hamiltonian take the form

$$|\Psi_n(\mathbf{q})\rangle = \sum_{j \in xyz} \sqrt{a_{n,j}(\mathbf{q})} e^{i\phi_{n,j}(\mathbf{q})} |j, \mathbf{k} = \mathbf{q} - \mathbf{k}_j\rangle, \quad (8.9)$$

where the quasimomentum  $\mathbf{q}$  is a good quantum number and the amplitudes are parametrized by  $a_{n,j}(\mathbf{q})$  and  $\phi_{n,j}(\mathbf{q})$ . We leveraged the wide momentum distribution of a non-condensed ensemble ( $T \approx 180$  nK and  $T/T_c \approx 1.1$ ) to sample a wide range of momentum states simultaneously. By starting separately in each of the  $|xyz\rangle$  states we sampled the range of quasimomentum states shown in Figure 8b, where the momentum distributions of an initial state  $|j, \mathbf{k}\rangle$  is shifted from  $\mathbf{q} = 0$  by the corresponding Raman wave vector  $\mathbf{k}_j$ .

Our measurement protocol consisted of abruptly removing the confining potential and the Raman lasers, initiating a 21 ms TOF. During this TOF we adiabatically transformed each of the  $|xyz\rangle$  states back to a corresponding  $|m_F\rangle$  state following the procedure described in Section 6.4 and spatially separated them using a Stern-Gerlach gradient. Finally, we used resonant absorption imaging to measure the resulting spin-momentum distributions.

The FWHM of the cloud after TOF was about  $700 \mu\text{m}$  which is much larger than the size of the in-situ cloud  $\sim 50 \mu\text{m}$  and the spatial density distribution atoms imaged after TOF represented the momentum distribution of the atoms. For the  $7.4 \mu\text{m}$  pixel size of our camera and the 3.283 magnification of our imaging

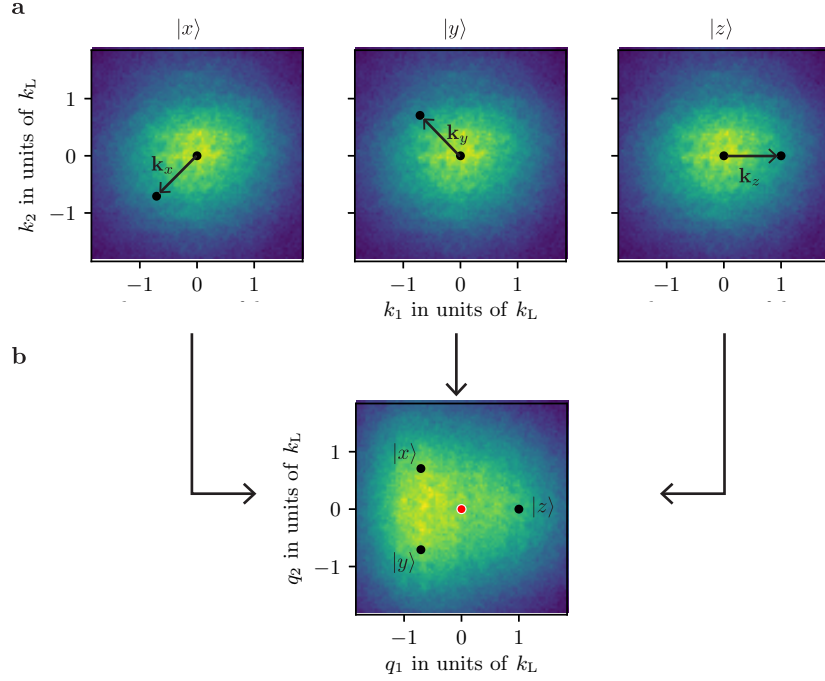


Figure 8: Mapping momentum into quasimomentum: **a** We used non-condensed atoms with a broad momentum distribution ( $T \approx 180$  nK and  $T/T_c \approx 1.1$ ). **b** Atoms in  $|j, \mathbf{k}\rangle$  are mapped to Raman dressed states with quasimomentum  $\mathbf{q} = \mathbf{k} + \mathbf{k}_j$ . The black dots in the bottom panel represent the location of  $\mathbf{k} = 0$  for each of the  $|xyz\rangle$  states and the red dot corresponds to  $\mathbf{q} = 0$ . We performed our experiments starting separately in each of the  $|xyz\rangle$  states, which allowed us to sample a larger range of quasimomentum states.

system, the momentum resolution of our images was  $0.018 k_L$  and the momentum distribution of a single state measured after TOF had a FWHM of  $\sim 2.2 k_L$ .

### 8.3.2.1 Correcting shears from Stern-Gerlach gradient

The magnetic field gradient used to separate the different  $m_f$  states during TOF additionally created a trap for atoms in the direction perpendicular to the Stern-Gerlach separation, causing a compression (expansion) of the  $m_f = -1$  ( $+1$ ) states (see Section 3.2.1). The faster moving atoms are subject to a stronger poten-



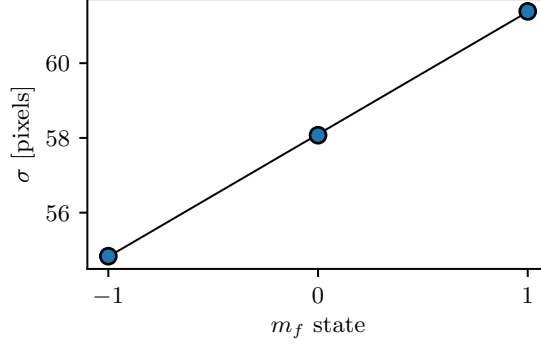


Figure 9: We measured the standard deviation of the momentum distribution along the axis perpendicular to the SG for 10 shots on each  $m_f$  state. From the slope of the linear fit we obtain a shearing parameter  $\alpha \approx \pm 0.067$  for  $m_f = \pm 1$ .

tial as they move further from the trap center. The projections of a given momentum state  $\mathbf{k}$  along and perpendicular to the Stern-Gerlach axis were transformed as  $k_{\parallel} \rightarrow k_{\parallel}$  and  $k_{\perp} \rightarrow (1 + \alpha)k_{\perp}$ , where  $\alpha = 0$  for  $m_f = 0$  and  $\text{sign}(\alpha) = \pm 1$  for  $m_f = \pm 1$ .

Since we were interested in mapping the momentum distribution of atoms it was important for us to quantify and correct the effect of these shears in the TOF images. We used two different methods: First, we prepared thermal atoms in all three of the  $m_f$  states and fit 2D Gaussians rotated by the angle of the SG displacement (63.8 degrees for our images). Figure 9 shows the standard deviation extracted from the Gaussian fits along the axis perpendicular to the SG direction as a function of  $m_f$  state. We performed a linear fit of  $\sigma$  and found that the  $m_f = \pm$  states are expanded/contracted by about  $\pm 6.7\%$  size of the compared to the  $m_f = 0$  cloud ( $\alpha = \pm 0.067$ ).

Alternatively we performed the Ramsey interferometer described in Section 8.5

but coupling only 2 states, either  $|z\rangle \leftrightarrow |x\rangle$  or  $|x\rangle \leftrightarrow |y\rangle$  (mapped to  $|-1\rangle \leftrightarrow |0\rangle$  and  $|0\rangle \leftrightarrow |+1\rangle$  after TOF). We looked at the oscillation frequencies of the density for each pixel of the CCD camera (each sheared momentum state) and fit them to Equation 8.17 for fixed value of the recoil momentum  $\mathbf{k}_{i,j}$  and with a free shear parameter that modifies  $\mathbf{q}$ . Using this method we extracted a shearing of the order of 7%, in good agreement with the Gaussian fitting method.

The transformed momentum coordinates were described by a function

$$g(\mathbf{k}) = (k_{\parallel}, (1 + \alpha)k_{\perp}) \quad (8.10)$$

and our observed data  $(y_i^{(\text{shear})}, \mathbf{k}^{(\text{shear})})$  was the density in the sheared coordinate system at the  $i$ th pixel of the CCD sensor. We estimated the density in the unsheared coordinate using the transformation

$$y_j = \frac{\sum_i \exp[-(g(\mathbf{k}_j) - \mathbf{k}_i^{(\text{shear})})^2 / 2\sigma^2] y_i^{(\text{shear})}}{\sum_i \exp[-(g(\mathbf{k}_j) - \mathbf{k}_i^{(\text{shear})})^2 / 2\sigma^2]}, \quad (8.11)$$

where  $\sigma$  is the spacing between two consecutive pixels ( $\sim 0.018 k_L$ ). Prior to any data analysis we applied this transformation to all of our images, where we used different values of  $\alpha$  that define  $g(\mathbf{k})$  for each of the  $m_f$  states.

## 8.4 Fourier spectroscopy of the Rashba dispersion

We directly measured the 2D dispersion relation using Fourier transform spectroscopy [25]. We considered the evolution of an initial state  $|i, \mathbf{k}\rangle$  suddenly sub-

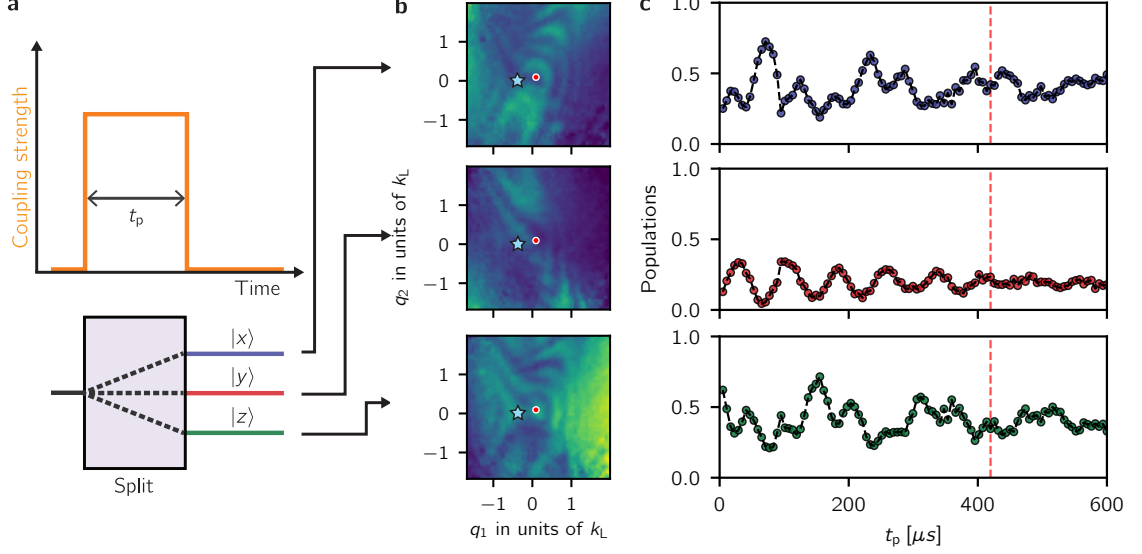


Figure 10: **a** Fourier spectroscopy protocol. We applied the Raman lasers for a variable time  $t_p$ : a Rabi-type atomic interferometer analogous to a three-port beam splitter. **b** Probabilities as a function of quasimomentum for a fixed Raman pulse time  $t_p = 420 \mu\text{s}$  **c** Dynamics of the final populations of the  $|xyz\rangle$  states with quasimomentum  $(q_1, q_2) = (-0.55, -0.18) k_L$  (blue star in panels **b**) after initializing the system in the  $|z\rangle$  state.

jected to the Raman coupling lasers. This atomic Rabi-type interferometer is analogous to the three-port beam-splitter depicted in Figure 10a. During a pulse time  $t_p$  we followed the dynamics of the populations in the  $|xyz\rangle$  states which evolved with oscillatory components proportional to  $\sum_{j \neq n} a_{n,j}(\mathbf{q}) \cos([E_n(\mathbf{q}) - E_j(\mathbf{q})]t_p / \hbar)$ , with frequencies determined by the eigenenergy differences  $E_n - E_j$ . Figure 10b shows the momentum dependent populations for a fixed pulse time  $t_p$  and Figure 10c shows representative final populations as a function of  $t_p$  for a fixed quasimomentum state. We Fourier transformed the populations with respect to  $t_p$  and for a given quasimomentum state for a total of 9 states accounting for each of the three initial  $|xyz\rangle$  states that were later split into 3 states, all of them with the same  $\mathbf{q}$ . Figure 11

shows the PSD computed for each of the 9 states for planes of constant  $q_1$ . The amplitude of the oscillatory components depends on the overlap integral between the initial state and the Raman dressed states (see Equation 5.1) so sampling all these states gave us access to a wider range of measurable frequencies. We produced the spectral maps in Figure 12b by averaging the PSDs from the 9 different states using  $\bar{n}$ , the mean population in  $t_p$ , as a weight:

$$\text{PSD}^{(\text{mean})}(\mathbf{q}) = \frac{\sum_{i,j} \text{PSD}_{i,j}(\mathbf{q}) \bar{n}_{i,j}(\mathbf{q})}{\sum_{i,j} \bar{n}_{i,j}(\mathbf{q})}, \quad (8.12)$$

where the indices  $i, j$  represent the different states of the grid shown in Figure 11. The extrema in the spectral maps are the energy differences  $E_n - E_j$  in the engineered dispersion (Figure 10a) and together they show the presence of a single Dirac point in the Rashba subspace, evidenced by the gap closing near  $\mathbf{q} = 0$  and the photon-like lower branch. The dashed curves correspond to the energy differences computed for our system using the dispersions shown in Figure 12a, and are in clear agreement with our experiment. This measurement directly confirms the expected set of energies, including the existence of a two-state subspace approximately described by the Rashba Hamiltonian.

## 8.5 Quantum state tomography with Ramsey interferometer

The Fourier spectroscopy measurement confirmed our quantum engineering of the Rashba Hamiltonian. However, the energies shed no light on the topology of the different branches of the dispersion, which instead requires knowledge of the eigen-

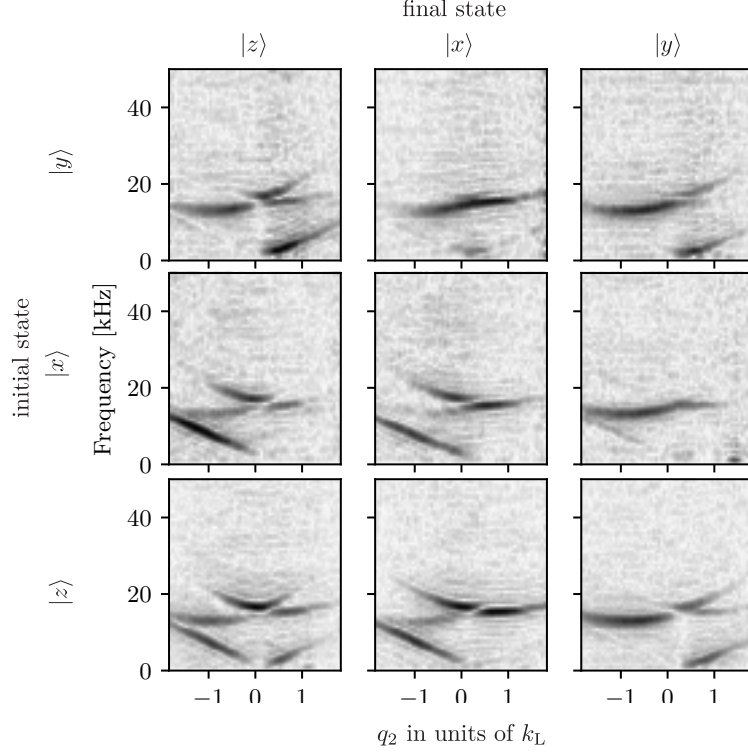


Figure 11: PSD of all the analyzed states as a function of  $q_2$  for fixed  $q_1 = 0.18 k_L$ . The different overlaps between the initial state, the Raman dressed states and the measured final state result in peaks with different amplitudes.

states. The Berry curvature present in the definition of the Chern number (Equation 7.11) can be derived from the Berry's connection  $\mathbf{A}_n(\mathbf{q}) = i \langle \Psi_n(\mathbf{q}) | \nabla_{\mathbf{q}} | \Psi_n(\mathbf{q}) \rangle$ , which as discussed in Chapter 7 behaves much like a vector potential in classical electromagnetism. The Berry curvature  $\mathbf{\Omega}_n(\mathbf{q}) = \nabla_{\mathbf{q}} \times \mathbf{A}(\mathbf{q})$  is the associated magnetic field and the flux through any surface is the line integral of  $\mathbf{A}(\mathbf{q})$  along its boundary, after neglecting the contributions of Dirac strings which I will discuss later. Using the expression for the Raman dressed eigenstates from Equation 8.9 we obtain

$$\mathbf{A}_n(\mathbf{q}) = - \sum_{j \in \{x,y,z\}} a_{n,j}(\mathbf{q}) \nabla_{\mathbf{q}} \phi_{n,j}(\mathbf{q}), \quad (8.13)$$

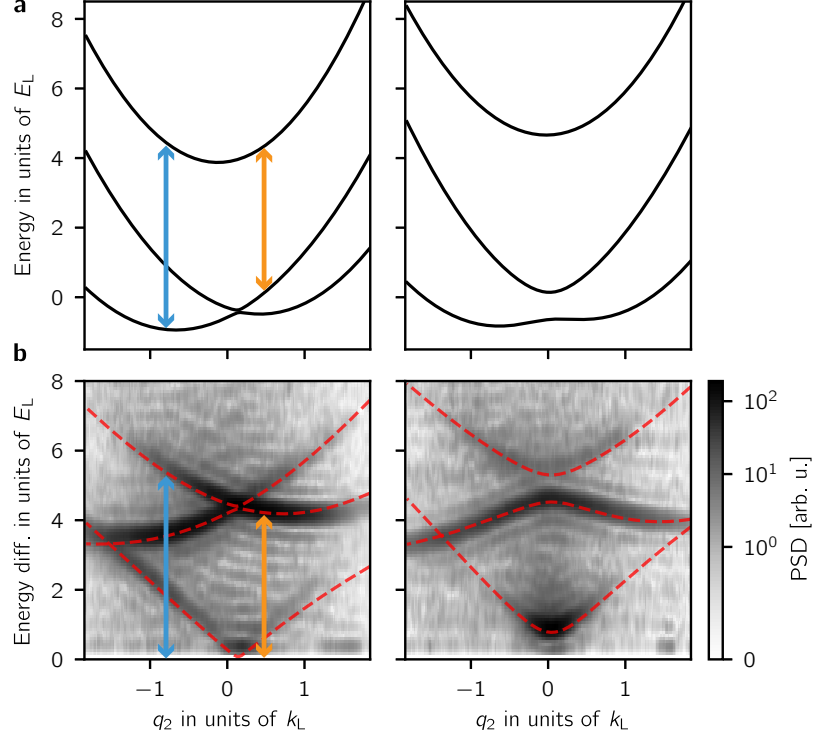


Figure 12: **a** Predicted dispersion relation as a function of  $q_2$  for fixed  $q_1 = -0.09 k_L$  (left) and  $0.65 k_L$  (right), computed for the experiment parameters. The energy differences between the branches enclosing the vertical arrows appear as peaks in the spectral maps below. **b** Power spectral density (PSD) for the same parameters as above which we obtained by Fourier transforming the populations in the  $|xyz\rangle$  states with respect to  $t_p$ . The dashed lines correspond to the energy differences computed using the dispersion curves on the top panel.

which depends on both the phase and amplitude of the wave function. We obtained  $a_{n,j}(\mathbf{q})$  and  $\phi_{n,j}(\mathbf{q})$  using a three-arm time-domain Ramsey interferometer (see Section 3.5.4), implementing a variant of quantum state tomography [142, 143]. The use of a multi-path interferometer allowed us to transduce information about phases into state populations, which we readily obtained from absorption images.

Figure 13 shows our experimental protocol which I will describe in more detail in the following section. We adiabatically mapped an initial  $|j, \mathbf{k}\rangle$  state into a corresponding eigenstate  $|n, \mathbf{q} = \mathbf{k} + \mathbf{k}_j\rangle$ , either in the topologically trivial highest

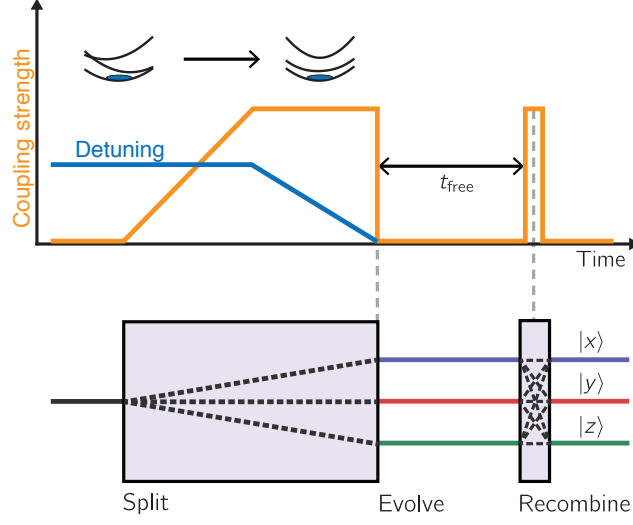


Figure 13: Experimental protocol for three-arm Ramsey interferometer (not to scale). (Top) We started with atoms in state  $|z, y, \mathbf{q}_i = \mathbf{k} + \mathbf{k}_j\rangle$  and with detuning  $\delta_y = \pm 5 E_L$  and  $\delta_z = \pm 5 E_L$ . We ramped the Raman lasers on in  $750 \mu s$  and then ramped the detuning to nominally zero. We let the system evolve in the dark for times between  $5 \mu s$  and  $400 \mu s$ , followed by a  $25 \mu s$  Raman pulse. (Bottom) The implemented experimental protocol was equivalent to a three-arm interferometer that split an initial state into three final states with amplitudes related to the initial wave function phases.

dispersion branch ( $n = 3$ ) or in the topological ground branch ( $n = 1$ ) by dynamically tailoring both the Raman coupling strength and detuning. We suddenly turned off the Raman coupling, allowing the three bare state components of the Rashba eigenstates to undergo free evolution for a time  $t_{\text{free}}$ , constituting the three arms of our time-domain interferometer. Finally, we applied a three-port beam splitter using a brief Raman ‘recombination’ pulse to interfere the three arms.

### 8.5.1 Wave function evolution in Ramsey interferometer

**Rashba dressed state preparation:** We started with  $|xyz\rangle$  states at different RF coupling strength  $\Omega_0 = \Omega_{\text{RF}}/\pi 2 \pm 20 \text{ kHz}$ , such that the energies of the

$|z\rangle$  and  $|y\rangle$  states were shifted by about  $\pm 18.8$  kHz. We used the same Raman frequencies as described earlier and the change in the  $|z\rangle$  and  $|y\rangle$  state eigenenergies corresponded to having non-zero values of  $\delta_z$  and  $\delta_y$  in Equation 8.8. We chose the detuning such that the initial state had a large overlap with either the  $n = 1$  or the  $n = 3$  eigenstates of Equation 8.8. We then ramped on the Raman coupling in  $750\ \mu\text{s}$ , adiabatically mapping the  $|z\rangle$  and  $|y\rangle$  states into the  $n = 1$  or  $n = 3$  eigenstates. Because our only experimental knob for dynamically changing the detuning was  $\Omega_{\text{RF}}$  we could not control  $\delta_x$  so when we initialized the system in  $|x\rangle$  the final dressed state always corresponded to the  $n = 2$  branch. After turning on the Raman, we ramped  $\Omega_{\text{RF}}$  to its final value in 1 ms, effectively ramping  $\delta_z$  and  $\delta_y$  close to zero. This detuning ramp had the additional effect of moving the location Dirac point through the atoms when loaded in the  $n = 1$  branch, creating a trajectory where the state preparation was not adiabatic. This trajectory depended on the sign of the detuning ramp so we combined data from different initial states that excluded the Dirac point trajectories. Near the final location of the Dirac point, the state preparation was not adiabatic regardless of the initial state or detuning used for the ground state preparation. Figure 14a shows an example absorption image of atoms initially prepared in the  $|y\rangle$  state and with  $\delta_y$  such atoms are loaded in the  $n = 1$  branch. The Dirac point was initially located near the lower-left edge of the cloud and when  $\delta_i$  was ramped the location of the Dirac point was dragged across the whole cloud leaving lines where non-adiabatic transitions occurred as can be seen in Figure 14b. The location of the Dirac point as a function of  $\delta_i$  can also be directly computed by numerically diagonalizing the SOC Hamiltonian from Equation 8.8.



At the end of this stage, excluding the points of non-adiabatic transitions, the state of the system was described by the eigenstates in Equation 8.9.

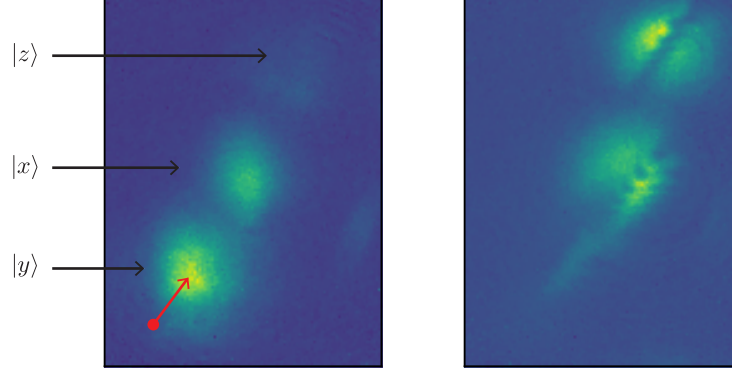


Figure 14: **a** We prepared atoms in the  $|y\rangle$  state such that they are adiabatically mapped to the ground dispersion branch. **b** We ramped  $\delta_i$  and dragged the Dirac point through the atoms.

**Free evolution:** We suddenly turned off the Raman coupling, projecting the Raman dressed states back into the  $|xyz\rangle$  basis. Each state represented a different branch of the interferometer and they acquired phases that are proportional to  $t_{\text{free}}$

$$|\Psi_n(\mathbf{q})\rangle \rightarrow \sum_{j \in xyz} \sqrt{a_{n,j}(\mathbf{q})} e^{i\phi_{n,j}(\mathbf{q})} e^{-iE_j(\mathbf{q})t_{\text{free}}/\hbar} |j, \mathbf{k} = \mathbf{q} - \mathbf{k}_j\rangle, \quad (8.14)$$

where  $E_j(\mathbf{q}) = \hbar^2 \mathbf{q}^2 / 2m$  is the free particle energy. The Rashba wavefunction phases  $\phi_{n,j}(\mathbf{q})$  that we were interested in measuring were imprinted during the loading procedure. The dynamical phases  $E_j(\mathbf{q})t_{\text{free}}/\hbar$  acquired in the different interferometer arms do not contribute to our knowledge of the Rashba eigenstates as they describe the evolution of the system in the absence of Raman dressing.

**Recombination pulse:** We applied a 25 us Raman pulse that acted as a

second beam splitter in our interferometer sequence. The wave function after the pulse was

$$|\Psi(\mathbf{q})\rangle = \sum_{j,j' \in xyz} \sqrt{a_{n,j}(\mathbf{q})} e^{i(\phi_{n,j}(\mathbf{q}) - E_j(\mathbf{q})t_{\text{free}}/\hbar)} U_{j,j'}(\mathbf{q}) |j, \mathbf{k} = \mathbf{q} - \mathbf{k}_j\rangle, \quad (8.15)$$

where  $U_{j,j'}(\mathbf{q}) = |U_{j,j'}(\mathbf{q})| \exp(i\phi_{j,j'}^{(\text{pulse})}(\mathbf{q}))$  is the matrix element of the unitary transformation  $\exp(i\hat{H}_R(\mathbf{q})t_{\text{pulse}})$  associated to the Raman pulse. At the end of this procedure, the population in a final state  $|l, \mathbf{q}\rangle$  was

$$P_l(\mathbf{q}, t) = \sum_{i \neq j} |U_{l,i}| |U_{j,l}| \sqrt{a_{n,i} a_{n,j}} \cos(\omega_{i,j}(\mathbf{q})t + \phi_{n,i}(\mathbf{q}) - \phi_{n,j}(\mathbf{q}) + \phi_{l,i,j}^{(\text{pulse})}(\mathbf{q})), \quad (8.16)$$

which directly reads out the phase differences. Here  $\phi_{l,i,j}^{(\text{pulse})}(\mathbf{q})$  is a smoothly varying phase imprinted by the recombination pulse and is independent of  $\mathbf{q}$  in the limit of short, strong pulses and does not affect the topological index of the system. The angular frequencies

$$\omega_{i,j}(\mathbf{q}) = \hbar \mathbf{q} \cdot \mathbf{k}_{i,j} / m + \delta_{i,j} \quad (8.17)$$

result from the known free particle kinetic energy, the recoil momenta and detuning  $\delta_{i,j}$  from the tripod resonance condition. Figure 13b shows the momentum-dependent populations in each output port at fixed  $t_{\text{free}} = 160 \mu\text{s}$  and Figure 13c shows the populations as a function of  $t_{\text{free}}$  for a representative quasimomentum state  $(q_1, q_2) = (0.55, -0.92) k_L$ .

We obtained the relative phases  $\Delta\phi_{n,i,j,l}(\mathbf{q}) = \phi_{n,i}(\mathbf{q}) - \phi_{n,j}(\mathbf{q}) + \phi_{l,i,j}^{(\text{pulse})}(\mathbf{q})$  from Equation 8.16 by fitting the measured populations to the sum of three cosines

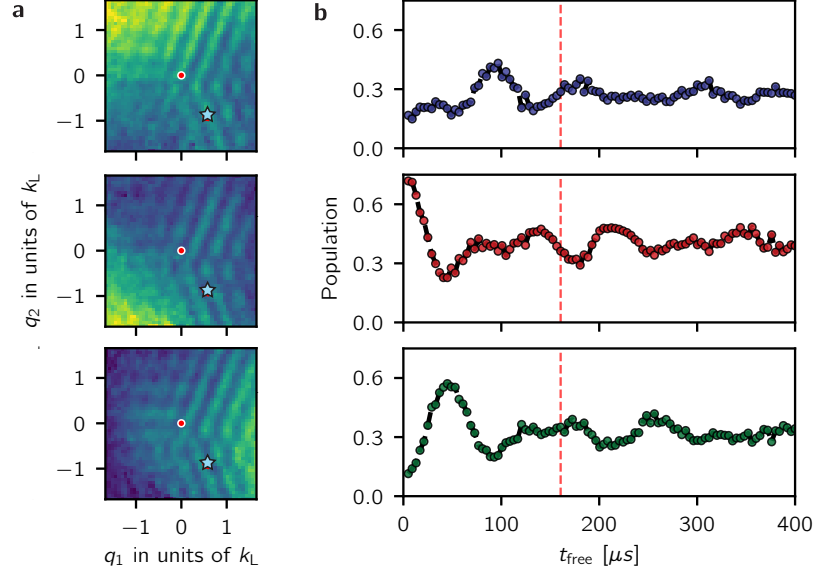


Figure 15: **a** Probabilities as a function of quasimomentum for the three output ports of the interferometer at  $t_{\text{free}} = 160 \mu\text{s}$  **b** Probabilities as a function of free evolution time  $t_{\text{free}}$  for an input state with quasimomentum  $(q_1, q_2) = (0.55, -0.92) k_L$  indicated by the blue star on **a** and in the topological ground branch ( $n = 1$ )

with the known free particle frequencies but unknown amplitudes and phases.

### 8.5.2 Combining phases from different measurements

We combined the phases  $\Delta\phi_{n,i,j,l}(\mathbf{q})$  obtained from fits to six different states (two initial states split into 3 states). Similar to the Fourier spectroscopy measurements, we performed a weighted average of the fitted relative phases

$$\Delta\phi_{n,i,j}(\mathbf{q}) = \frac{\sum_l \Delta\phi_{n,i,j,l}(\mathbf{q}) w_{i,j,l}(\mathbf{q})}{\sum_l w_{i,j,l}(\mathbf{q})}, \quad (8.18)$$

where the weights were determined using fit uncertainties and when loading atoms in the topological branch we additionally accounted for the trajectory of the Dirac

point during the loading procedure. Figure 16 shows an example of two different weight arrays used to combine the phase difference associated to the  $z \rightarrow x$  transition  $i, j = z, x$  for the topological branch  $n = 1$  ( $\Delta_{1,z,x}$ ):

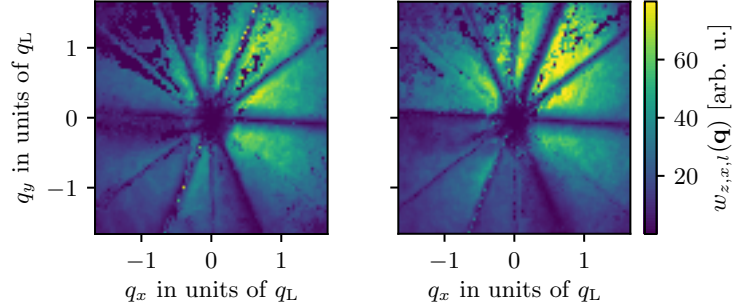


Figure 16: Two sample weight arrays  $w_{i,j,l}(\mathbf{q})$  for  $i, j = z, x$  and atoms prepared in the topological dispersion branch. We obtained the weights using the uncertainties from the fits and the trajectory of the Dirac point during the state preparation. We combined the weights and the phase differences  $\Delta\phi_{n,i,j,l}$  obtained from the time dependent fits using Equation 8.18 to obtain the phase maps displayed in Figure 17a

The ‘spokes’ in the weight arrays correspond to high uncertainty regions. This uncertainty comes from our inability to resolve the phases of low frequencies  $\omega_{ij}(\mathbf{q})$  as well as when two different frequencies  $\omega_{ij}(\mathbf{q})$  and  $\omega_{i'j'}(\mathbf{q})$  are close to each other which is limited by the largest value of  $t_{\text{free}}$  in the experiment. The phase maps in Figure 17a represent fits to a combined a total of 120,000 different time traces (60,000 for each dispersion branch).

### 8.5.3 Measuring the topological index

Figure 17a shows typical phase-maps for both the non-topological and topological branches. In the non-topological phase-maps, the momentum dependence of the recombination pulse  $\phi_{l,i,j}^p(\mathbf{q})$  causes a smooth variation of the phases along

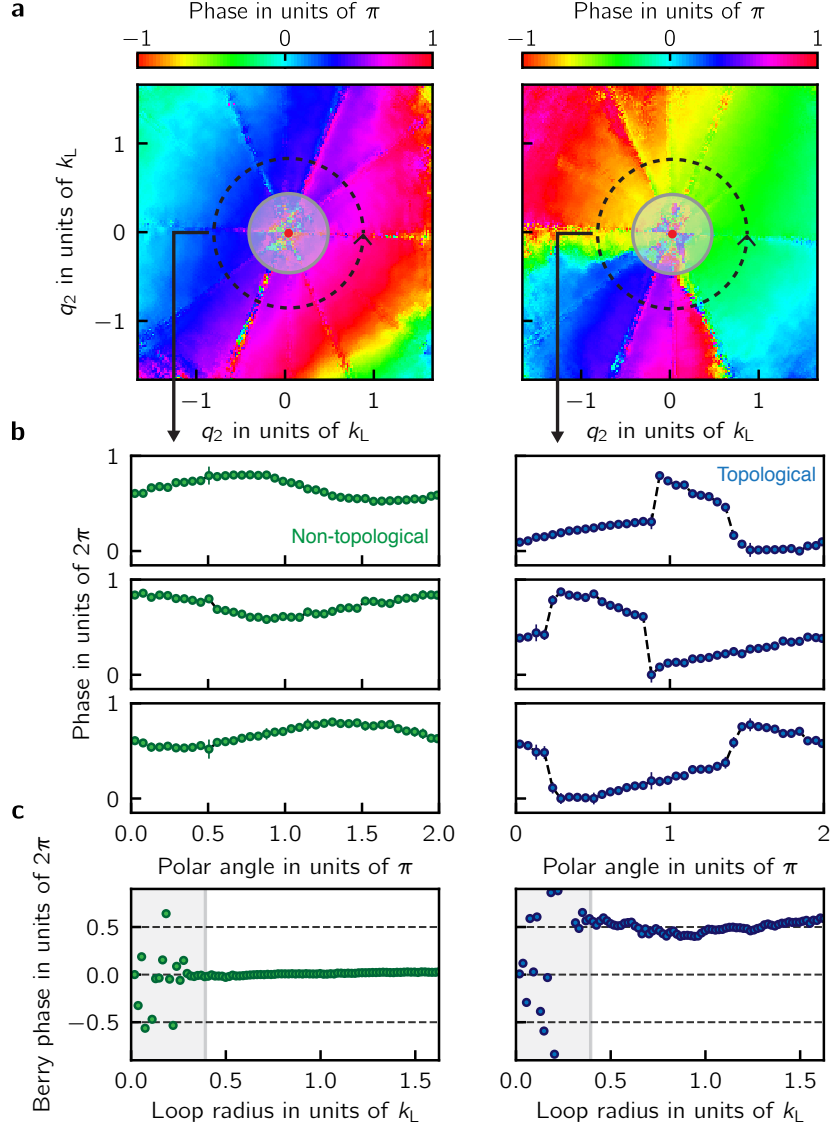


Figure 17: Topological invariants from quantum state tomography, for the non-topological branch ( $n = 3$ , left) and the topological branch ( $n = 1$ , right). **a** Phase differences as a function of quasimomentum from the  $z \rightarrow x$  transition **b** Phase differences as a function of polar angle for a loop radius  $0.77 k_L$  from the  $z \rightarrow x$  (top),  $x \rightarrow y$  (middle) and  $y \rightarrow z$  (bottom) transitions. The phases associated with the topological branch are characterized by two  $\pi$  valued discontinuities. Each row of phases was shifted by a constant value so that the three rows of phases share the same vertical axis. All phases shown here were binned and averaged using the fit uncertainties as weights. **c** Inferred Chern number as a function of loop radius. For loops with  $q > 0.4 k_L$  we obtained an integrated Berry phase and asymptotic Chern number of  $\Phi_B/2\pi = 0.01(1)$  for the non-topological branch and  $\Phi_B/2\pi = 0.5(5)$  for the topological branch.

the Raman recoil axes that does not affect the evaluation topological index of our system. We recovered the phases  $\phi_{n,j}$  of the full spinor wave function from the relative phases  $\Delta\phi_{n,i,j}(\mathbf{q})$  by choosing a particular gauge such that  $\phi_{n,3} = 0$ . We then used the values of  $a_{n,i}$  obtained from measuring the populations in the  $|xyz\rangle$  states at  $t_{\text{free}} = 0$  in combination with the phases of the wave function to compute the Berry connection [144]. Figure 17b shows the three phase differences as a function of polar angle for a loop of radius  $q \approx 0.77 k_L$ . In addition to the smooth variations induced by the recombination which are present in both columns, the phases of the topological branch have two  $\pi$  valued jumps that lead to non-zero Berry phases when the Berry connection is integrated along a closed loop in momentum space. Figure 17c shows the integrated Berry phase as a function of loop radius. As mentioned earlier the largest value of  $t_{\text{free}}$  limits how well we can resolve the phases of small frequencies and this is reflected in the large variation in the Berry phase depicted in the shaded region of Figure 17c near  $q = 0$ . For loops with  $q > 0.4 k_L$  we obtain an integrated Berry phase that suggests an asymptotic Chern number of  $\Phi_B/2\pi = 0.01(1)$  for the non-topological branch and  $\Phi_B/2\pi = 0.5(5)$  for the topological branch. However, Berry's phase measurements like ours include the (potential) contribution of any Dirac strings traversing the integration area. In our system, these are possible at the Dirac point (red dot near  $q = 0$ ), and each contributes  $\pm 2\pi$  to  $\Phi_B$  (see Section 7.6). Even with this  $2\pi$  ambiguity we are can associate a half-integer Chern number with the topological branch, which is possible only for a topological dispersion branch in the continuum.

## 8.6 Conclusion

In conventional lattices — for example graphene, or the topological Haldane model — it is well established that Dirac points each contribute a Berry’s phase of  $\Phi_B/2\pi = \pm 1/2$  [145], but crystalline materials conspire for these to appear in pairs [127], always delivering integer Chern numbers. In contrast, our continuum system contains a single Dirac point, resulting in a non-integer Chern number. This leads to intriguing questions about edge states at interfaces with non-integer Chern numbers with non-integer Chern number differences. Initial studies in the context of electromagnetic waveguides [146] and atmospheric waves [107] have applied Chern invariants and the bulk-edge correspondence to continuous media.

While the true Rashba Hamiltonian features a ring of degenerate eigenstates, our implementation including the quadratic and cubic Dresselhaus-like SOC lifts this macroscopic degeneracy giving three nearly degenerate minima [135]. Already these three minima could allow the study of rich ground state physics in many body systems of bosons, for example the formation of fragmented BECs [28] when the system does not condense into a single-particle state. Furthermore, the use of additional spin states or larger Raman couplings can partially restore this degeneracy allowing the possible realization of fractional Hall like states [147].

## Chapter 9: Conclusions and outlook

This thesis presented new experimental techniques that have proven to be useful in the control and characterization of ultracold atomic systems and applied them in a new implementation Rashba-type spin-orbit coupling.

We developed a Fourier transform spectroscopy technique [25] which is based on measuring the quantum coherent evolution of a single particle system under a quench of a Hamiltonian of interest. This technique was successfully used to measure the dispersion relation of a BEC with tunable 1D (equal combination of Rashba and Dresselhaus) spin-orbit coupling. The use of this technique was extended to thermal gases with broad momentum distributions to perform a parallelized measurement of the dispersion relation of a system with Rashba-type SOC [26] as well as the band structure of a fractional period adiabatic superlattice [27].

We implemented CDD in the ground hyperfine manifold of  $^{87}\text{Rb}$  by applying a strong RF magnetic field [81]. The CDD states are first-order insensitive to magnetic field fluctuations, making them effective clock states, and additionally, have non-zero matrix coupling elements that allow cyclical couplings that are not possible in the bare hyperfine  $|m_F\rangle$  states. The clock states have made our experiments more robust against magnetic field noise and were a necessary ingredient for the implementation



of Rashba spin-orbit coupling as well as the engineering of fractional period adiabatic superlattice and an ongoing project involving Hofstadter cylinders

Finally, we showed a new implementation of Rashba spin-orbit coupling using Raman induced transitions of the CDD states [78, 135] and characterized the topology underlying this system [26]. We presented a protocol for performing quantum state tomography which involves a three-arm Ramsey like interferometer and used it to reconstruct the momentum-dependent wave function and calculate topological invariants. Unlike conventional materials with an underlying crystalline structure where topological invariants take integer values, we found that our system in the continuum was characterized by half-integer invariants. Our Rashba implementation offers the possibility of studying new ground state physics at the nearly degenerate minima like the formation of fragmented condensates [28] or possible realizations of fractional Hall like states [147]. One open question for a half-integer Chern number system like ours is what kind of edge states emerge at the interfaces where the Chern numbers differ by a non-integer number.

All the current experimental efforts focus on understanding the physics of a Hofstadter cylinder under different magnetic fluxes and the role that disorder can play in driving phase transitions. Additionally, a considerable effort is going to the fabrication of a new apparatus that will allow the creation of ultracold samples of  $^{87}\text{Rb}$  and  $^{39}\text{K}$ . The use of bosonic  $^{39}\text{K}$  atoms will allow the tuning of scattering lengths using Feshbach resonances [148], enabling new kinds of experiments where atomic interactions can play an important role. The new apparatus will feature better optical access that will enable high resolution imaging and the projection of

arbitrary potentials.

## Appendix A: The good, the bad and the ugly of RbLi

This appendix summarizes the best, the worst and the meh aspects of the RbLi apparatus. Hopefully the items presented can help guide some decisions of future students building experimental apparatuses for ultracold atoms.

### A.1 The good

It is very easy to come up with a list of bad things that don't work quite well in the lab. Coming up with a list of good things that work well is harder as we do not tend to think too much about the things that we are not currently fixing. The list below summarizes what I believe are the outstanding good players in the lab.

**Overkill transistor banks:** Large currents in the lab (quadrupole and Zeeman slower) are controlled with transistor banks formed by a group of MOSFETS whose drain and source are connected in parallel and sharing the same gate voltage that is controlled by a PI servo. The Zeeman slower always operates at a fixed current but the current in the quadrupole coils is dynamically changed throughout the experimental sequence and a fast response is desirable. In 2013 we replaced the quadrupole MOSFET bank with a new unit that contains 20 IXFN 520N075T2 transistors rated for 75 V and 480 A (left panel of Figure 1). Even though our currents

are well below the 480 A limit, the performance of the transistors really decays as the drain to source voltage is increased as can be seen in the right panel of Figure 1. The use of more transistors reduces the power dissipation of each individual transistor which allows us to operate the power supply at a higher voltage of 15 V that helps counteract the inductive kickback of the coils. With the new transistor bank the turn on time of the coils was reduced from 100 ms to 5 ms leading to improved magnetic trapping and better Stern-Gerlach pulses for imaging, only with an unavoidable small number of blown off transistors. This was a one time problem that happens typically when the turn-on voltage for one or a few transistors is slightly lower than the rest and therefore few number of transistors end up dissipating all the power when they are first turned on. Using transistors from the same batch is advisable so that they operate as similarly as possible.

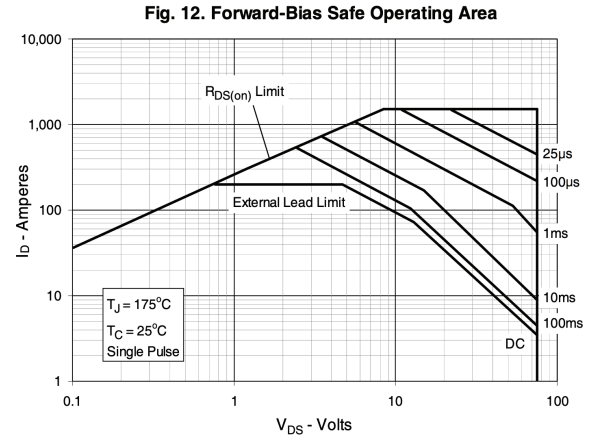
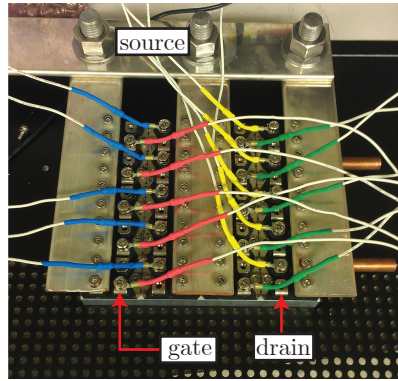


Figure 1: Left: New MOSFET bank. Right: Safe operation regime of the IXFN520N075T2 MOSFET. Even though they are in principle rated for up to 480 A the maximum safe current is greatly reduced at larger drain to source voltages  $V_{DS}$ . A high  $V_{DS}$  is desirable to reduce the inductive kickback during turn on.

**Oven chamber including hand made in vacuum shutters:** Before going into the Zeeman slower, the atoms that are heated in the Rb oven travel to the main oven chamber that is pictured in Figure 2b containing a cold-cup and an oven shutter. The cold-cup is a cylindrical shaped copper piece that is attached to the cold end of a thermo-electric cooler (TEC) via a copper rod. We keep the cold-cup temperature at  $-30$  C in order to capture excess Rb atoms in the chamber and prevent damaging the ion pumps. The oven shutter allows us to block after the MOT loading stage to prevent unwanted heating. We use a homemade (by Ian) device, made from a re-purposed hard drive disk shutter with a metallic flag attached to its end. The shutter is electrically connected to an electric feedthrough with vacuum-compatible Kapton sealed wires. Other apparatus within the JQI [149] have commercial shutters from Uniblitz and some of them have failed in the past. Overall we have found this setup to be very reliable. The only problem we experienced once was some accumulation of Rb on the cold cup that started blocking the atomic beam. To remedy this we reversed the polarity of the TEC and heated the cold cup barely enough so that the accumulated Rb atoms melted and moved away from the aperture of the atomic beam.

**Ultraviolet LEDs:** We have two 3 W ultraviolet LEDs from Mightex placed at the glass cell side of the vacuum system. One is aimed at the vacuum window where the slower beam enters and the other is placed aiming at the glass cell. The LEDs prevent Rubidium from depositing on the vacuum system and can conveniently be turned on and off with a TTL signal from the computer. We have found that routinely turning them on (for example, leaving them on overnight) leads to a

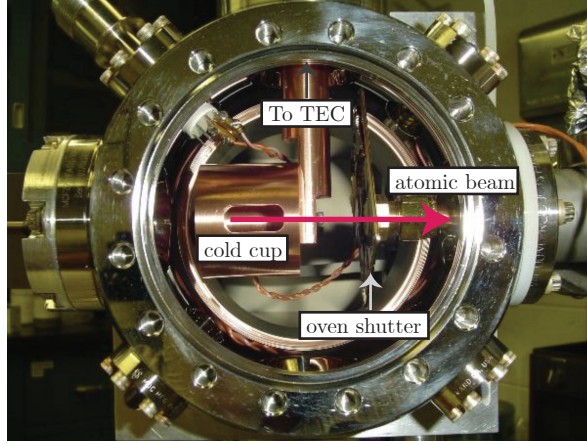


Figure 2: The RbLi oven chamber. We use a cold cup to prevent excess Rb atoms from damaging the ion pumps and a homemade in-vacuum shutter to block the atomic beam after the MOT stage to prevent heating of the atoms at later stages.

smoother operation of the system.

**Mirror mounts with picomotor actuators:** We use 8816-6 picomotor optical mounts from New Focus Optics whose deflection angle can be electronically adjusted on the order of microradians. The addition of picomotor mounts has made alignment of laser beams to the atoms significantly easier. We use this mounts on the last tunable mirror before the atoms for beam paths whose alignment is critical, for example in optical dipole trap and Raman beams.

**Polarizers on MOT beams:** This item is a bad thing disguised as a good thing. The light of our MOT beams is coupled to polarization maintaining optical fibers. We found that despite our best efforts to align the polarization of the incoming light to the slow/fast axis of the fiber the fluctuations in the output polarization could cause considerable instabilities in the BEC production. To keep the polarization clean we placed polarizers at the output of the fibers. We found that despite the power hit we get from the changes in polarization, this solution leads to a much

more stable production of BECs.

**Lab couch:** When the experiment is functional enough that data can be taken long hours in the lab are often required. If it gets late, the lab couch allows the person running the experiment to take small naps as the data keeps coming while still being close to the apparatus in case something needs to be fixed. A slightly rested grad student tends to make less poor decisions than an exhausted grad student.

**Other elements already mentioned in the main text:** The new master laser from Vescent photonics has been very stable and reliable. The new Mako camera has been very helpful to get rid of unwanted fringes in absorption images. Labscript makes writing experimental sequences very straightforward.

## A.2 The bad

The bad, these are elements of the apparatus that were constant sources of pain and if considering a new experimental design should be avoided.

**Water cooling shared between two labs:** The quadrupole and Zeeman slower coils as well as the transistor banks require water cooling due to Joule heating. Our lab space is shared with a Rubidium-Ytterbium ultracold mixtures apparatus [150] and amongst the things we share is the water cooling system. The schematic in Figure 3 illustrates a simplified layout of the water cooling system. The water was filtered at two different points, first each line has a  $440\text{ }\mu\text{m}$  particulate filter from Swagelok and then the water returning to the heat exchanger is

filtered with a low-impedance cellulose cartridge (McMaster 7191K11). Both filters only capture impurities in the water for one given flow direction. One of the failure modes which occurs when one of the booster pumps is turned on before the heat exchanger, causing water to flow from one experiment to the other and bringing a collection of nasty things that escapes the filters into the coils. Over the years our system has suffered of clogged filters, clogged coils and broken booster pumps. For best operation it is highly recommended that the cartridge filter is changed and that the Swagelok filters be cleaned at least once a year and that a 10% solution of an anti-corrosive **Optishield Plus** in water is used as a coolant. Even when following this practices, we managed to find lots of gunk and unidentified objects (sand? glass? mud? oxide? dead bacteria?) in the water, just at a slower rate. Besides this issue, just having to interrupt the operation of two experiments whenever any plumbing works needs to be done on one experiment is quite inconvenient.

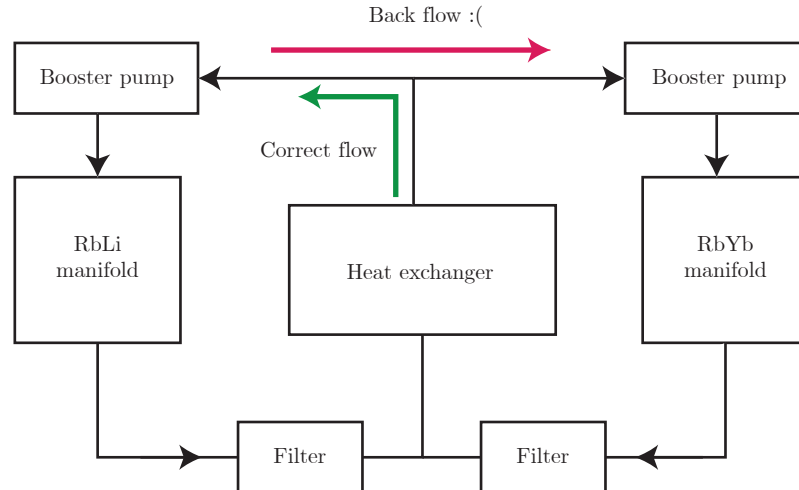


Figure 3: Simplified schematic of the shared water cooling manifold.



**Flipper mirrors:** The optical path of the MOT beams near the atoms is very close to that of Raman, optical dipole trap and probe beams. Since the MOT beams are only used at the early stages of the experiment it is tempting to use flipper mirror mounts so that once they are no longer needed they can be moved away to make space for other beams. This was the approach originally taken in the lab and we used 8893-K motorized optical flipper mounts from Newport in multiple locations. As they break over and over again, they have been slowly replaced by more stable solutions such as periscopes or polarizing beam splitters wherever it is possible. Flipper mirrors are always bound to break, it is only a matter of time before it happens. Avoid using them unless you absolutely have no alternative.

**Optical fibers right below air vents:** The optical fibers connecting the main experiment optical table and the laser optical table are routed close to a pair of AC vents in the lab. The changes in air temperature result in polarization fluctuations at the output of optical fibers (see previous section about MOT polarizers), a constant cause of pain and instability in our BEC production. We have tried to remedy this issue by partially blocking vents and enclosing the fibers in a large PVC tube.

**Free spaced dipole laser:** The laser system providing 1064 nm light for the optical dipole trap is not fiber coupled and is setup in the same optical table as the vacuum system; we are not able to change the laser without destroying the alignment of the dipole trap with the atoms. This issue became relevant while setting up a 1D optical lattice by retro-reflecting one of the dipole trap beams as we noticed that the laser mode changes on a very fast timescale, leading to large fluctuations

in the optical lattice. In the original design of the laser the light was meant to be fiber coupled using high-power photonic crystal fibers but they did not have built in mode expanders which resulted in the tip of the fibers repeatedly getting burnt after some time of use. In short, mode expanders are recommended in applications involving large optical powers.

### A.3 The ugly

The ugly elements are not quite bad but they don't function flawlessly either. If given the option to replace them with something better I definitely would.

**Kepco bipolar power supplies:** We use three Kepco BOP 20-20M bipolar power supplies to provide the current for the bias coils. While it is nice to have a commercially available power supply that can provide  $\pm 20$  A they come with a few drawbacks. First the current they provide has 60 Hz noise in it and in order to suppress it and stabilize the currents we must use a PI feedback circuit. The power supplies has multiple banks of NPN and PNP transistors inside mounted on a big heat sink with fans attached to it making them quite noisy; it is not optimal to place them close to the main experiment chamber and long connections open the door to unwanted ground loops. Additionally they have a few failure modes. All the problems we experienced whenever they stopped working (e.g. the output current railing) were symptoms of malfunctioning transistors, something that we experienced on multiple occasions.

**Toptica’s BoosTA:** Our cooling light comes from a Toptica DL Pro laser that is amplified using a Toptica BoosTA tapered amplifier system. The output power of this TA has been relatively stable over the years, in fact it has been operating for over 7 years now without the need of replacing the TA chip, unlike a homemade TA setup (see [62,63]). The downside is it has a tendency to turn itself off and on its worst days it would turn off so often that it would be impossible to operate the experiment. We have not been able to identify the problem despite our best efforts to look into the TA controller, the TA itself, multiple conversations with Toptica engineers, etc.

**Too many USB devices connected to the same computer:** We use multiple USB-6229 data acquisition (DAQ) devices from National Instruments. They are located at different points of the lab and then connected to the control computer through USB to optical fiber adapters that break the ground between the computer and the rest of the lab equipment (a practice we always try to follow when connecting things to the computer). We have a total of 6 NI devices in addition to other equipment like oscilloscopes all connected to the computer through BNC cables. Often times we struggled with the computer failing to detect one or multiple devices and it would take a very special (and different every time) combination of plugging and unplugging, turning off and turning back on things until all devices were recognized by the computer. We observe that the problem occurs less often when we don’t have as many USB devices connected to the computer.

## Appendix B: New apparatus

As mentioned in the main text, the construction of a new apparatus for producing BECs of  $^{87}\text{Rb}$  and  $^{39}\text{K}$  is underway. The design of the apparatus is intended to be a bug fix version of the RbChip [151] lab at NIST. The new apparatus does not have a Zeeman slower and instead will use magnetic transport coils to move atoms from a MOT cell to the main science cell.

This Appendix describes some aspects of the design and construction of the new apparatus where I was involved. Disclaimer: none of these things have been tested yet so we don't know yet if it will all work horribly. I have tried to include an extensive list of the part numbers used so they can be used as a future reference.

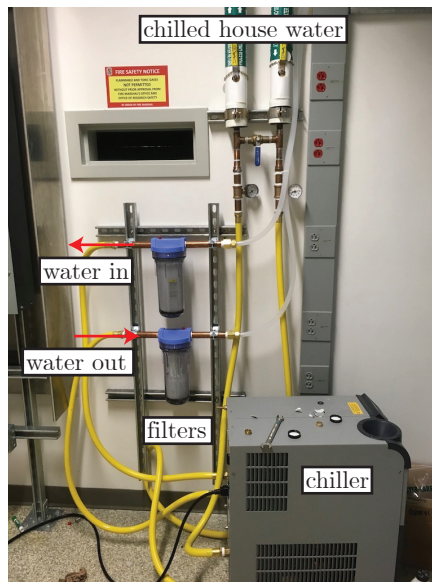
### B.1 Water cooling

The lack of a Zeeman slower in the apparatus greatly simplifies the water cooling system compared to that of RbLi. Since we don't anticipate to have any coils with high flow impedance we expect that the pressure from a recirculating chiller will be enough to provide water cooling to the transistor banks and the magnetic transport coils.

Our choice of chiller was the TF1LN400-LN 1.4 kW recirculating chiller from

Thermo Fisher Scientific. The water is filtered both at the output and the return with a high-impedance filter with a cellulose cartridge filter(replacement filter: McMaster 4422K3, filter housing: McMater 7191K11). A breakout manifold divides the chilled water into 5 different lines, each one with a flow meter (Proteus Industries 0101C110) that can be used to interlock the current of water cooled applications to the flow of water. Based on previous experiences with plumbing (see Appendix A) I highly encourage replacing the filters at least once a year and to use a solution of 10% corrosion inhibitor (e.g. OptiShield Plus) and distilled water as a coolant.

a. Service corridor side



b. Experiment side

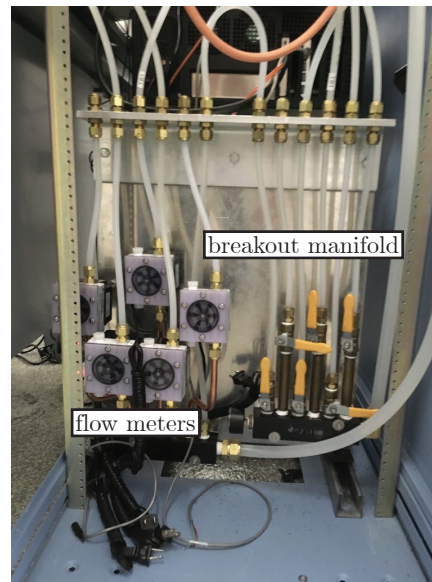


Figure 1: Water cooling

## B.2 Electrical installation

We have two Agilent 6690A (440 A at 15 V max. current) to provide all the necessary currents. The power supplies are located in the service corridor and are connected to three copper bars corresponding to  $\pm 15$  V and ground using welding cable (McMaster 7818A17) that is laid on cable trays (McMaster 30065T11 and similar). The copper bars serve as a hub for power distribution inside the lab where lab devices such as transistor banks can be connected. The positive and negative terminals of the power supplies have two cables attached to them and they are all arranged in the pattern shown in Figure 2 so that the magnetic field produced by currents in them is closer to a magnetic quadrupole which decays faster than the field of a magnetic dipole. We are trying to minimize unwanted magnetic fields at the experiment and being nice to our neighbors as well. We are not planing to use commercial bipolar power supplies in this lab (see Appendix A) and instead we will be using a MOSFET based homemade supply that will draw current from the  $\pm 15$  V rails.

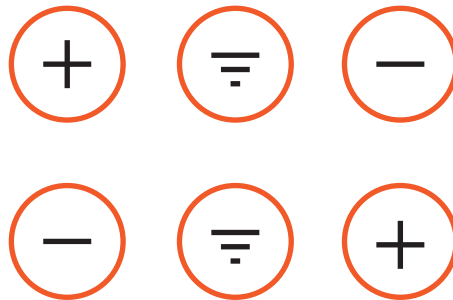


Figure 2: Configuration of the welding cables connecting the power supplies to equipment in the lab.

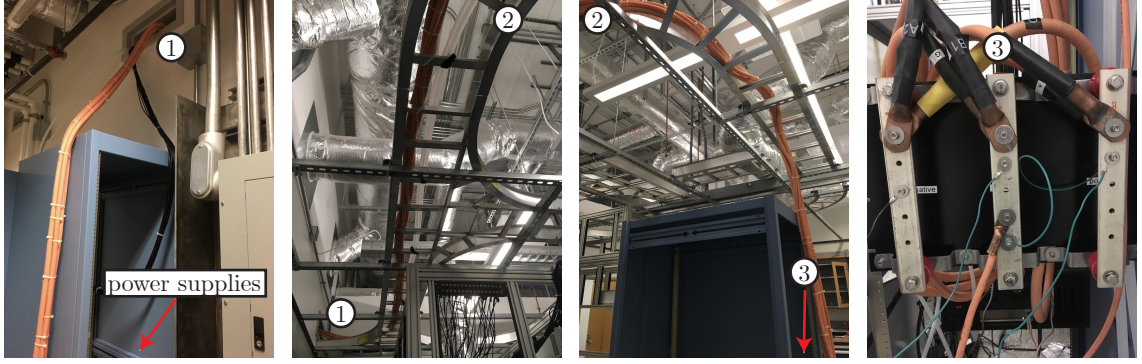


Figure 3: A roller coaster ride, from the power supplies in the service corridor to three copper bars that distribute the power.

### B.3 Coil winding

All the coils in the apparatus including magnetic transport, bias and gradient cancellation will be made of ribbon wire (**Laminax** from **Bridgeport Magnetics**). We followed the the coil fabrication process described in [151] which involves first winding a fixed number of turns around a prefabricated form with a particular geometry. The coils were then covered with a machinable epoxy (**Stycast 1266**) to fill in any air gaps. A lesson we learned while doing this is that only room temperature epoxy should be mixed. We keep the epoxy in a fridge to extend its lifetime but if it is cold some tiny drops of water will condense in it as it is being mixed and it will not properly be cured. To minimize the air bubbles inside the epoxy we placed the coils on a vacuum bell and we pumped the air out using an electric vacuum pump (**McMaster 4396K21**). After the epoxy has cured (overnight if it is left at room temperature or less if it is left at higher temperature) the coils

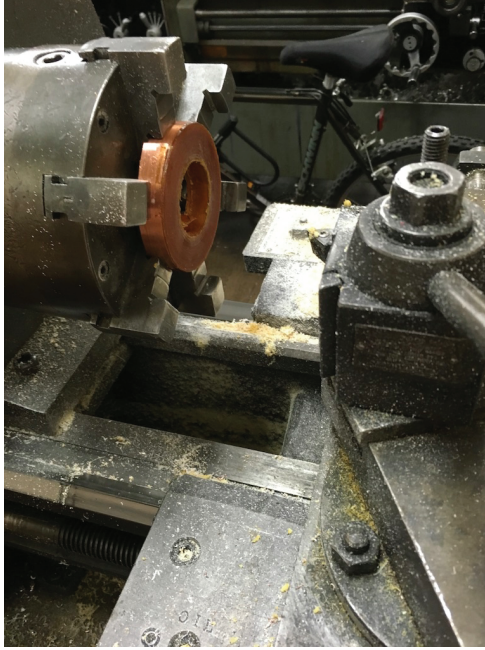
are ready for lathing to remove all excess epoxy and kapton tape up to the surface of the copper. After some trial and error (and lots of frustration) we found that using a diamond tip cutter (McMaster 3316A32) and spinning the lathe not faster than 150 rpm gives the best results. Using a cutter that is not sharp enough or cutting too aggressively close to the soft copper results in deformed traces that merge into each other causing unwanted shorts. The coils of the RbChip apparatus were machined at NIST using a special aluminum form to mount the coils on the lathe (see. The machinist at UMD considered this was not safe enough so I instead had to mount the coils using a 6 jaw chuck as shown in Figure 4a. For anyone making coils in the future: it is sort of an art to get it right and screwing up many coils at the beginning is part of learning the art. That being said, an important aspect of learning the ‘art’ right be as careful and consistent as possible from the start.

#### B.4 Rb source and ‘oven’

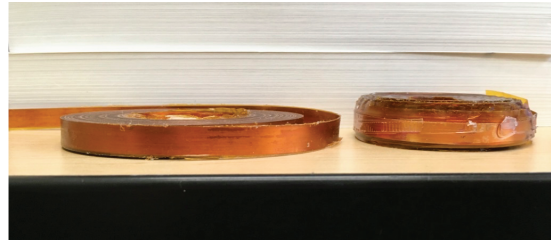
The new apparatus will have a cold ‘oven’ to keep the vapor pressure low connected to a glass cell where light induced atomic desorption (LIAD) [152] will be used to increase the Rb vapor pressure for MOT loading using non-thermal means. The Rb source consists of a 1.33” CF flanged bellow (Kurt J. Lesker MH-CF-A03) with a Rb ampule. The bellow is housed in the ‘oven’ which is designed to keep the source at temperatures near 1 C. The oven is made of hollow aluminum cylinder with a slit on one side with tapped holes so that 1/4 – 20 screws can be used to tighten it and fix the oven to the source. The bottom of the oven attaches to the



**a.** Coil mounted on lathe



**b.** Coils before and after lathing



**c.** Coil labeling system



Figure 4: **a.** I used a six jaw chuck and a diamond cutter on the lathe to remove the excess epoxy and kapton on the coils. **b.** Coils before (left) and after lathing (right). **c.** A good labeling system is important to keep to ensure uniformity of coils. The ‘Wartortle’ coil shown here has 64 turns. During fabrication we keep track of all the number of turns and resistance of all coils in a table.

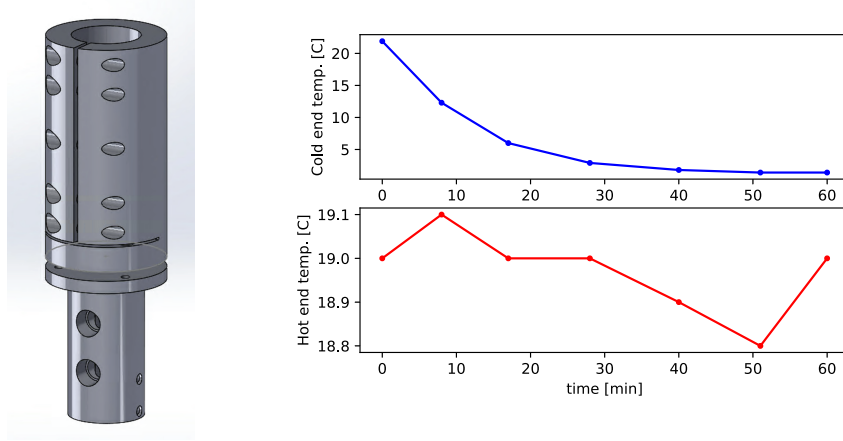
cold end of a TEC that provides the cooling. The hot side of the TEC is attached to a heat sink made of a hollow brass piece with tapped holes for 1/4” NPT pipes that will be used to provide water cooling. Figure 5a shows CAD drawings of both of this parts.

We tested the performance of a prototype without any vacuum systems attached to it. We applied a 2 A current to two TECs (Digikey 102-1664-ND) sandwiched in between the heat sink and oven. The heat sink was water cooled using  $\approx [18]C$  water. Figure 5b shows the temperature of both the oven side (top) and the heat sink side (bottom) of the assembly. We did not use any insulation for this

test to prevent condensation (see Figure 5c right) which should be done once its installed on the apparatus.

Our initial plan was to control the temperature using a linear temperature controller designed at the JQI (the design is available at the [JQI GitHub](#)) interfaced to the lab computer and Labscript using a serial to ethernet adapter (WIZnet WIZ107SR). This project is not completed to this date.

**a.** Rb ‘oven’ and heat sink    **b.** Bench test temperatures



**c.** Testing prototype oven and heat sink

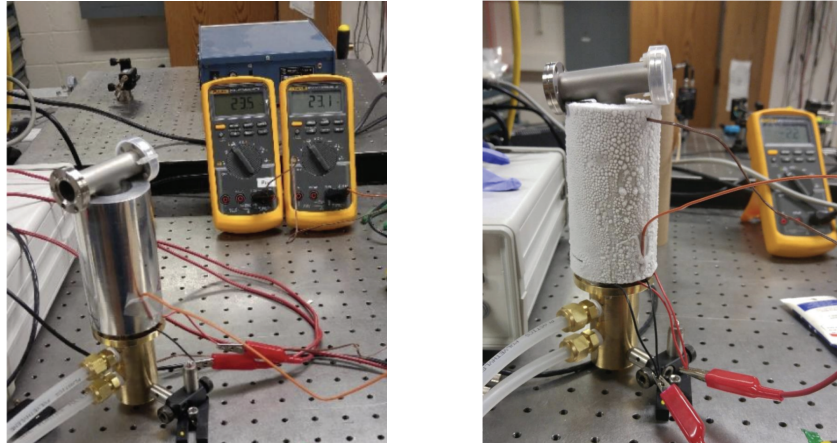


Figure 5: Rubidium oven assembly

## B.5 Table enclosures

The enclosures of the optical tables are made of **Alumalite** from **Laminators Inc.** mounted on frames made out of aluminum extrusions from 80/20 and sliding tracks (2220 and 2210). Alumalite is a sandwich of a corrugated corrugated polypropylen material in between two thin sheets of aluminum. We chose this material because it is strong and lightweight. Its laser safety properties remain to be tested but we anticipate it is better than the acrylic panels at the RbLi lab which are essentially transparent at 1064 nm.

To the new and future members of the lab: I sincerely hope the things I designed and built don't cause you much pain!

## Appendix C: Full derivation of the Raman coupled $|xyz\rangle$ states

In this Appendix I derive the full time-dependent Hamiltonian describing the  $|xyz\rangle$  states coupled by three Raman beams to produce Rashba type SOC (see Chapter 8. It takes a bit of thinking to understand what the frequencies of the different Raman coupling terms correspond to in the RF rotating frame where the  $|xyz\rangle$  states are defined. Hopefully this Appendix helps to clarify our specific choices for the laser geometry and polarization.

Our system is based on the proposal described in [78] to engineer a system with Rashba-like SOC. We consider  $^{87}\text{Rb}$  atoms in the ground hyperfine  $F = 1$  manifold subject to a constant magnetic field  $B_0\mathbf{e}_z$  and an RF magnetic field  $B_{\text{RF}}\cos(\omega_{\text{RF}}t)\mathbf{e}_x$  as in Chapter 6. The system is described by the Hamiltonian

$$\hat{H}_{\text{RF}} = \omega_0\hat{F}_z - \frac{\epsilon}{\hbar}(\hat{F}_z^2 - \mathbb{1}) + 2\Omega_{\text{RF}}\cos(\omega_{\text{RF}}t)\hat{F}_x, \quad (\text{C.1})$$

where  $\omega_0 = g_F\mu_B B_0$  is the Larmor frequency,  $\epsilon$  is a quadratic Zeeman<sup>1</sup> shift that breaks the degeneracy of the  $|m_F = -1\rangle \leftrightarrow |m_F = 0\rangle$  and  $|m_F = 1\rangle \leftrightarrow |m_F = 0\rangle$  transitions,  $\Omega_{\text{RF}} = g_F\mu_B B_{\text{RF}}/2$  is the RF coupling strength and  $\mathbb{1}$  is the identity

---

<sup>1</sup>the quadratic dependence of  $\epsilon$  on the applied bias field is not explicitly indicated on the symbol but is important to take into account to get the detuning just right

matrix. We transform the Hamiltonian into the RF rotating frame using the unitary transformation  $\hat{U}(t) = \exp(-i\omega_{\text{RF}}t\hat{F}_z)$ . The spin-1 operators under this transformation become

$$\begin{aligned}
\hat{F}_x &\rightarrow \cos(\omega_{\text{RF}}t)\hat{F}_x - \sin(\omega_{\text{RF}}t)\hat{F}_y \\
&= e^{i\omega_{\text{RF}}t}\hat{F}_+ + e^{-i\omega_{\text{RF}}t}\hat{F}_- \\
\hat{F}_y &\rightarrow \sin(\omega_{\text{RF}}t)\hat{F}_x + \cos(\omega_{\text{RF}}t)\hat{F}_y \\
&= \frac{1}{i}(e^{i\omega_{\text{RF}}t}\hat{F}_+ - e^{-i\omega_{\text{RF}}t}\hat{F}_-) \\
\hat{F}_z &\rightarrow \hat{F}_z.
\end{aligned} \tag{C.2}$$

The unitary evolution in the rotating frame is described by the transformed Hamiltonian  $\hat{U}^\dagger(t)(\hat{H}_{\text{RF}} - i\hbar\partial_t)\hat{U}(t)$ , which after neglecting terms that are oscillating with angular frequency  $2\omega_{\text{RF}}$  is

$$\hat{H}_{\text{RWA}} = \Delta\hat{F}_z - \frac{\epsilon}{\hbar}(\hat{F}_z^2 - \mathbb{I}) + \Omega_{\text{RF}}\hat{F}_x \tag{C.3}$$

The eigenstate of Equation C.3 are the  $|xyz\rangle$  states described in Chapter 6. Now we apply three Raman beams as shown in Figure 1. This configuration differs from the one proposed in [78] for technical reasons as we wanted all the Raman recoil vectors to lie within the imaging plane of the experiment which corresponds to the  $xy$  plane.

The electric field at the atoms in the lab (not rotating) frame is

$$\mathbf{E}(x, t) = \sum_{i=1}^3 E_i \mathbf{e}_i e^{i(\mathbf{k}_i \cdot \mathbf{x} - \omega_i t)}, \tag{C.4}$$

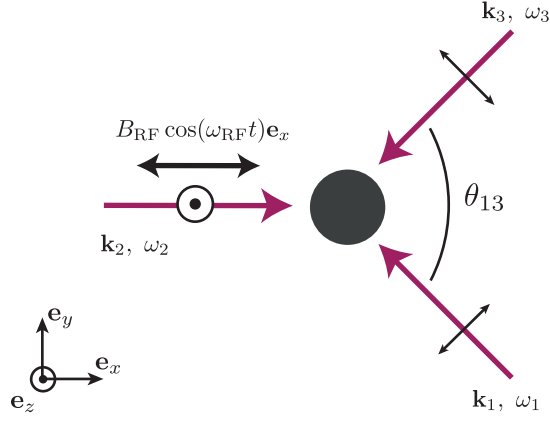


Figure 1: Laser layout: We use a strong RF field and three linearly polarized Raman beams propagating in the  $xy$  plane couple the  $|xyz\rangle$  states and engineer the Rashba Hamiltonian.

where  $E_i$  is the field amplitude,  $\omega_i$  is the angular frequency,  $\mathbf{k}_i$  is the wave vector and  $\mathbf{e}_i$  is the polarization of each of the beams. In order to have the right coupling matrix elements to cyclically couple all three states in a ring-like coupling as described in [135] we need a Hamiltonian of the form

$$\hat{H}_{\text{SOC}} = (\Omega_x, \Omega_y, \Omega_z) \cdot \hat{\mathbf{F}} \quad (\text{C.5})$$

(see Section 6.3.3). This is possible if we choose two Raman beams to have in plane polarization and one vertically polarized beam:

$$\begin{aligned} \mathbf{e}_1 &= \frac{(k_{1y}, -k_{1x}, 0)}{||\mathbf{k}_1||^2}, \\ \mathbf{e}_2 &= (0, 0, 1), \\ \mathbf{e}_3 &= \frac{(k_{3y}, -k_{3x}, 0)}{||\mathbf{k}_3||^2}, \end{aligned} \quad (\text{C.6})$$

The Raman coupling strength is proportional to the vector polarizability (see Section 3.3.2) and the Hamiltonian describing the atom-light coupling is

$$\hat{H}_R = (iu_v \mathbf{E} \times \mathbf{E}^*) \cdot \hat{F}, \quad (\text{C.7})$$

where  $u_v$  is the vector polarizability. The expression for the cross product of the electric field at the atoms is quite messy, lets rewrite it in a more convenient way:

$$\begin{aligned} \mathbf{E} \times \mathbf{E}^* &= (E_1^* \mathbf{e}_1 e^{-i(\mathbf{k}_1 \cdot \mathbf{x} - \omega_1 t)} + E_2^* \mathbf{e}_2 e^{-i(\mathbf{k}_2 \cdot \mathbf{x} - \omega_2 t)} + E_3^* \mathbf{e}_3 e^{-i(\mathbf{k}_3 \cdot \mathbf{x} - \omega_3 t)}) \times c.c \\ &= E_1^* E_2 (\mathbf{e}_1 \times \mathbf{e}_2) e^{i[(\mathbf{k}_2 - \mathbf{k}_1) \cdot \mathbf{x} - (\omega_2 - \omega_1)t]} + E_1^* E_3 (\mathbf{e}_1 \times \mathbf{e}_3) e^{i[(\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{x} - (\omega_3 - \omega_1)t]} \\ &\quad + E_2^* E_1 (\mathbf{e}_2 \times \mathbf{e}_1) e^{i[(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{x} - (\omega_1 - \omega_2)t]} + E_2^* E_3 (\mathbf{e}_2 \times \mathbf{e}_3) e^{i[(\mathbf{k}_3 - \mathbf{k}_2) \cdot \mathbf{x} - (\omega_3 - \omega_2)t]} \\ &\quad + E_3^* E_1 (\mathbf{e}_3 \times \mathbf{e}_1) e^{i[(\mathbf{k}_1 - \mathbf{k}_3) \cdot \mathbf{x} - (\omega_1 - \omega_3)t]} + E_3^* E_2 (\mathbf{e}_3 \times \mathbf{e}_2) e^{i[(\mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{x} - (\omega_2 - \omega_3)t]} \\ &= 2i \left[ (\mathbf{e}_1 \times \mathbf{e}_2) \text{Im}\{E_1^* E_2 e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21}t)}\} \right. \\ &\quad + (\mathbf{e}_1 \times \mathbf{e}_3) \text{Im}\{E_1^* E_3 e^{i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32}t)}\} \\ &\quad \left. + (\mathbf{e}_2 \times \mathbf{e}_3) \text{Im}\{E_2^* E_3 e^{i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32}t)}\} \right] \end{aligned} \quad (\text{C.8})$$

and using the definition of the polarization vectors (Equation C.6)

$$\begin{aligned} \mathbf{e}_1 \times \mathbf{e}_2 &= \frac{(-k_{1x}, -k_{1y}, 0)}{||\mathbf{k}_1||^2} = -\hat{\mathbf{k}}_1 \\ \mathbf{e}_1 \times \mathbf{e}_3 &= \frac{(0, 0, -k_{1y}k_{3x} + k_{3y}k_{1x})}{||\mathbf{k}_1||^2 ||\mathbf{k}_3||^2} = \mathbf{e}_z \sin \theta_{13} \\ \mathbf{e}_2 \times \mathbf{e}_3 &= \frac{(k_{3x}, k_{3y}, 0)}{||\mathbf{k}_3||^2} = \hat{\mathbf{k}}_3, \end{aligned} \quad (\text{C.9})$$

we obtain the the desired Hamiltonian describing the atom light interaction

$$\begin{aligned}
iu_v \mathbf{E}^* \times \mathbf{E} \cdot \hat{\mathbf{F}} &= -2u_v \left[ -\hat{\mathbf{k}}_1 \text{Im}\{12\} + \mathbf{e}_z \sin \theta_{13} \text{Im}\{13\} + \hat{\mathbf{k}}_3 \text{Im}\{23\} \right] \cdot \hat{\mathbf{F}} \\
&= (\Omega_x, \Omega_y, \Omega_z) \cdot \hat{\mathbf{F}},
\end{aligned} \tag{C.10}$$

where

$$\begin{aligned}
\Omega_x &= \frac{k_{1x}}{||\mathbf{k}_1||} \text{Im}\{\Omega_{12} e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21} t)}\} + \frac{k_{3x}}{||\mathbf{k}_3||} \text{Im}\{\Omega_{23} e^{i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32} t)}\} \\
\Omega_y &= \frac{k_{1y}}{||\mathbf{k}_1||} \text{Im}\{\Omega_{12} e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21} t)}\} + \frac{k_{3y}}{||\mathbf{k}_3||} \text{Im}\{\Omega_{23} e^{i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32} t)}\} \\
\Omega_z &= \text{Im}\{\Omega_{13} e^{i(\mathbf{k}_{31} \cdot \mathbf{x} - \omega_{31} t)}\},
\end{aligned} \tag{C.11}$$

and

$$\begin{aligned}
\Omega_{12} &= 2u_v E_1^* E_2 \\
\Omega_{13} &= -2u_v E_1^* E_3 \sin \theta_{13} \\
\Omega_{23} &= -2u_v E_2^* E_3.
\end{aligned} \tag{C.12}$$

Now we need to transform Eq. C.10 into the rotating frame, this is where things start getting fun. The ‘slow’ or ‘fast’ nature of a given term depends on the specific choice of frequencies on each Raman beam which should be such that the frequency differences  $\omega_{ij}$  are resonant with dressed state transitions in the rotating frame as shown in Figure 2a.

I showed in Equation C.2 that in the rotating frame the  $\hat{F}_x$  and  $\hat{F}_y$  operators get additional factors of  $\exp(\pm i\omega_{\text{RF}})$  while  $\hat{F}_z$  remains unchanged. We must therefore have the frequencies of beams giving rise to  $\hat{F}_x$  and  $\hat{F}_y$  coupling to differ in



frequency by an additional  $\omega_{\text{RF}}$  and the beams that give the  $\hat{F}_z$  coupling to be close in frequency. The two possible ways of doing so are shown in Figure 2b and they determine whether  $\omega_{21}$  and  $\omega_{31}$  are positive or negative.

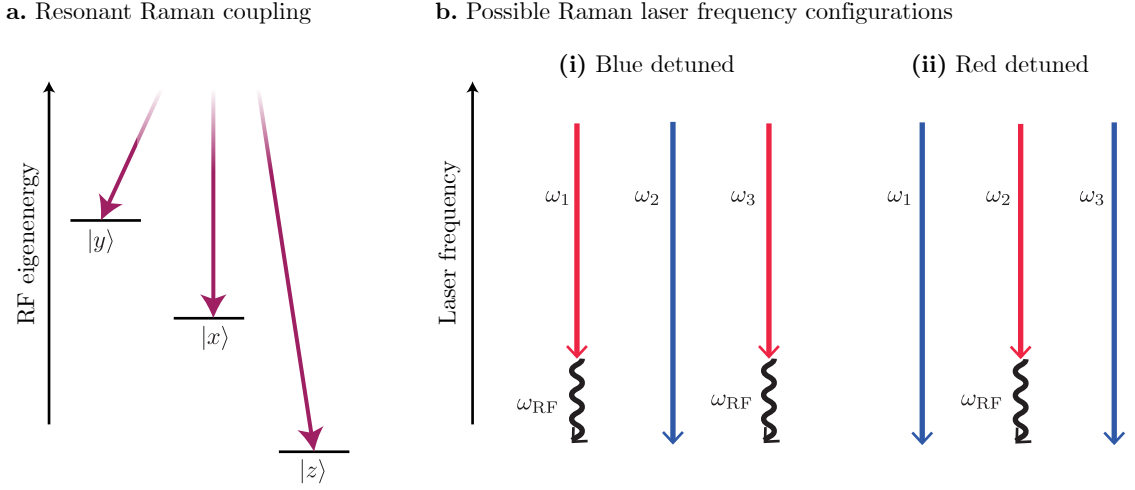


Figure 2: **a** The choice of laser frequencies should be such that in the frame rotating with frequency  $\omega_{\text{RF}}$  we get resonant Raman coupling of the  $|xyz\rangle$  states. **b.** Possible laser frequency configurations: **i)** Blue detuned configuration: There are 2 frequencies smaller by about  $\omega_{\text{RF}}$  and one larger frequency. **ii)** The red detuned configuration: there are 2 frequencies that are larger by about  $\omega_{\text{RF}}$  and one smaller frequency.

Lets look at the first term of the  $\Omega_x \hat{F}_x$  coupling to get an idea of how the RWA

will work here:

$$\begin{aligned}
\Omega_x^{(1)} \hat{F}_x &\rightarrow \frac{1}{4i} \frac{k_{1x}}{||\mathbf{k}_1||} \left( \Omega_{12} e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21})t} - \Omega_{12}^* e^{-i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21})t} \right) \left( e^{i\omega_{\text{RF}}t} \hat{F}_+ + e^{-i\omega_{\text{RF}}t} \hat{F}_- \right) \\
&\approx \frac{1}{4i} \frac{k_{1x}}{||\mathbf{k}_1||} \left( \Omega_{12} e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}})t)} \hat{F}_{\pm} - \Omega_{12}^* e^{-i(\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}})t)} \hat{F}_{\mp} \right) \\
&= \frac{1}{2} \frac{k_{1x}}{||\mathbf{k}_1||} |\Omega_{12}| \left( \sin[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}})t + \phi_{12}] \hat{F}_x \right. \\
&\quad \left. \pm \cos[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}})t + \phi_{12}] \hat{F}_y \right).
\end{aligned} \tag{C.13}$$

Here the upper sign corresponds to the  $\omega_{21} > 0$  case (blue detuned) and the lower sign to  $\omega_{21} < 0$  (red detuned) and I performed a RWA approximation in the second line by neglecting the terms oscillating with frequency close to  $2\omega_{\text{RF}}$ . Similarly, the second term of  $\Omega_x \hat{F}_x$  is

$$\begin{aligned}
\Omega_x^{(2)} \hat{F}_x &\rightarrow \frac{1}{4i} \frac{k_{3x}}{||\mathbf{k}_3||} \left( \Omega_{23} e^{i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32})t} - \Omega_{23}^* e^{-i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32})t} \right) \left( e^{i\omega_{\text{RF}}t} \hat{F}_+ + e^{-i\omega_{\text{RF}}t} \hat{F}_- \right) \\
&\approx \frac{1}{2} \frac{k_{3x}}{||\mathbf{k}_3||} |\Omega_{23}| \left( \sin[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \mp \omega_{\text{RF}})t + \phi_{23}] \hat{F}_x \right. \\
&\quad \left. \pm \cos[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \mp \omega_{\text{RF}})t + \phi_{23}] \hat{F}_y \right)
\end{aligned} \tag{C.14}$$

where I used the same sign convention as before. It is important to keep in mind that if  $\omega_{21}$  is positive then  $\omega_{32}$  must be negative and vice versa. Lets keep cranking

the algebra!

$$\begin{aligned}
\Omega_y^{(1)} \hat{F}_y &\rightarrow -\frac{1}{4} \frac{k_{1y}}{||\mathbf{k}_1||} \left( \Omega_{12} e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21} t)} - \Omega_{12}^* e^{-i(\mathbf{k}_{21} \cdot \mathbf{x} - \omega_{21} t)} \right) \left( e^{i\omega_{\text{RF}} t} \hat{F}_+ - e^{-i\omega_{\text{RF}} t} \hat{F}_- \right) \\
&\approx \mp \frac{1}{4} \frac{k_{1y}}{||\mathbf{k}_1||} \left( \Omega_{12} e^{i(\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}}) t)} \hat{F}_{\pm} + \Omega_{12}^* e^{-i(\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}}) t)} \hat{F}_{\mp} \right) \\
&= \mp \frac{1}{2} \frac{k_{1y}}{||\mathbf{k}_1||} |\Omega_{12}| \left( \cos[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}}) t + \phi_{12}] \hat{F}_x \right. \\
&\quad \left. - \sin[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}}) t + \phi_{12}] \hat{F}_y \right),
\end{aligned} \tag{C.15}$$

$$\begin{aligned}
\Omega_y^{(2)} \hat{F}_y &\rightarrow -\frac{1}{4} \frac{k_{3y}}{||\mathbf{k}_3||} \left( \Omega_{23} e^{i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32} t)} - \Omega_{23}^* e^{-i(\mathbf{k}_{32} \cdot \mathbf{x} - \omega_{32} t)} \right) \left( e^{i\omega_{\text{RF}} t} \hat{F}_+ - e^{-i\omega_{\text{RF}} t} \hat{F}_- \right) \\
&\approx \mp \frac{1}{2} \frac{k_{3y}}{||\mathbf{k}_3||} |\Omega_{23}| \left( \cos[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \mp \omega_{\text{RF}}) t + \phi_{23}] \hat{F}_x \right. \\
&\quad \left. - \sin[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \mp \omega_{\text{RF}}) t + \phi_{23}] \hat{F}_y \right).
\end{aligned} \tag{C.16}$$

The complete Hamiltonian in the rotating frame after doing the rotating wave ap-

proximation is then

$$\begin{aligned}
\hat{H} = & \frac{1}{2} \frac{|\Omega_{12}|}{||\mathbf{k}_1||} \left( \left( k_{1x} \sin[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}})t + \phi_{12}] \pm k_{1y} \cos[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \pm \omega_{\text{RF}})t + \phi_{12}] \right) \hat{F}_x \right. \\
& \left. + \left( \pm k_{1x} \cos[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \mp \omega_{\text{RF}})t + \phi_{12}] \mp k_{1y} \sin[\mathbf{k}_{21} \cdot \mathbf{x} - (\omega_{21} \pm \omega_{\text{RF}})t + \phi_{12}] \right) \hat{F}_y \right) \\
& \frac{1}{2} \frac{|\Omega_{23}|}{||\mathbf{k}_3||} \left( \left( k_{3x} \sin[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \mp \omega_{\text{RF}})t + \phi_{23}] \pm k_{3y} \cos[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \pm \omega_{\text{RF}})t + \phi_{23}] \right) \hat{F}_x \right. \\
& \left. + \left( \pm k_{3x} \cos[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \mp \omega_{\text{RF}})t + \phi_{23}] \mp k_{3y} \sin[\mathbf{k}_{32} \cdot \mathbf{x} - (\omega_{32} \pm \omega_{\text{RF}})t + \phi_{23}] \right) \hat{F}_y \right) \\
& + |\Omega_{13}| \sin(\mathbf{k}_{31} \cdot \mathbf{x} - \omega_{31}t + \phi_{13}) \hat{F}_z
\end{aligned} \tag{C.17}$$

In order to go from this rather complicated looking Hamiltonian to the effective time independent Hamiltonian used in Chapter 8 we need to take two steps: first the off resonant coupling terms need to be neglected. This can be more or less safely done since they will be detuned by something on the order of tens to hundreds of kHz. Second we need to go into a second transformed frame using the unitary transformation

$$\hat{U} = \sum_{i \in \{xyz\}} e^{i(\mathbf{k}_i \cdot \mathbf{x} - \omega_i t)} \tag{C.18}$$

and eliminate the ‘fast’ terms that are proportional to  $\exp(i\omega_{ij}t)$ . The neglected terms of the Hamiltonian in Equation C.17 have the effect of slightly shifting the eigenenergies of the effective Hamiltonian from Equation 8.8. We interpret this shifts in energy as coming from new effective Raman coupling strengths that slightly differ from our calibrations performed by measuring the Rabi frequencies of individual

pairs of Raman beams.

## Bibliography

- [1] Bose. Plancks Gesetz und Lichtquantenhypothese. *Zeitschrift für Physik*, 26(1):178–181, December 1924.
- [2] A. Einstein. Quantentheorie des einatomigen idealen Gases. In *Albert Einstein: Akademie-Vorträge*, pages 237–244. John Wiley & Sons, Ltd, 2006.
- [3] F. London. On the Bose-Einstein Condensation. *Physical Review*, 54(11):947–954, December 1938.
- [4] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell. Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor. *Science*, 269(5221):198–201, July 1995.
- [5] K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle. Bose-Einstein Condensation in a Gas of Sodium Atoms. *Physical Review Letters*, 75(22):3969–3973, November 1995.
- [6] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet. Evidence of Bose-Einstein Condensation in an Atomic Gas with Attractive Interactions. *Physical Review Letters*, 75(9):1687–1690, August 1995.
- [7] Ketterle W., Durfee D.S., and Stamper-Kurn D.M. Making, probing and understanding Bose-Einstein condensates. *Proceedings of the International School of Physics “Enrico Fermi”*, 140(Bose-Einstein Condensation in Atomic Gases):67–176, 1999.
- [8] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell. Collective Excitations of a Bose-Einstein Condensate in a Dilute Gas. *Physical Review Letters*, 77(3):420–423, July 1996.

- [9] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle. Collective Excitations of a Bose-Einstein Condensate in a Magnetic Trap. *Physical Review Letters*, 77(6):988–991, August 1996.
- [10] M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle. Observation of Interference Between Two Bose Condensates. *Science*, 275(5300):637–641, January 1997.
- [11] B. DeMarco and D. S. Jin. Onset of Fermi Degeneracy in a Trapped Atomic Gas. *Science*, 285(5434):1703–1706, September 1999.
- [12] Dan M. Stamper-Kurn and Masahito Ueda. Spinor Bose gases: Symmetries, magnetism, and quantum dynamics. *Reviews of Modern Physics*, 85(3):1191–1244, July 2013.
- [13] Xibo Zhang and Jun Ye. Precision measurement and frequency metrology with ultracold atoms. *National Science Review*, 3(2):189–200, June 2016.
- [14] Immanuel Bloch, Jean Dalibard, and Sylvain Nascimbène. Quantum simulations with ultracold quantum gases. *Nature Physics*, 8(4):267–276, April 2012.
- [15] Markus Greiner, Olaf Mandel, Tilman Esslinger, Theodor W. Hänsch, and Immanuel Bloch. Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms. *Nature*, 415(6867):39–44, January 2002.
- [16] N. Goldman, G. Juzeliūnas, P. Öhberg, and I. B. Spielman. Light-induced gauge fields for ultracold atoms. *Reports on Progress in Physics*, 77(12):126401, November 2014.
- [17] A. Celi, P. Massignan, J. Ruseckas, N. Goldman, I. B. Spielman, G. Juzeliūnas, and M. Lewenstein. Synthetic gauge fields in synthetic dimensions. *Phys. Rev. Lett.*, 112:043001, Jan 2014.
- [18] Y.-J. Lin, R. L. Compton, K. Jiménez-García, J. V. Porto, and I. B. Spielman. Synthetic magnetic fields for ultracold neutral atoms. *Nature*, 462(7273):628–632, December 2009.
- [19] Y.-J. Lin, R. L. Compton, K. Jiménez-García, W. D. Phillips, J. V. Porto, and I. B. Spielman. A synthetic electric force acting on neutral atoms. *Nature Physics*, 7(7):531–534, July 2011.
- [20] Hirokazu Miyake, Georgios A. Siviloglou, Colin J. Kennedy, William Cody Burton, and Wolfgang Ketterle. Realizing the Harper Hamiltonian with Laser-Assisted Tunneling in Optical Lattices. *Physical Review Letters*, 111(18):185302, October 2013.

- [21] Nigel R. Cooper and Jean Dalibard. Reaching Fractional Quantum Hall States with Optical Flux Lattices. *Physical Review Letters*, 110(18):185301, April 2013.
- [22] Victor Galitski and Ian B. Spielman. Spin-orbit coupling in quantum gases. *Nature*, 494(7435):49–54, February 2013.
- [23] M. Z. Hasan and C. L. Kane. Colloquium: Topological insulators. *Reviews of Modern Physics*, 82(4):3045–3067, November 2010.
- [24] Yu A. Bychkov and E. I. Rashba. Oscillatory effects and the magnetic susceptibility of carriers in inversion layers. *Journal of Physics C: Solid State Physics*, 17(33):6039, 1984.
- [25] A. Valdés-Curiel, D. Trypogeorgos, E. E. Marshall, and I. B. Spielman. Fourier transform spectroscopy of a spin-orbit coupled Bose gas. *New Journal of Physics*, 19(3):033025, 2017.
- [26] A. Valdés-Curiel, D. Trypogeorgos, Q.-Y. Liang, R. P. Anderson, and I. B. Spielman. Unconventional topology with a Rashba spin-orbit coupled quantum gas. *arXiv:1907.08637 [cond-mat, physics:quant-ph]*, July 2019. arXiv: 1907.08637.
- [27] R. P. Anderson, D. Trypogeorgos, A. Valdés-Curiel, Q.-Y. Liang, J. Tao, M. Zhao, T. Andrijauskas, G. Juzeliūnas, and I. B. Spielman. Realization of a fractional period adiabatic superlattice. *arXiv:1907.08910 [cond-mat, physics:physics, physics:quant-ph]*, July 2019. arXiv: 1907.08910.
- [28] Tudor D. Stanescu, Brandon Anderson, and Victor Galitski. Spin-orbit coupled Bose-Einstein condensates. *Physical Review A*, 78(2):023616, August 2008.
- [29] Tigran A. Sedrakyan, Alex Kamenev, and Leonid I. Glazman. Composite fermion state of spin-orbit-coupled bosons. *Physical Review A*, 86(6):063639, December 2012.
- [30] Hui Hu, Lei Jiang, Xia-Ji Liu, and Han Pu. Probing Anisotropic Superfluidity in Atomic Fermi Gases with Rashba Spin-Orbit Coupling. *Physical Review Letters*, 107(19):195304, November 2011.
- [31] C.J. Pethick and H. Smith. *Bose-Einstein Condensation of Dilute Gases*. Oxford University Press, Oxford, UK, 2005.
- [32] *Bose-Einstein Condensation*. International Series of Monographs on Physics. Oxford University Press, Oxford, New York, June 2003.
- [33] Franco Dalfovo, Stefano Giorgini, Lev P. Pitaevskii, and Sandro Stringari. Theory of Bose-Einstein condensation in trapped gases. *Reviews of Modern Physics*, 71(3):463–512, April 1999.



- [34] Y. Castin and R. Dum. Bose-Einstein Condensates in Time Dependent Traps. *Physical Review Letters*, 77(27):5315–5319, December 1996.
- [35] G. F. Gribakin and V. V. Flambaum. Calculation of the scattering length in atomic collisions using the semiclassical approximation. *Physical Review A*, 48(1):546–553, July 1993.
- [36] S. De, D. L. Campbell, R. M. Price, A. Putra, Brandon M. Anderson, and I. B. Spielman. Quenched binary Bose-Einstein condensates: Spin-domain formation and coarsening. *Physical Review A*, 89(3):033631, March 2014.
- [37] Daniel Adam Steck. Rubidium 87 D line data. Available online, <http://steck.us/alkalidata>, April 2018. revision 2.1.5.
- [38] Charles Schwartz. Theory of Hyperfine Structure. *Physical Review*, 97(2):380–395, January 1955.
- [39] P. Zeeman. The effect of magnetisation on the nature of light emitted by a substance. *Nature*, 55(1424):347–347, 1897.
- [40] G. Breit and I. I. Rabi. Measurement of Nuclear Spin. *Physical Review*, 38(11):2082–2083, December 1931.
- [41] W. Gerlach and O. Stern. Der experimentelle Nachweis des magnetischen Moments des Silberatoms. *Zeitschrift für Physik*, 8(1):110–111, December 1922.
- [42] J. Stark. Beobachtungen über den Effekt des elektrischen Feldes auf Spektrallinien. I. Quereffekt. *Annalen der Physik*, 348(7):965–982, 1914.
- [43] Jacques Dupont-Roc Claude Cohen-Tannoudji and Gilbert Grynberg. *Atom-Photon Interactions*. Wiley-VCH, Weinheim, 2006.
- [44] Daniel Adam Steck. Quantum and atom optics. Available online at <http://steck.us/teaching>, January 2015. revision 0.12.2.
- [45] Ivan H. Deutsch and Poul S. Jessen. Quantum control and measurement of atomic spins in polarization spectroscopy. *Optics Communications*, 283(5):681–694, March 2010.
- [46] J. J. Sakurai. *Modern Quantum Mechanics*. Addison Wesley Publishing Company, Menlo Park, California, 1994.
- [47] Rudolf Grimm, Matthias Weidemüller, and Yurii B. Ovchinnikov. Optical Dipole Traps for Neutral Atoms. In Benjamin Bederson and Herbert Walther, editors, *Advances In Atomic, Molecular, and Optical Physics*, volume 42, pages 95–170. Academic Press, January 2000.

- [48] Rui Han, Hui Khoon Ng, and Berthold-Georg Englert. Raman transitions without adiabatic elimination: a simple and accurate treatment. *Journal of Modern Optics*, 60(4):255–265, February 2013.
- [49] Bindiya Arora, M. S. Safronova, and Charles W. Clark. Tune-out wavelengths of alkali-metal atoms and their applications. *Physical Review A*, 84(4):043401, October 2011.
- [50] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman. Spin-orbit-coupled Bose-Einstein condensates. *Nature*, 471(7336):83–86, March 2011.
- [51] G. Dresselhaus. Spin-Orbit Coupling Effects in Zinc Blende Structures. *Physical Review*, 100(2):580–586, October 1955.
- [52] Christopher J. Foot. *Atomic Physics*. Oxford University Press, Cambridge, UK, 2006.
- [53] G. Reinaudi, T. Lahaye, Z. Wang, and D. Guéry-Odelin. Strong saturation absorption imaging of dense clouds of ultracold atoms. *Optics Letters*, 32(21):3143–3145, November 2007.
- [54] D. Genkina, L. M. Ayccock, B. K. Stuhl, H.-I. Lu, R. A. Williams, and I. B. Spielman. Feshbach enhanced-wave scattering of fermions: direct observation with optimized absorption imaging. *New Journal of Physics*, 18(1):013001, December 2015.
- [55] I. I. Rabi. Space Quantization in a Gyration Magnetic Field. *Physical Review*, 51(8):652–654, April 1937.
- [56] Clarence Zener and Ralph Howard Fowler. Non-adiabatic crossing of energy levels. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 137(833):696–702, September 1932.
- [57] Erin Marshall Seroka, Ana Valdés Curiel, Dimitrios Trypogeorgos, Nathan Lundblad, and Ian B. Spielman. Repeated Measurements with Minimally Destructive Partial-Transfer Absorption Imaging. *arXiv:1907.05372 [physics]*, July 2019. arXiv: 1907.05372.
- [58] Norman F. Ramsey. A Molecular Beam Resonance Method with Separated Oscillating Fields. *Physical Review*, 78(6):695–699, June 1950.
- [59] D. L. Campbell, R. M. Price, A. Putra, A. Valdes-Curiel, D. Trypogeorgos, and I. B. Spielman. Magnetic phases of spin-1 spin-orbit-coupled Bose gases. *Nature Communications*, 7:10897, March 2016.
- [60] C. Silber, S. Günther, C. Marzok, B. Deh, Ph. W. Courteille, and C. Zimmermann. Quantum-Degenerate Mixture of Fermionic Lithium and Bosonic Rubidium Gases. *Physical Review Letters*, 95(17):170408, October 2005.

- [61] B. Deh, C. Marzok, C. Zimmermann, and Ph. W. Courteille. Feshbach resonances in mixtures of ultracold  $^6\text{Li}$  and  $^{87}\text{Rb}$  gases. *Physical Review A*, 77(1):010701, January 2008.
- [62] Daniel L. Campbell. *Engineered potentials in ultracold Bose-Einstein condensates*. PhD thesis, University of Maryland College Park, Digital Repository at the University of Maryland, 6 2015.
- [63] Ryan Price. *Phase transitions in engineered ultracold quantum systems*. PhD thesis, University of Maryland College Park, Digital Repository at the University of Maryland, 6 2016.
- [64] William D. Phillips and Harold Metcalf. Laser Deceleration of an Atomic Beam. *Physical Review Letters*, 48(9):596–599, March 1982.
- [65] Paul D. Lett, Richard N. Watts, Christoph I. Westbrook, William D. Phillips, Phillip L. Gould, and Harold J. Metcalf. Observation of Atoms Laser Cooled below the Doppler Limit. *Physical Review Letters*, 61(2):169–172, July 1988.
- [66] Christoph Becker. *Multi component Bose-Einstein condensates From mean field physics to strong correlations*. PhD thesis, Universität Hamburg, <http://citeseerx.ist.psu.edu>, 6 2008.
- [67] P. T. Starkey, C. J. Billington, S. P. Johnstone, M. Jasperse, K. Helmersson, L. D. Turner, and R. P. Anderson. A scripted control system for autonomous hardware-timed experiments. *Review of Scientific Instruments*, 84(8):085111, August 2013.
- [68] Doppler-Limited Absorption and Fluorescence Spectroscopy with Lasers. In Wolfgang Demtröder, editor, *Laser Spectroscopy: Vol. 2 Experimental Techniques*, pages 1–75. Springer Berlin Heidelberg, Berlin, Heidelberg, 2008.
- [69] Lawrence W. Cheuk, Ariel T. Sommer, Zoran Hadzibabic, Tarik Yefsah, Waseem S. Bakr, and Martin W. Zwierlein. Spin-Injection Spectroscopy of a Spin-Orbit Coupled Fermi Gas. *Physical Review Letters*, 109(9):095302, 2012.
- [70] David M. Jonas. Two-Dimensional Femtosecond Spectroscopy. *Annual Review of Physical Chemistry*, 54(1):425–463, 2003.
- [71] Bryce Yoshimura, W. C. Campbell, and J. K. Freericks. Diabatic-ramping spectroscopy of many-body excited states. *Physical Review A*, 90(6):062334, 2014.
- [72] Limei Wang, Hao Zhang, Linjie Zhang, Georg Raithel, Jianming Zhao, and Suotang Jia. Atom-interferometric measurement of Stark level splittings. *Physical Review A*, 92(3):033619, September 2015.

- [73] Anders Sørensen and Klaus Mølmer. Entanglement and quantum computation with ions in thermal motion. *Physical Review A*, 62(2):022311, July 2000.
- [74] G. Floquet. Sur les équations différentielles linéaires à coefficients périodiques. *Annales scientifiques de l'École Normale Supérieure*, 12:47–88, 1883.
- [75] Chunqing Deng, Jean-Luc Orgiazzi, Feiruo Shen, Sahel Ashhab, and Adrian Lupascu. Observation of Floquet States in a Strongly Driven Artificial Atom. *Physical Review Letters*, 115(13):133601, September 2015.
- [76] J. Eisert, M. Friesdorf, and C. Gogolin. Quantum many-body systems out of equilibrium. *Nature Physics*, 11(2):124–130, February 2015.
- [77] K. Jiménez-García, L. J. LeBlanc, R. A. Williams, M. C. Beeler, C. Qu, M. Gong, C. Zhang, and I. B. Spielman. Tunable spin-orbit coupling via strong driving in ultracold-atom systems. *Phys. Rev. Lett.*, 114:125301, Mar 2015.
- [78] D. L. Campbell and I. B. Spielman. Rashba realization: Raman with RF. *New J. Phys.*, 18(3):033035, 2016.
- [79] Douglas R. Hofstadter. Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields. *Physical Review B*, 14(6):2239–2249, September 1976.
- [80] D. Trypogeorgos, A. Valdés-Curiel, I. B. Spielman, and C. Emary. Perpetual emulation threshold of  $\mathcal{PT}$ -symmetric Hamiltonians. *Journal of Physics A: Mathematical and Theoretical*, 51(32):325302, June 2018.
- [81] D. Trypogeorgos, A. Valdés-Curiel, N. Lundblad, and I. B. Spielman. Synthetic clock transitions via continuous dynamical decoupling. *Physical Review A*, 97(1):013407, January 2018.
- [82] R. P. Anderson, M. J. Kewming, and L. D. Turner. Continuously observing a dynamically decoupled spin-1 quantum gas. *Physical Review A*, 97(1):013408, January 2018.
- [83] E. L. Hahn. Spin Echoes. *Physical Review*, 80(4):580–594, November 1950.
- [84] Lorenza Viola and Seth Lloyd. Dynamical suppression of decoherence in two-state quantum systems. *Phys. Rev. A*, 58(4):2733–2744, October 1998.
- [85] I. Cohen, N. Aharon, and A. Retzker. Continuous dynamical decoupling utilizing time-dependent detuning. *Fortschr. Phys.*, 65(6-8):1600071, June 2017.
- [86] F. F. Fanchini, J. E. M. Hornos, and R. d J. Napolitano. Continuously decoupling single-qubit operations from a perturbing thermal bath of scalar bosons. *Physical Review A*, 75(2), February 2007. arXiv: quant-ph/0611188.

- [87] N. Aharon, I. Cohen, F. Jelezko, and A. Retzker. Fully robust qubit in atomic and molecular three-level systems. *New J. Phys.*, 18(12):123012, 2016.
- [88] Michael J. Biercuk, Hermann Uys, Aaron P. VanDevender, Nobuyasu Shiga, Wayne M. Itano, and John J. Bollinger. Optimized dynamical decoupling in a model quantum memory. *Nature*, 458(7241):996–1000, April 2009.
- [89] J.-M. Cai, B. Naydenov, R. Pfeiffer, L. P. McGuinness, K. D. Jahnke, F. Jelezko, M. B. Plenio, and A. Retzker. Robust dynamical decoupling with concatenated continuous driving. *New Journal of Physics*, 14(11):113023, November 2012. arXiv: 1111.0930.
- [90] A. Bermudez, P. O. Schmidt, M. B. Plenio, and A. Retzker. Robust trapped-ion quantum logic gates by continuous dynamical decoupling. *Phys. Rev. A*, 85(4):040302, April 2012.
- [91] I. Baumgart, J.-M. Cai, A. Retzker, M. B. Plenio, and Ch. Wunderlich. Ultrasensitive magnetometer using a single atom. *Phys. Rev. Lett.*, 116:240801, Jun 2016.
- [92] G. A. Kazakov and T. Schumm. Magic radio-frequency dressing for trapped atomic microwave clocks. *Phys. Rev. A*, 91(2):023404, February 2015.
- [93] L. Sárkány, P. Weiss, H. Hattermann, and J. Fortágh. Controlling the magnetic-field sensitivity of atomic-clock states by microwave dressing. *Phys. Rev. A*, 90(5):053416, November 2014.
- [94] Arne Laucht, Rachpon Kalra, Stephanie Simmons, Juan P. Dehollain, Juha T. Muhonen, Fahd A. Mohiyaddin, Solomon Freer, Fay E. Hudson, Kohei M. Itoh, David N. Jamieson, Jeffrey C. McCallum, Andrew S. Dzurak, and A. Morello. A dressed spin qubit in silicon. *Nat Nano*, 12(1):61–66, January 2017.
- [95] D. Farfurnik, N. Aharon, I. Cohen, Y. Hovav, A. Retzker, and N. Bar-Gill. Experimental realization of time-dependent phase-modulated continuous dynamical decoupling. *arXiv:1704.07582 [quant-ph]*, April 2017. arXiv: 1704.07582.
- [96] Atsushi Noguchi, Shinsuke Haze, Kenji Toyoda, and Shinji Urabe. Generation of a Decoherence-Free Entangled State Using a Radio-Frequency Dressed State. *Phys. Rev. Lett.*, 108(6):060503, February 2012.
- [97] D. Andrew Golter, Thomas K. Baldwin, and Hailin Wang. Protecting a Solid-State Spin from Decoherence Using Dressed Spin States. *Phys. Rev. Lett.*, 113(23):237601, December 2014.
- [98] N. Timoney, I. Baumgart, M. Johanning, A. F. Varón, M. B. Plenio, A. Retzker, and Ch Wunderlich. Quantum gates and memory using microwave-dressed states. *Nature*, 476(7359):185–188, August 2011.

- [99] S. C. Webster, S. Weidt, K. Lake, J. J. McLoughlin, and W. K. Hensinger. Simple Manipulation of a Microwave Dressed-State Ion Qubit. *Phys. Rev. Lett.*, 111(14):140501, October 2013.
- [100] A. Barfuss, J. Teissier, E. Neu, A. Nunnenkamp, and P. Maletinsky. Strong mechanical driving of a single electron spin. *Nat Phys*, 11(10):820–824, October 2015.
- [101] S. Rohr, E. Dupont-Ferrier, B. Pigeau, P. Verlot, V. Jacques, and O. Arcizet. Synchronizing the Dynamics of a Single Nitrogen Vacancy Spin Qubit on a Parametrically Coupled Radio-Frequency Field through Microwave Dressing. *Phys. Rev. Lett.*, 112(1):010502, January 2014.
- [102] Gediminas Juzeliūnas, Julius Ruseckas, and Jean Dalibard. Generalized Rashba-Dresselhaus spin-orbit coupling for cold atoms. *Phys. Rev. A*, 81(5):053403, May 2010.
- [103] D. Bacon, J. Kempe, D. A. Lidar, and K. B. Whaley. Universal Fault-Tolerant Quantum Computation on Decoherence-Free Subspaces. *Phys. Rev. Lett.*, 85(8):1758–1761, August 2000.
- [104] Pablo Solano, Jeffrey A. Grover, Jonathan E. Hoffman, Sylvain Ravets, Fredrik K. Fatemi, Luis A. Orozco, and Steven L. Rolston. Chapter Seven - Optical Nanofibers: A New Platform for Quantum Optics. In Chun C. Lin and Susanne F. Yelin Ennio Arimondo, editor, *Advances In Atomic, Molecular, and Optical Physics*, volume 66, pages 439–505. Academic Press, 2017. DOI: 10.1016/bs.aamop.2017.02.003.
- [105] Ze-Liang Xiang, Sahel Ashhab, J. Q. You, and Franco Nori. Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems. *Rev. Mod. Phys.*, 85(2):623–653, April 2013.
- [106] Tomoki Ozawa, Hannah M. Price, Alberto Amo, Nathan Goldman, Mohammad Hafezi, Ling Lu, Mikael C. Rechtsman, David Schuster, Jonathan Simon, Oded Zilberberg, and Iacopo Carusotto. Topological photonics. *Reviews of Modern Physics*, 91(1):015006, March 2019.
- [107] Pierre Delplace, J. B. Marston, and Antoine Venaille. Topological origin of equatorial waves. *Science*, 358(6366):1075–1077, November 2017.
- [108] V. Peano, C. Brendel, M. Schmidt, and F. Marquardt. Topological Phases of Sound and Light. *Physical Review X*, 5(3):031011, July 2015.
- [109] Zhaoju Yang, Fei Gao, Xihang Shi, Xiao Lin, Zhen Gao, Yidong Chong, and Baile Zhang. Topological Acoustics. *Physical Review Letters*, 114(11):114301, March 2015.
- [110] N. R. Cooper, J. Dalibard, and I. B. Spielman. Topological bands for ultracold atoms. *Rev. Mod. Phys.*, 91:015005, Mar 2019.

- [111] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs. Quantized Hall Conductance in a Two-Dimensional Periodic Potential. *Physical Review Letters*, 49(6):405–408, August 1982.
- [112] F. D. M. Haldane. Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly". *Physical Review Letters*, 61(18):2015–2018, October 1988.
- [113] C. L. Kane and E. J. Mele.  $\mathbb{Z}_2$  Topological Order and the Quantum Spin Hall Effect. *Physical Review Letters*, 95(14):146802, September 2005.
- [114] D. B. Newell, F. Cabiati, J. Fischer, K. Fujii, S. G. Karshenboim, H. S. Margolis, E. de Mirandés, P. J. Mohr, F. Nez, K. Pachucki, T. J. Quinn, B. N. Taylor, M. Wang, B. M. Wood, and Z. Zhang. The CODATA 2017 values of  $h$ ,  $e$ ,  $k$ , and  $N_A$  for the revision of the SI. *Metrologia*, 55(1):L13–L16, January 2018.
- [115] Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma. Non-Abelian anyons and topological quantum computation. *Reviews of Modern Physics*, 80(3):1083–1159, September 2008.
- [116] Eduardo Nahmad-Achar. *Differential Topology and Geometry with Applications to Physics*. 2053-2563. IOP Publishing, 2018.
- [117] L. Landau. The Theory of Phase Transitions. *Nature*, 138(3498):840–841, November 1936.
- [118] K. v. Klitzing, G. Dorda, and M. Pepper. New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance. *Physical Review Letters*, 45(6):494–497, August 1980.
- [119] David Tong. Lectures on the Quantum Hall Effect. *arXiv:1606.06687 [cond-mat, physics:hep-th]*, June 2016. arXiv: 1606.06687.
- [120] Mikio Nakahara. CRC Press, New York, NY, 2003.
- [121] Barry Simon. Holonomy, the Quantum Adiabatic Theorem, and Berry's Phase. *Physical Review Letters*, 51(24):2167–2170, December 1983.
- [122] Berry Michael Victor. Quantal phase factors accompanying adiabatic changes. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 392(1802):45–57, March 1984.
- [123] S. Pancharatnam. Generalized theory of interference, and its applications. *Proceedings of the Indian Academy of Sciences - Section A*, 44(5):247–262, November 1956.
- [124] Y. Aharonov and D. Bohm. Significance of Electromagnetic Potentials in the Quantum Theory. *Physical Review*, 115(3):485–491, August 1959.

- [125] Dirac Paul Adrien Maurice. Quantised singularities in the electromagnetic field,. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 133(821):60–72, September 1931.
- [126] Ralph M. Kaufmann, Dan Li, and Birgit Wehefritz-Kaufmann. Notes on topological insulators. *Reviews in Mathematical Physics*, 28(10):1630003, November 2016. arXiv: 1501.02874.
- [127] H. B. Nielsen and Masao Ninomiya. The Adler-Bell-Jackiw anomaly and Weyl fermions in a crystal. *Physics Letters B*, 130(6):389–396, November 1983.
- [128] M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch. Realization of the Hofstadter Hamiltonian with Ultracold Atoms in Optical Lattices. *Physical Review Letters*, 111(18):185301, October 2013.
- [129] Gregor Jotzu, Michael Messer, Rémi Desbuquois, Martin Lebrat, Thomas Uehlinger, Daniel Greif, and Tilman Esslinger. Experimental realization of the topological Haldane model with ultracold fermions. *Nature*, 515(7526):237–240, November 2014.
- [130] H.-I Lu, M. Schemmer, L. M. Aycok, D. Genkina, S. Sugawa, and I. B. Spielman. Geometrical pumping with a bose-einstein condensate. *Phys. Rev. Lett.*, 116:200402, May 2016.
- [131] M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch. A Thouless quantum pump with ultracold bosonic atoms in an optical superlattice. *Nature Physics*, 12(4):350–354, April 2016.
- [132] Zhan Wu, Long Zhang, Wei Sun, Xiao-Tian Xu, Bao-Zong Wang, Si-Cong Ji, Youjin Deng, Shuai Chen, Xiong-Jun Liu, and Jian-Wei Pan. Realization of two-dimensional spin-orbit coupling for Bose-Einstein condensates. *Science*, 354(6308):83–88, October 2016.
- [133] Wei Sun, Bao-Zong Wang, Xiao-Tian Xu, Chang-Rui Yi, Long Zhang, Zhan Wu, Youjin Deng, Xiong-Jun Liu, Shuai Chen, and Jian-Wei Pan. Highly Controllable and Robust 2d Spin-Orbit Coupling for Quantum Gases. *Physical Review Letters*, 121(15):150401, October 2018.
- [134] Frank Wilczek and A. Zee. Appearance of Gauge Structure in Simple Dynamical Systems. *Physical Review Letters*, 52(24):2111–2114, June 1984.
- [135] D. L. Campbell, G. Juzeliūnas, and I. B. Spielman. Realistic Rashba and Dresselhaus spin-orbit coupling for neutral atoms. *Physical Review A*, 84(2):025602, August 2011.
- [136] Tudor D. Stanescu and Victor Galitski. Spin relaxation in a generic two-dimensional spin-orbit coupled system. *Physical Review B*, 75(12):125307, March 2007.



- [137] Y.-J. Lin, A. R. Perry, R. L. Compton, I. B. Spielman, and J. V. Porto. Rapid production of  $^{87}\text{Rb}$  Bose-Einstein condensates in a combined magnetic and optical potential. *Physical Review A*, 79(6):063631, June 2009.
- [138] Lianghai Huang, Zengming Meng, Pengjun Wang, Peng Peng, Shao-Liang Zhang, Liangchao Chen, Donghao Li, Qi Zhou, and Jing Zhang. Experimental realization of two-dimensional synthetic spin-orbit coupling in ultracold Fermi gases. *Nature Physics*, 12(6):540–544, June 2016.
- [139] Zengming Meng, Lianghai Huang, Peng Peng, Donghao Li, Liangchao Chen, Yong Xu, Chuanwei Zhang, Pengjun Wang, and Jing Zhang. Experimental Observation of a Topological Band Gap Opening in Ultracold Fermi Gases with Two-Dimensional Spin-Orbit Coupling. *Physical Review Letters*, 117(23):235304, December 2016.
- [140] Satoshi Tojo, Taro Hayashi, Tatsuyoshi Tanabe, Takuya Hirano, Yuki Kawaguchi, Hiroki Saito, and Masahito Ueda. Spin-dependent inelastic collisions in spin-2 Bose-Einstein condensates. *Physical Review A*, 80(4):042704, October 2009.
- [141] N. Goldman and J. Dalibard. Periodically Driven Quantum Systems: Effective Hamiltonians and Engineered Gauge Fields. *Physical Review X*, 4(3):031027, 2014.
- [142] N. Fläschner, B. S. Rem, M. Tarnowski, D. Vogel, D.-S. Lühmann, K. Sengstock, and C. Weitenberg. Experimental reconstruction of the Berry curvature in a Floquet Bloch band. *Science*, 352(6289):1091–1094, May 2016.
- [143] Clément Godfrin, Rafik Ballou, Edgar Bonet, Mario Ruben, Svetlana Klyatskaya, Wolfgang Wernsdorfer, and Franck Balestro. Generalized Ramsey interferometry explored with a single nuclear spin qubit. *npj Quantum Information*, 4(1):53, October 2018.
- [144] Takahiro Fukui, Yasuhiro Hatsugai, and Hiroshi Suzuki. Chern Numbers in Discretized Brillouin Zone: Efficient Method of Computing (Spin) Hall Conductances. *Journal of the Physical Society of Japan*, 74(6):1674–1677, June 2005.
- [145] L. Duca, T. Li, M. Reitter, I. Bloch, M. Schleier-Smith, and U. Schneider. An Aharonov-Bohm interferometer for determining Bloch band topology. *Science*, 347(6219):288–292, January 2015.
- [146] Mário G. Silveirinha. Chern invariants for continuous media. *Physical Review B*, 92(12):125153, September 2015.
- [147] Tigran A. Sedrakyan, Victor M. Galitski, and Alex Kamenev. Statistical Transmutation in Floquet Driven Optical Lattices. *Physical Review Letters*, 115(19):195301, November 2015.

- [148] Cheng Chin, Rudolf Grimm, Paul Julienne, and Eite Tiesinga. Feshbach resonances in ultracold gases. *Reviews of Modern Physics*, 82(2):1225–1286, April 2010.
- [149] Roger Brown. *Nonequilibrium many body dynamics with ultracold atoms in optical lattices and selected problems in atomic physics*. PhD thesis, University of Maryland College Park, Digital Repository at the University of Maryland, 6 2014.
- [150] Creston D. Herold. *Ultracold mixtures of Rubidium and Ytterbium for open system quantum engineering*. PhD thesis, University of Maryland College Park, Digital Repository at the University of Maryland, 6 2014.
- [151] Abigail R. Perry. *An apparatus for light-less artificial gauge fields and new imaging techniques*. PhD thesis, University of Maryland College Park, Digital Repository at the University of Maryland, 6 2015.
- [152] B. P. Anderson and M. A. Kasevich. Loading a vapor-cell magneto-optic trap using light-induced atom desorption. *Physical Review A*, 63(2):023404, January 2001.