ABSTRACT

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We present the design, testing, and implementation of a minimallydestructive, partial-transfer absorption imaging system. Partial-transfer absorption imaging in ⁸⁷Rb utilizes a microwave transition to transfer a fraction of the atoms in a Bose-Einstein condensate (BEC) prepared in the F = 1hyperfine state into the F = 2 hyperfine state, where they can be imaged on a cycling transition. The F = 1 and F = 2 hyperfine states are far apart enough in frequency that the F = 1 BEC is essentially unaffected by the imaging probe beam. The modulation transfer function, spot diagram, and point spread function for the imaging optics are simulated and measured on a bench model. We demonstrate the use of the imaging system, and we characterize the atom number and decay rate in a series of images of a repeatedly imaged BEC as a function of one of the imaging parameters, the microwave pulse time.

PARTIAL-TRANSFER ABSORPTION IMAGING OF $^{87}\mathrm{Rb}$ BOSE-EINSTEIN CONDENSATES

by

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Dedication

This thesis is dedicated to my father, Todd Marshall, who inspired my love of learning and has always encouraged me to keep growing.

Acknowledgments

First of all, I'd like to thank my family for their love and encouragement. My parents, Todd and Kelly, have been incredibly supportive in every possible way. My sisters, Morgan, Katy, and Kristen, are my best friends, and the time we spend laughing together is a great stress reliever. I wouldn't have made it this far without the five of you.

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Chapter 1: Background

1.1 Introduction

This thesis describes the design, testing, and implementation of a partialtransfer absorption imaging (PTAI) system in the RbLi laboratory at the University of Maryland.

Chapter 1 introduces a variety of background material: Bose-Einstein condensation theory, the level structure of ⁸⁷Rb, the RbLi experiment, the absorption time-of-flight imaging technique, and a number of minimally destructive imaging techniques.

Chapter 2 describes the imaging system that was already in use in the RbLi lab, as well as the design, testing, and construction of the new partialtransfer absorption optical system. We also describe the metrics we used to test the optical system.

Chapter 3 demonstrates the use of partial-transfer absorption imaging. We describe the imaging procedure and characterize the atom number and decay rate in a series of images of a repeatedly imaged Bose-Einstein condensate (BEC) as a function of one of the imaging parameters.

1.2 Bose-Einstein Condensation

A Bose-Einstein condensate is a quantum state of matter that was first predicted in the 1920s by Albert Einstein and Satyendra Bose. They predicted that, at very low temperatures, a gas of indistinguishable particles with what we now called bosonic quantum statistics would condense into the ground state. The condensation occurs at a critical temperature [2]

$$T_C \simeq \frac{2\pi\hbar^2 n^{2/3}}{mk_B} \tag{1.1}$$

where \hbar is the reduced Planck constant, n = N/V is the number density of N particles in a volume V, m is the particle mass, and k_B is Boltzmann's constant.

This phase transition can also be intuitively understood in terms of the phase space density. The phase space density is $\rho = n\lambda_{dB}^3$, where $\lambda_{dB} = \frac{h}{p}$ is the thermal de Broglie wavelength of the particles, h is the Planck constant, and p is the particle momentum. Above T_C , the product of the number density and the volume $\sim \lambda_{dB}^3$ occupied by an atom is $\ll 1$. The thermal-to-BEC phase transition occurs when $\rho \sim 1$ at the critical temperature.

This formulation has the benefit of having a nice qualitative description, depicted in Figure 1.1, a graphic created by Wolfgang Ketterle's group. At room temperature, the behavior of a cloud of atoms can be approximated as a collection of hard-shell spheres bouncing around and colliding. This is often called the "billiard ball" picture. As the temperature is decreased, the average velocity of the atoms decreases, and therefore their de Broglie wavelength increases. The atoms begin to behave more like wavepackets than hard-shell spheres. At the critical temperature, λ_{dB} is on the order of the interparticle spacing $n^{1/3}$. It becomes difficult to follow the trajectory of a single wavepacket, and a macroscopic fraction of the atoms condense into the p = 0momentum state, creating a matter wave. As long as the temperature is above zero, a fraction of the atoms remain thermal; at T = 0, the atoms form a pure condensate.

The experimental difficulty of working at such low temperatures rendered the creation of weakly interacting BECs impossible until the late 20th century. After many advances in laser cooling, laser trapping, and magnetic trapping techniques, the first BEC was created in Rubidium by Carl Wieman and Eric Cornell at NIST Boulder in 1995 [3]. This breakthrough was followed shortly thereafter by the creation of BECs in Sodium at MIT [7] and Lithium at Rice University [5].



Figure 1.1: Different temperature regimes of an atomic cloud. At high temperature, the motion of atoms can be approximated as hard-shell spheres moving and colliding. As the temperature drops, the de Broglie wavelength of each atom increases. Finally, at the critical temperature, the de Broglie wavelength is on the order of the interparticle spacing, and condensation occurs.

1.3 Rubidium 87

Rubidium continues to be the most common species used to study BECs, and it is the one used in this experiment. Rubidium has two isotopes, $^{85}\mathrm{Rb}$ and ⁸⁷Rb. Because ⁸⁵Rb has a negative scattering length, its interactions are attractive, and the cloud will collapse when the number of atoms exceeds a critical value. Therefore, ⁸⁷Rb is the preferred isotope of Rubidium for studying BECs. Due to coupling between the valence electron's orbital angular momentum \vec{L} and its spin angular momentum \vec{S} , the $L = 0 \rightarrow L = 1$ transition is split into two transitions: the $(5^2 S_{1/2} \rightarrow 5^2 P_{1/2})$ transition (giving the socalled D1 spectral line) and the $(5^2S_{1/2} \rightarrow 5^2P_{3/2})$ transition (giving the socalled D2 spectra line). The energy difference between these transitions results from the fine structure term in the atomic Hamiltonian and is called fine structure splitting. The D2 line for ⁸⁷Rb, used for imaging BECs, is shown in Figure 1.2. The energy levels are further split by the coupling of the total nuclear angular momentum \vec{I} with the total electron angular momentum $\vec{J} =$ $\vec{L} + \vec{S}$. This so-called hyperfine structure splits the $|J = 1/2\rangle \otimes |I = 3/2\rangle$ states into the total angular momentum F = 1 and F = 2 hyperfine ground states and the $|J = 3/2\rangle \otimes |I = 3/2\rangle$ states into the F' = 0, F' = 1, F' = 2, and F' = 3 excited states. In the presence of a magnetic field \vec{B} , the Hamiltonian also takes on the magnetic perturbation term $-\vec{\mu} \cdot \vec{B}$, where $\vec{\mu}$ is the magnetic moment of the atom. If *B* is small enough that this term is smaller than the fine structure splitting, then the splitting of each magnetic sublevel due to the magnetic field is [18]

$$\Delta E = \mu_B g_F m_F B, \tag{1.2}$$

to lowest order, where μ_B is the Bohr magneton, g_F is the hyperfine Landé factor (shown for each level in Figure 1.2), and m_F takes on any value from -F to F. This level splitting is called the Zeeman splitting.

1.4 RbLi Experiment

The RbLi experiment at the University of Maryland is described in detail in the theses of previous graduates of this lab [15] [6]. Therefore, it is described only briefly in this thesis. We create our Bose-Einstein condensates using what is now a fairly conventional collection of laser cooling, laser trapping, and magnetic trapping techniques.

First, we heat a Rubidium source in an oven to about 400 K, collimate the atoms into a beam, and send the beam toward the main part of the experiment, where the science chamber, shown in Figure 1.3, is located. The atomic beam travels down a Zeeman slower (Figure 1.3), a stainless steel tube surrounded



Figure 1.2: Level diagram of the D1 (red) and D2 (orange) line of Rubidium 87, showing fine and hyperfine structure. The hyperfine splitting between F = 1 and F = 2 is also shown (purple) [18].

by two copper coils. At the same time, a 780 nm slower laser beam propagates from the experiment toward the source, slowing the Rubidium atoms using the scattering force. As the atoms slow down, they experience a Doppler shift and fall out of resonance with the slower beam. The spacing of the Zeeman slower coils is designed to counteract this shift using the Zeeman effect to keep the atoms in resonance [14]. This step in the sequence slows the atoms' mean velocity to about 50 m/s.



Figure 1.3: Photograph of the experiment, showing the Zeeman slower (A) and the science chamber (B).

After passing through the Zeeman slower, the atoms arrive in the science chamber, where we load them into the magneto-optical trap (MOT) for several seconds. The MOT combines a number of the early breakthroughs in magnetic trapping and laser cooling, including optical molasses [11] and Sisyphus cooling [20], to trap and cool the atoms. A quadrupole magnetic field creates a spatially-dependent Zeeman shift, which provides a restoring force that pushes the atoms toward the trap minimum.

In order to slow atoms moving in all directions, we use six MOT beams, with two counter-propagating beams along each of the Cartesian axes. The beams create an optical molasses, in which each photon imparts a momentum kick to an atom when absorbed. The beams are red-detuned, which puts them in resonance with Doppler-shifted atoms moving toward the laser source, but out of resonance with atoms moving away from the laser source.

Each pair of counter-propagating laser beams has opposite circular polarization. The overlap of the counter-polarized beams creates a polarization standing wave, where the polarization varies in space from right-circular to linear to left-circular and back again. The Zeeman levels experience a spatially oscillating light shift, and each atom can adiabatically ride the polarization wave to a higher energy. After excitation to a higher energy level, the atom can decay to a ground state with a lower energy. The emitted photon is more energetic than the absorbed one, so this phenomenon, called Sisyphus cooling, continues the cooling process.

After the MOT, we continue cooling the atoms using radio frequency

(RF) evaporation. In this process, we trap the atoms in a quadrupole magnetic field, and we turn on an RF field of frequency $\omega_{RF} = \mu_B g_F B/\hbar$, where B is the magnitude of the magnetic field at a radial distance from the trap center where the atoms are on resonance with the RF field. An atom in this region will be transferred from the magnetically trappable $m_F = -1$ state to a magnetically untrappable state and ejected from the cloud. We lower the frequency of the RF signal, decreasing the spatial extent of the trap and allowing more and more atoms to escape. Because the most energetic atoms are released, the average kinetic energy and therefore temperature of the remaining atoms decrease.

From here, we relax and then turn off the magnetic trap while we transfer the atoms into a one-beam optical dipole trap, then into a two-beam crossed dipole trap. The dipole potential of a beam of intensity I and detuning Δ from an optical transition scales as I/Δ , so the beam is red-detuned to create a potential minimum at the region of highest intensity. In order to trap the atoms but minimize the scattering rate, which scales as I/Δ^2 , we use 1064 nm beams with a combined power of 11 W. We further relax the dipole beam in order to evaporate atoms and further cool the cloud until it reaches condensation. The final temperature of the Rubidium cloud is about 90 nK.

We create the BEC in the $|F = 1, m_F = -1\rangle$ hyperfine ground state, but

we image the atoms using a laser resonant to the $F = 2 \rightarrow F' = 3$ cycling transition, also called the cooling transition. F = 1 and F = 2 are separated by 6.8 GHz, enough to make F = 1 inaccessible for imaging by the cooling light. Therefore we use a repump laser, resonant to the $F = 1 \rightarrow F' = 2$ transition, to excite atoms into F' = 2, where they can decay to F = 2.

1.5 Absorption Time-of-Flight Imaging

In ultracold atom experiments, information about the BEC is obtained primarily through imaging data. By far the most common imaging method used is absorption imaging following a period of nominally ballistic time-offlight (TOF).

Absorption imaging uses the absorption of resonant light to obtain information about the spatial density distribution of the atomic cloud. The intensity I(x, y, z) of a laser beam passing through an absorbing medium of density n(x, y, z) and cross-section σ at z = 0, where z is the distance along the optical axis, is

$$I(x, y, z) = I(x, y, 0)e^{-OD},$$
(1.3)

where the $OD = \int n(x, y, z)\sigma dz$ is the integrated density along the imaging axis and I_{sat} is the saturation intensity. A single absorption image of an atomic cloud will only show a shadow in the middle where the beam was absorbed, so in order to obtain the optical depth of the BEC, three images are typically taken. This process is demonstrated in Figure 1.4. First, a resonant probe beam is shone on the atoms. The photons are absorbed into the condensate, leaving a dark spot in the image of the beam that arrives at the camera. Second, once the energy from the resonant photons has heated and destroyed the BEC, an image is taken of just the probe beam. Third, the probe is turned off and a background image is taken. The optical depth of the cloud can be calculated as

$$OD = -ln \left(\frac{I_{atoms} - I_{background}}{I_{probe} - I_{background}} \right), \tag{1.4}$$

assuming $I \ll I_{sat}$. If I is comparable to or larger than I_{sat} , then a correction term must be added [17].

Typically, the time-of-flight technique is used as well. Here, the confining dipole trap potential is turned off and the BEC is allowed to expand and fall under the influence of gravity. The spatial distribution is mapped onto a momentum distribution, as atoms will expand more rapidly along directions with tighter trap confinement. In the RbLi experiment, the time-of-flight is typically set to 21 ms.

Because the atoms are released from their trap and scattered by a reso-



Figure 1.4: Absorption imaging. **a)** A series of three pictures is taken: one with atoms while the probe beam is on (forming an image I_{atoms}), one with the probe beam only (forming an image I_{probe}), and one with the probe beam off (forming an image $I_{background}$). **b)** A sample image of a BEC, where the optical depth is calculated using the three images.

nant beam, the BEC is destroyed during the absorption TOF imaging process. Therefore, only one image per experimental cycle may be captured. This is the primary disadvantage of this technique, and it means that, though absorption TOF imaging is widely used, sometimes less destructive imaging techniques would be preferable. For one thing, TOF absorption imaging slows down the data acquisition in BEC experiments considerably because only one data point may be taken in an experimental cycle, which has a duration of about 30 seconds. Being able to take 10 shots per cycle, for example, would increase the rate of data acquisition by a factor of 10. Additionally, for experiments that require stability of variable BEC parameters such as atom number, or for experiments involving stochastic processes, it is preferable to sample the same BEC repeatedly. In these cases, a minimally destructive imaging technique is required. A number of such methods have already been developed and implemented [16] [9] [10] [13] [19].

1.6 Minimally Destructive Imaging Methods

Most minimally destructive imaging techniques rely on the dispersive part of the atomic ensemble's susceptibility. The atom is modeled as a dipole in an electric field,

$$\hat{H} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}} \tag{1.5}$$

where $\hat{\mathbf{d}}$ is the electric dipole moment of the atom and $\hat{\mathbf{E}}$ is the laser electric field. With a bit of math [10], we see that the effective Hamiltonian can be decomposed into three terms: a scalar term, a vector term, and a tensor term. Most nondestructive imaging methods rely on one of these three terms.

$$\hat{H}^{(0)} = g \sum_{J'F'} \frac{\alpha_{J'F'}^{(0)}}{\Delta_{J'F'}} \frac{2}{3} \hat{S}_0 \hat{1}_F$$
(1.6)

$$\hat{H}^{(1)} = g \sum_{J'F'} \frac{\alpha_{J'F'}^{(1)}}{\Delta_{J'F'}} \hat{S}_z \hat{F}_z$$
(1.7)

$$\hat{H}^{(2)} = g \sum_{J'F'} \frac{\alpha_{J'F'}^{(2)}}{\Delta_{J'F'}} \left(\hat{S}_x (\hat{F}_x^2 - \hat{F}_y^2) + \hat{S}_y (\hat{F}_x \hat{F}_y + \hat{F}_y \hat{F}_x) + \hat{S}_0 [3\hat{F}_z^2 - F(F+1)\hat{1}_F]/3 \right)$$
(1.8)

Here $g = \pi c/\lambda \epsilon_0 V$, and the $\alpha_{J'F'}$ are the components of the polarizability tensor $\overleftarrow{\alpha}$. \hat{S}_i are the Stokes operators describing the light field, and \hat{F}_i are the Pauli spin operators for i = (x, y, z). The summations are made over all excited states $|J'F'\rangle$, and $\Delta_{J'F'}$ is the detuning of the laser from the $|J, F\rangle \longleftrightarrow |J'F'\rangle$ transition.

The first term, the scalar term, creates a scalar phase shift of the light that depends on the atoms' hyperfine state. It is proportional to \hat{S}_0 , the total intensity of the beam. The second term, the vector term, couples \hat{F}_z to \hat{S}_z , the polarization projection in the circular σ^{\pm} basis. This results in a differential phase shift of the circularly-polarized components of the light field. The third term, the tensor term, causes a rotation in the probe's polarization state in a way that does not preserve ellipticity. Typically this term is small compared to the scalar and vector terms, and it is rarely used in dispersive imaging.

Typically, dispersive methods require the probe beam to be far-detuned from the atomic transition. This is due to the fact that, in the dipole model of the light-atom interaction described above, the scattering rate of photons scales as I/Δ^2 . However, the imaging signal scales as I/Δ and thus decreases with increasing detuning. Therefore, a detuning can be chosen such that a good signal-to-noise ratio is achieved while minimizing the perturbation to the BEC.

Phase-contrast imaging was made into a work-house tool for cold-atom experiments by Wolfgang Ketterle's group at MIT [4]. The BEC imparts a phase shift (Eq. 1.6) to the electric field of the probe beam, which can then be measured. In the earliest implementation, called dark-field phase-contrast imaging, two lenses are used to magnify the image of the atoms, and a small bit of opaque material called a phase dot is placed between the two lenses at the focal plane of the laser to block the unperturbed laser light. The only light that passes to the detector is the light that was phsae-shifted, and the intensity of the signal is proportional to the square of the atom-induced phase shift. In another implementation, a object with an index of refraction larger than 1 (often a raised bit of glass on a glass pane) is placed at the focal plane of the laser to shift the phase of the unperturbed probe beam by $\pi/2$. A polarizer after this phase object is set such that the probe beam and the light from the atoms constructively interfere, creating an image of the BEC on the camera.

Faraday rotation imaging [9] takes advantage of the differential phase shift (Eq. 1.7) between the two circularly-polarized components of the light field. For a linearly polarized probe beam, the BEC induces a rotation of the polarization of the light. A polarizing beam-splitter cube (PBS) is placed between the BEC and the camera to separate out the horizontal and vertical polarization components. The polarization of the probe beam is set such that the transmission through the cube is minimized in the absence of atoms. When atoms are present, the polarization rotation imparted to the beam allows some light to pass through to the camera [9].

Non-dispersive methods for minimally-destructive imaging have also been implemented. For example, fluorescence imaging [12] detects the light scattered from the BEC, but the acquired signal is typically weak due to the near-isotropy of the scattered light and the small solid angle over which that light can be collected. Diffraction-contrast imaging [19] uses the diffraction of the probe beam due to the atoms to image the BEC. For various reasons, in the search for robust minimally-destructive imaging techniques, non-dispersive techniques have been less popular than dispersive ones.

Partial-transfer absorption imaging (PTAI) combines aspects of standard absorption imaging and minimally-destructive dispersive imaging in a simple and rather clever scheme [8] [16]. In PTAI, the BEC is prepared in a ground state that is not part of a cycling transition. Every time an image of the same BEC is taken, a subset of the atoms are transferred into a state in a cycling transition, which can then be resonantly imaged. For the untransferred atoms, the probe beam is far-detuned from any optical transition, so they remain effectively unperturbed.

For ⁸⁷Rb, this means preparing the BEC in $|F, m_F\rangle = |1, -1\rangle$ and transferring a fraction of the atoms into $|2, -2\rangle$, where they can then be imaged on the cycling transition. As in standard absorption imaging, these atoms are scattered and expelled from the trap. The untransferred atoms are 6.8 GHz off-resonant from the cycling transition, so their scattering rate due to the probe beam is negligible. One of the challenges of imaging in situ is that in situ BECs have high optical depths, which can cause the probe beam to be completely absorbed, providing no information about the column density of the cloud. PTAI can be implemented in situ because the atoms imaged on each shot have a smaller OD than the whole BEC.

We chose to implement partial-transfer absorption imaging in the RbLi experiment. The design, testing, and construction of the imaging system is described in Chapter 2.

Chapter 2: Design and Testing of Partial-Transfer Absorption Imaging

2.1 Characterization Metrics

The image formed by an optical system will always be an imperfect representation of the object due to aberrations in the system, such as spherical aberration, field curvature, coma, astigmatism, and distortion [1]. These aberrations arise from higher-order corrections to the paraxial approximation $sin(\theta) \approx \theta$, where θ is the angle between the optical axis and a ray. The goal of optical design is to create a system where aberrations are minimized.

Because of diffraction, aberrations can never be entirely eliminated; a point object will always create an image spot of finite size. An optical system is called diffraction-limited if the resolution of the system reaches the theoretical limit. The minimum separation between two points in the image that can be resolved is given by the radius of the Airy disk,

$$r = \frac{1.22f\lambda}{D},\tag{2.1}$$

where λ is the wavelength of the imaging light, f the focal length of the imaging lens, and and D is the diameter of the imaging optics [1]. This resolution limit is called the Rayleigh criterion. For a diffraction-limited system, most of the intensity of the beam falls within the first minimum of the Airy function.

A wide variety of metrics are used to characterize the magnitude of the aberrations in an imaging system. For our system, we chose three metrics: the modulation transfer function, the spot diagram, and the point spread function.

The modulation transfer function is a measure of an imaging system's ability to resolve features of different spatial frequencies. Typically, a resolution test target such as the 1951 United States Air Force test pattern, shown in Figure 2.5a, is placed in the object plane and observed in the image plane. The image of the bars in the test pattern will be sharp for low spatial frequencies, but for higher frequencies, the contrast between the bars and the spaces in between will decrease. The modulation is defined as

$$Modulation = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$
(2.2)

where I_{max} is the maximum intensity of the pattern and I_{min} is the minimum

intensity of the pattern. The modulation is almost 1 for low spatial frequencies, and for an in-focus system, it decreases with increasing spatial frequency.

The spot diagram is the cross-section in the image plane of a set of rays launched from the object through the imaging system. This is a purely geometrical, ray optical approach; diffraction and other effects of Gaussian optics are not described by the spot diagram. In a simulation of this metric, for example, in Zemax OpticStudio (Figure 2.2b and 2.3c), rays are launched from a point source in the object plane of the imaging system and traced through the system to the image plane. The image of the point source will have some finite width due to the presence of aberrations. The imaging system is considered diffraction-limited if all the rays fall within the Airy disk.

The point spread function (PSF) is similar to the spot diagram, except that it incorporates the effects of diffraction. The point spread function is simply the intensity of the cross-section of the light from the object in the image plane along a line that runs through the most intense part of the beam. The PSF is therefore intensity as a function of position.

2.2 Pre-Existing Absorption Time-of-Flight Imaging System

Before the addition of the partial-transfer absorption imaging system to the experiment, an absorption TOF imaging system had been designed, characterized, and implemented. This imaging system consists of two compound lenses with a magnification M_1 of 3.16. The numerical aperture NA_O in the object plane is about 0.225, constrained by the aperture of the first lens. The first compound lens consists of a 60 mm achromatic doublet (Thorlabs part number AC254-060-B) and a 300 mm plano-convex singlet (Thorlabs LA1484). The second compound lens consists of a 500 mm plano-convex singlet (Thorlabs 1380) and a 250 mm achromatic doublet (Thorlabs AC508-250-B). The numerical aperture NA_I in the image plane is about 0.0728. The resolution of the imaging system is about 2 μ m. This imaging system is shown in Figure 2.1.

The probe beam used for imaging comes to the experiment through a fiber-optic cable, is collimated with a 7.5 mm aspheric lens and a 300 mm lens, and passes through a $\lambda/2$ waveplate, a Glan-Taylor polarizer, and a $\lambda/4$ waveplate to ensure σ - polarization. The beam reflects off two gold-coated mirrors, which preserve the polarization, passes through the glass cell, passes through the first compound lens, reflects off a mirror on the lowest level of the experiment, and passes through the second compound lens before illuminating the camera. The camera is a PointGray Flea3 with a 648 x 488 pixel CCD and 5.6 μ m pixels. The camera is mounted on a XYZ micrometer translation stage for precise focusing.



Figure 2.1: Diagram of the pre-existing TOF absorption imaging system. The beam passes through the BEC in the science chamber (a glass cell under vacuum), passes through the first compound lens, reflects off a 2" circular mirror, passes through the second compound lens, and ends at the Flea3 camera.

The magnification of the preexisting time-of-flight imaging system was measured using an optical lattice formed by the counter-propagating dipole trap beams. Each lattice site receives a momentum kick, and the difference in momentum between two adjacent lattice sites is $2k_R = 2 \times 2\pi/\lambda_R$, where λ_R is the wavelength of 1064 nm. We can use the spread in momentum space Δk between two sites to calculate their spread in position space for a given time-of-flight as

$$\Delta x = \frac{\hbar \Delta k t_{TOF}}{m_{Rb}},\tag{2.3}$$

where t_{TOF} is the time-of-flight time. We then measure the spatial separation between the two sites on the camera. The magnification of the imaging system is the quotient of the distance between lattice sites as measured on the camera and Δx . From an average of five measurements, the magnification was determined to be 3.16 ± 0.12 , where the uncertainty is the standard deviation of the mean.

The modulation transfer function, spot diagram, and point spread function of the imaging system, simulated using Zemax OpticStudio, are provided in Figure 2.2.



Figure 2.2: Simulation of the pre-existing TOF absorption imaging systemin Zemax OpticStudio. a) The modulation transfer function. b) The spotdiagram. c) The point spread function.

2.3 Design

The partial transfer absorption imaging system was built in series with the time-of-flight imaging system. We chose to use two achromatic lenses for the imaging system, one with a focal length of $f_1 = 150$ mm and one with a focal length of $f_2 = 750$ mm. The combination gives a magnification of $M = f_2/f_1 = 5$. We decided to magnify by a factor of 5, in addition to the pre-existing magnification, because we wanted the size of a diffraction-limited spot in the BEC to register as two (16 μ m) pixels on a camera we originally intended to use. The imaging system is designed to be compatible with both that camera and the Flea3. Due to geometric constraints in the experiment, we did not follow the typical $f_1 + f_2$ configuration for a beam expander. The 150 mm lens is 14.3 cm from the image plane of the first imaging system, which is also the object plane of the second imaging system. The 150 mm and 750 mm lenses are 69.5 cm apart, and the camera is 79.3 cm from the 750 mmlens. The object plane numerical aperture of this imaging system is 0.0728, limited to this value by the absorption TOF imaging system. The image plane numerical aperture is 0.0146.

In order to model the imaging system theoretically before constructing it, we constructed and analyzed the system in Zemax. A side view of the system in Figure 2.3a shows a point source imaged through the two lenses. The modulation transfer function is shown in Figure 2.3b. The spot diagram of the PTAI imaging system is shown in Figure 2.3c. The rays (shown in blue) launched from the object all fall within the Airy disk (shown in black), indicating that the system is diffraction-limited. The point spread function is shown in Figure 2.3d.

2.4 Testing

In order to test the imaging system before deployment in the main experiment, we constructed a bench model on a spare optical table in the lab. The bench model is depicted in Figure 2.4. Unlike the full imaging system in the experiment, the numerical aperture of the bench model was limited only by the aperture of the first lens, not by the pre-existing absorption TOF imaging system. Therefore the tests on the bench model were conducted with an object plane NA of 0.085. We measured the three metrics defined in Section 2.1: the modulation transfer function, the spot diagram, and the point spread function.

In order to measure the MTF, the United States Air Force (USAF) test pattern (Figure 2.5a) is placed at the object plane of the optical system and observed at the image plane. The test pattern contains sets of bars of different



Figure 2.3: Simulation of the proposed imaging system in Zemax OpticStudio.
a) A side view of the imaging system, with rays tracing a point in the object plane to a point in the image plane. The numerical aperture of the system is 0.0728 in the object plane.
b) The modulation transfer function of the imaging system. Unlike in Figure 2.2a, the y-axis shows the square wave MTF, which shows the response of the imaging system to square waves, rather than sines waves, of different spatial frequencies.
c) The spot diagram of the imaging system.



Figure 2.4: Photo of the bench model of the PTAI imaging system. The object plane of the PTAI imaging system, the lenses, and the CCD are represented by a star, circle segments, and a rectangle, respectively.

widths and spacings. Each group, numbered from 2 to 7, contains 6 elements, or groups of three bars. The widths of the bars decrease with increasing group number and element number.

As the sets of bars increase in spatial frequency and their spacing approaches the resolution of the imaging system, the image of the bars becomes smoothed. The intensity difference between the bars and spaces decreases. By fitting the images of the different sets of bars to a sine curve, we can calculate the modulation for each spatial frequency. The procedure for measuring the modulation transfer function is illustrated in Figure 2.5.

It is worth noting that the Nyquist condition, which states that an analog signal of frequency f must be measured with a sampling rate of 2f, is satisfied for all bar widths except the two smallest. Two pixels per bar are needed,



Figure 2.5: Measurement of the modulation transfer of the imaging system. a) A positive image of the 1951 United States Air Force test pattern is placed in the object plane of the imaging system. b) An image of the test pattern is captured at the object plane. A set of three bars is isolated and the intensity is averaged along the direction of the bars. c) A sine curve is fit to the data (here Group 2, Element 1 was chosen). The green point were excluded from the fitting data by the fit algorithm used in MATLAB. The modulation for each spatial frequency is calculated using the maximum and minimum of the sine curve.

which, with 6.45 μ m pixels, is 12.9 μ m. The last two groups of bars on the test pattern (Group 7, Elements 5 and 6) have line widths of 2.46 and 2.19 μ m. With a magnification of 5, this corresponds to 12.3 μ m and 10.95 μ m in the object plane. These sets of bars have widths less than two pixels; therefore, they cannot be included in the measurement.

The simulated and measured modulation transfer function are shown in Figure 2.6. The modulation transfer is high for low spatial frequencies on the bench model and drops more suddenly than in the simulation. It replicates the bend in the MTF simulation between 6.6 and 12.9 lp/mm, but the bend in the bench model is more pronounced. The two points with the highest spatial frequency were left out, as already described, but the modulation transfer functions appear to approach 0 at about the same frequency. The quick drop in modulation between 15 and 20 lp/mm is likely due to astigmatism in the imaging system, which will be discussed in more detail later. When the MTF was measured, the focal plane of the USAF plate was chosen to be where the combination of the defocus along the two transverse directions was minimal.

We could not measure the spot diagram of the imaging system because it is a ray optics measurement. However, we measured what would be the Gaussian optics equivalent: a cross-section in the image plane of a laser beam propagating from a 1 μ m pinhole in the object plane through the imaging



Figure 2.6: The modulation transfer function of the bench model of the partial-transfer absorption system, with an NA of 0.085. **a)** The modulation transfer function as simulated in Zemax. **b)** The modulation transfer function as measured on the bench model. The MTF is a function of spatial frequency in line pairs (lp), that is, one black line and one space, per millimeter.

system. This is compared to the spot diagram as simulated in Zemax OpticStudio, which is shown in 2.7a. In the bench model, the focal point along the vertical direction and the focal point along the horizontal direction fall at different positions along the optical axis, indicating that there is some astigmatism in the imaging system. The beam cross-sections for the horizontal and vertical directions are shown in 2.7b and 2.7c, respectively. The separation between the two focal points is 7.24 mm. When aligning the imaging system, particularly when measuring the modulation transfer function using the USAF plate, the image plane was chosen to be halfway between these two focal points.

There are a number of possible reasons for the astigmatism. Differences in lens curvature along the two transverse directions could cause the beam to focus in different places along the optical axis. Warping of the mirrors due to over-tightened mounts could have the same effect. Another possibility is oblique astigmatism, which occurs when the object is displaced from the optical axis by some distance.

The Airy radius is displayed in the simulated spot diagram and can be determined from the beam cross-sections. The theoretical value of the Airy radius is 28.10 μ m. The simulation value of 28.66 μ m is different by 2.0%. The measured value of 25.80 ± 3.23 μ m is different by 8.2%.



Figure 2.7: The spot diagram. **a**) The spot diagram of the imaging system as simulated in Zemax OpticStudio. This represents the cross-section of the beam path near the image where the beam waist is smallest. **b**) The crosssection of the beam when the imaging system is in focus along the horizontal direction. **c**) The cross-section of the beam when the imaging system is in focus along the vertical direction.

The point spread function was also measured in order to characterize the quality of the imaging system. We plotted the relative intensity of the beam as a function of position along the line that runs through the intensity peak along the direction that is in focus. We did this for the two spot diagrams in Figure 2.7. The PSF as simulated in Zemax is shown in 2.8a. For the bench model, the point spread functions were measured for the focal points along the horizontal and vertical directions and are shown in 2.8b and 2.8c, respectively. The Airy radii here are the same as in the spot diagram and beam cross-sections.

2.5 Construction

The fully constructed imaging system is shown in Figure 2.9. Due to geometric constraints, the first lens is 14.4 cm from the image plane of the first imaging system, the 150 mm and 750 mm lenses are 69.5 cm apart, and the camera is 79.3 cm from the 750 mm lens. The same camera model was used for PTAI imaging as for TOF imaging.

The deployed PTAI imaging system differs from the bench model in a couple of ways. First, the bench model used 1" elliptical mirrors, which appear to be 1" in diameter along both directions to a beam incident at 45°. If circular mirrors were used, the projection along one direction would appear



Figure 2.8: The point spread function. **a)** The point spread function of the imaging system as simulated in Zemax OpticStudio. **b)** The point spread function of the beam when the imaging system is in focus along the horizontal direction. **c)** The point spread function of the beam when the imaging system is in focus along the vertical direction.

shorter by a factor of $\sqrt{2}$ to the incident beam. This creates an aperture along one dimension, introducing asymmetry into the system. The PTAI imaging system was constructed before the importance of this effect was recognized. The first mirror, a few centimeters after the 150 mm lens, is a 1" circular mirror. The next mirror, about 15 cm before the 750 mm lens, is a 2" circular mirror. The final mirror, between the second lens and the camera, is a 2" elliptical mirror.

Another difference, as mentioned in Section 2.4, lies in the numerical aperture of the system. The numerical aperture of the bench model is 0.085, defined by the radius of the first lens and the distance between the object plane and the first lens. However, the NA of the imaging system in the experiment is limited by the NA of the time-of-flight imaging system. Zemax reports the image plane NA of the TOF system to be 0.0728, and the object plane NA of the PTAI system must be the same. Simulations of the modulation transfer function, spot diagram, and point spread function for the PTAI imaging system with an object plane NA of 0.0728 are provided in Figure 2.3.

The absorption TOF imaging is still in use, so we installed a flipper mirror between the two imaging systems in order to be able to flip between them at will. The flipper mirror was placed in the beam line a few centimeters after the last lens in the TOF imaging system. When the flipper mirror is



Figure 2.9: The deployed imaging system, displayed in sequence. **a)** The probe beam comes down from a higher level in the experiment, reflects off a 2" mirrors, and passes through the last two lenses (cage-mounted) in the TOF imaging system (right). A flipper mirror, when up, redirects light to the TOF camera (center). When down, the probe beam passes through a lens tube (left) containing the first PTAI imaging lens. **b)** The beam path passes through the first PTAI lens, reflects off a 1" mirror, reflects off a 2" mirrors, and passes through the second PTAI imaging lens. **c)** The beam reflects off a 45°, 2" mirror and passes through a hole in a raised breadboard into the camera, which is mounted on an XYZ translation stage.

up, it redirects the beam to the camera used for TOF imaging. When it is down, the beam continues through the TOF imaging line. This choice prevents small angular displacements in the mirror, which can occur when it flips, from propagating through the long PTAI imaging line and causing misalignment. The flip-to-flip uncertainty in the mirror angle is 50 μ rad, which corresponds to 82 μ m or 15 pixels for the long beam path. Having the beam bypass the flipper mirror when it is down avoids this problem, and the beam reflected from the flipper to the TOF camera is too short for angular displacements in the flipper mirror to be noticeable.

The same camera model was used for the PTAI imaging as for the TOF imaging. The camera is placed on a raised breadboard for two reasons. First, the long focal lengths of the lenses necessitate a long beam path, and raising the camera provides extra distance. Second, we had originally planned to do experiments which required using a different (much heavier) camera and being able to rotate it about the optical axis, and this would have been easier to achieve by placing the camera face-down on a rotation stage over the hole in the breadboard.

2.6 Magnification of the Imaging System

The magnification of the second imaging system was measured by placing a piece of glass with the 1951 United States Air Force test pattern at the image plane of the first imaging system, which is also the object plane of the second imaging system. The width of a set of bars in the image was divided by the width of a set of bars in the object plane to get the magnification. From an average of eight measurements, the magnification was determined to be 5.00 \pm 0.15. When the absorption TOF imaging system and the partial-transfer absorption imaging system are combined, the overall magnification is thus 15.80 \pm 1.07. Chapter 3: Deployment of Partial-Transfer Absorption Imaging

3.1 Imaging Procedure

In the partial-transfer absorption imaging technique, the BEC is prepared in a state outside a cycling transition. For every image in a cycle, a subset of the atoms is transferred into a state that is part of a cycling transition. In our system, the BEC is prepared in $|1, -1\rangle$ and transferred to $|2, -2\rangle$ using a microwave pulse.

The microwave pulses are generated by a Stanford Research Systems SG384 signal generator. Coupling the microwave signal and a ~100 MHz signal from a Novatech into a Marki IRW0618 mixer allows us to vary the microwave signal across tens of MHz. The signal then passes through an analog voltage-controlled General Microwave Herley D1956 attenuator and a Microwave Amplifiers AM53 amplifier, which allow us to control the signal amplitude. A MCLI CS-57 circulator-isolator prevents power from being reflected back toward the source by dumping a portion of the signal into a Minicircuits ZX47-40-S+ power detector. This prevents damage to upstream components, and we can maximize the power sent to the atoms by tuning the downstream Maury Microwave 1819C stub tuner to minimize the output in the power detectors after the circular. Doing this matches the impedance of the microwave components to the impedance of the microwave antenna. Finally, the signal is delivered to the antenna, a waveguide which is aimed at the atoms. The microwave transition is depicted by the purple arrow in Figure 3.1.

The atoms transferred into $|2, -2\rangle$ are then imaged on the cycling transition with the probe beam, which draws from the cooling laser. As in typical absorption imaging, three images are taken. Here the background image is taken first, followed by the probe beam image, and concluding with the atom images. The resonant probe beam heats and scatters the atoms, destroying the $|2, -2\rangle$ BEC. The BEC in $|1, -1\rangle$ sees the probe with a detuning of 6.8 GHz, so it is effectively invisible to the probe beam, and it remains in the trap to be imaged again. We take six PTAI images per experimental cycle in our experiment. A sample series of shots from one experimental cycle is shown in Figure 3.2.



Figure 3.1: Relevant levels and transitions in partial-transfer absorption imaging in ⁸⁷Rb. The Zeeman splitting is determined by the magnitude of the magnetic field, which is about 20 Gauss. The purple arrows shows the microwave transition, $|1, -1\rangle \rightarrow |2, -2\rangle$. The red arrow shows the cycling transition, $|2, -2\rangle \rightarrow |3, -3\rangle$ with σ - polarization.



Figure 3.2: A series of six PTAI shots taken in a single cycle. In this sequence, each microwave pulse lasted for 12 μ s.

3.2 Image Analysis

As described in Chapter 1, the optical depth of a BEC is

$$OD = -ln \left(\frac{I_{atoms} - I_{probe}}{I_{background} - I_{background}} \right), \tag{3.1}$$

where I_{atoms} is the intensity of the shot with the atoms, I_{probe} is the intensity of the probe shot, and $I_{background}$ is the intensity of the background shot. The OD of the BEC in each of the six PTAI shots in a cycle was calculated by referring to the background shot and the probe shot.

Additionally, a spatial lowpass filter was applied to the images in postprocessing in MATLAB in order to eliminate high-frequency shot noise. The resolution of the combined TOF and PTAI imaging systems is about 30 μ m, which is 5.4 pixels, so filtering out spatial frequencies higher than 1/30 μ m = 0.033 μ m⁻¹ will not eliminate any information about the spatial distribution



Figure 3.3: The effect of a spatial lowpass filter. **a**) Optical depth of the first BEC in a sequence. **b**) The same image with a lowpass spatial filter applied.

of the BEC. We chose a cutoff wavelength of 1 pixel because most of the noise seemed to be pixel-to-pixel variation. The effects of this filter are shown in Figure 3.3.

3.3 Optimizing Microwave Pulse Time

For every partial-transfer absorption image, we applied a microwave pulse to the system in order to transfer some of the atoms from $|1, -1\rangle$ into $|2, -2\rangle$. Figure 3.4 shows the population of the $|1, -1\rangle$ state as a function of the time for which the microwaves were on. As can be seen from the data, the Rabi period is about 80 μ s.

Clearly the length of time for which the microwaves are applied influences



Figure 3.4: Relative population of $|F = 1, m_F = -1\rangle$ as a function of microwave pulse time. The populations of the two states were measured by applying a microwave pulse of variable duration and then imaging the BECs after 21 ms TOF on the absorption TOF camera.

the destructivity of the imaging technique. On every PTAI shot, a fraction ϵ atoms are transferred from the BEC in $|F = 1\rangle$ into the $|F = 2\rangle$ hyperfine state, where

$$\epsilon = \sin^2(\omega_R t), \tag{3.2}$$

where ω_R is the Rabi frequency and t is the microwave pulse time. The Rabi frequency is typically about 90 kHz for this system. Therefore, the N^{th} PTAI image in a sequence draws atoms from an $|F = 1\rangle$ BEC with a population of

$$P_N = P_0 \left(1 - \epsilon \right)^{N-1}, \tag{3.3}$$

where P_0 is the original number of atoms in the BEC. The number of atoms that one can expect to observe in the N^{th} PTAI shot is therefore

$$M_N = \epsilon P_N = \epsilon P_0 (1 - \epsilon)^{N-1}.$$
(3.4)

This can be expressed as an exponential function

$$M_N = M_0 \epsilon \exp\left[(N-1)ln(1-\epsilon)\right] = N_0 \exp(-\frac{N-1}{\tau_N}),$$
 (3.5)

where M_0 is the population of atoms in the F = 1 BEC before any images are taken. N_0 and τ_N are the fit parameters of an exponential function: the initial population and time constant, respectively. M_0 is approximately the number of atoms in the first PTAI shot, and $\tau_N = ln(1-\epsilon) \approx -\epsilon$ for $\epsilon \ll 1$. An exponential fit was applied to the atom number in each PTAI shot. N_0 and τ_N are plotted in Figure 3.5 as a function of the time used to pulse atoms into $|2, -2\rangle$ for each shot using the microwaves.

Figure 3.5a shows the fit parameter N_0 in each experimental cycle as a function of the time for which the microwave pulse was applied. As expected, the data bears a resemblance to Rabi flopping. For short pulse times, the atom number is small because only a small percentage of the atoms are transferred into $|2, -2\rangle$. The initial population reaches a peak near half the value of the Rabi period, due to the fact that close to 100% of the atoms have been transferred into $|2, -2\rangle$. The initial population falls again as the atoms have been transferred into $|2, -2\rangle$ and are being transferred back into $|1, -1\rangle$ again.

Additionally, for the six partial-transfer absorption images taken in a cycle, we should expect the decay rate to depend on the microwave pulse length. For short times, when few atoms are transferred into $|2, -2\rangle$ on every shot, the decay rate should be small. For times near $(n + \frac{1}{2})T_R$, where T_R is the Rabi period and n is an integer, we should expect the decay rate to be large because the majority of the BEC was transferred into $|2, -2\rangle$ on the first shot. We did an exponential fit to the atom number as a function of PTAI shot



Figure 3.5: For the partial-transfer absorption imaging shots in each cycle, an exponential fit was applied to the atom number as a function of shot number. **a)** The M_0 fit parameter, theory (black) and experiment (blue). **b)** The τ_N fit parameter, theory (black) and experiment (blue).

number for different values of the microwave pulse length. The time constant of the fit is shown in Figure 3.5b. As expected, the time constant is large for short pulse times and near the Rabi period.

We can also look at the atom number in each PTAI shot compared to theory. This is displayed in Figure 3.6. The atom number in each shot is normalized to the atom number in the first shot of the sequence in order to avoid noise due to shot-to-shot number fluctuations. This is displayed as a function of the microwave pulse time, along with the theoretical normalized



Figure 3.6: Atom number in each PTAI shot, normalized to the atom number in the first shot. Plots show second to fifth shot from left to right and top to bottom row.

atom number

$$\frac{M_N}{M_1} = (1 - \epsilon)^{N-1} = (1 - \sin^2(\omega_R t_\mu)^{N-1}.$$
(3.6)

As expected, when the microwave pulse time is low and a small fraction of atoms is transferred per shot, the normalized atom number in shots 2 to 5 approaches 1. Near 40 μ s, about half the Rabi period, nearly all of the atoms were transferred into F = 2 on the first shot, so the normalized atom number is near zero.

3.4 Outlook and Future Work

We have demonstrated the design, testing, and implementation of a partial-transfer absorption imaging system. We have also characterized the atom number in each shot and the number decay rate as a function of one of the imaging parameters, the length of time for which the microwave signal was applied.

A number of improvements could be made to the imaging system. First, we could replace the 1" and 2" circular mirrors with 2" elliptical mirrors to avoid asymmetric apertures in the reflected light field. Second, we could replace the Flea3 camera with a Princeton Instruments ProEM camera already owned by RbLi. This camera uses an electron-multipled CCD and frametransfer to achieve kHz imaging rates.

This camera would be necessary to study collective modes in BECs, such as breathing, dipole, and scissors modes. The frequencies of collective modes are on the order of the BEC trap frequencies, or up to an order of magnitude higher. The trap frequencies in the RbLi experiment are $(\omega_x, \omega_y, \omega_z)/2\pi =$ (42(3), 34(2), 133(3)) Hz. The Flea3 camera is limited to a rate of 120 fps, so a faster camera is needed for a serious study of collective modes.

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