Strong Atom-Light Interactions in Nano-Photonics
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Senior Collaborators:
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**Outline: Strong Atom-Light Interactions in Nano-Photonics**

**H. Jeff Kimble**  
Institute for Quantum Information and Matter  
California Institute of Technology

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**Lecture 1 - Atom-photon interactions along a 1D waveguide**

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**Lecture 2 - Optical traps in nano-photonics**

~ 300nm

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**Lecture 3 - Atom-atom interactions mediated by photons in 1D and 2D photonic crystals**

\[
H = \sum_{i,j} \left[ J_{ij} \sigma_i^z \sigma_j^z + J_{ij}^{xy} \sigma_i^x \sigma_j^y \right]
\]
A Grand Challenge for Quantum Information Science - Building Exotic Quantum Systems

- "Lego blocks" for the realization of complex quantum systems
- Fundamental scientific question and diverse technical challenges

Laboratory realization of physical systems different in kind than have heretofore existed

- Characterization and verification of entanglement for multipartite systems
- The physical consequences of entanglement

- Quantum information processing
- Quantum measurement
- Quantum simulation
Quantum interactions between atoms and light

A frontier to achieve 1), 2), 3) in one setting with strong interactions of single photons & atoms

What's inside here?

1. Multi-pass interaction and small mode volume in an optical cavity (cQED)

   cavity QED

2. Large optical depth
   • atomic ensembles
   • quantum gases

3. Strong focusing (localization) of light
A New Way Forward - 
The integration of atomic physics and nano-photonics

- Nano-scopic dielectric waveguides and resonators -
  - Low optical loss ($Q \sim 10^6$), efficient transport and coupling (>99%)  

- Quantum functionality -
  - Trap single atoms within dielectric nano-structures
  - Optical depth for one atom $\sim 6$: $A_{\text{eff}} < \lambda^2$
  - Critical photon number in cQED $< 10^{-7}$: $V_{\text{eff}} < \lambda^3$

- Photonic & phononic quantum circuits -
  - Photon transport entangled with atomic quantum state
  - Atom-atom interactions mediated by single photons
  - Quantum many-body physics
Observation of Quantized Motion of Rb Atoms in an Optical Field

National Institute of Standards and Technology, U.S. Department of Commerce, Technology Administration,
PHYS A167, Gaithersburg, Maryland 20899
(Received 6 May 1992)

Photonic band gaps in optical lattices

National Institute of Standards and Technology, PHYS A167, Gaithersburg, Maryland 20899
(Received 31 August 1994; revised manuscript received 5 December 1994)

FIG. 1. The self-consistent lattice arises from a redistribution of photons in a standing wave due to phase shifts induced by periodically trapped atoms. (a) shows the standing wave in the absence of atoms. For a standing wave detuned to the blue of the atomic resonance shown in (b), atoms are trapped at the nodes and thus the period of the standing wave is equal to its vacuum value. A standing wave detuned to the red of resonance traps atoms at the antinodes, which leads phase shifts of the field. The solution shown in (c) is such that the nodes are “pulled” closer to the atomic planes and the resulting standing wave has a reduced effective wavelength with cusps at its antinodes [Eq. (2b)].
**Observation of Quantized Motion of Rb Atoms in an Optical Field**


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(Received 6 May 1992)

**Photonic band gaps in optical lattices**

I. H. Deutsch, R. J. C. Spreeuw,* S. L. Rolston, and W. D. Phillips

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(Received 31 August 1994; revised manuscript received 5 December 1994)

**Photon Recoil Momentum in Dispersive Media**

Gretchen K. Campbell, Aaron E. Leanhardt,* Jongchul Mun, Micah Boyd, Erik W. Streed, Wolfgang Ketterle, and David E. Pritchard†

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(Received 31 January 2005; published 4 May 2005)

A systematic shift of the photon recoil momentum due to the index of refraction of a dilute gas of atoms has been observed. The recoil frequency was determined with a two-pulse light grating interferometer using near-resonant laser light. The results show that the recoil momentum of atoms caused by the absorption of a photon is \( n \hbar k \), where \( n \) is the index of refraction of the gas and \( k \) is the vacuum wave vector of the photon. This systematic effect must be accounted for in high-precision atom interferometry with light gratings.
Photonic Band Gaps in One-Dimensionally Ordered Cold Atomic Vapors

Alexander Schilke, Claus Zimmermann, Philippe W. Courteille, and William Guerin

FIG. 2 (color online). Spectra for different atom numbers \( N \) in the lattice (constant length, varying density) with \( \Delta \lambda_{\text{ap}} = 0.15 \) nm. Inset: Maximum reflectance as a function of \( N \).
Optical transport along a 1D waveguide coupled to a linear lattice of atoms

Single atom coupling

\[ E_{in} \xrightarrow{r_1 E_{in}} \Gamma_{1D} \xrightarrow{t_1 E_{in}} \]

Atomic Bragg mirror with \( N_M \) atoms

\[ E_{in} \xrightarrow{r_{N_M} E_{in}} j=1,2 \ldots \ x \rightarrow \xrightarrow{t_{N_M} E_{in}} j=N_M \]
Transfer Matrices from Lecture 1 Blackboard

Transfer matrices for 1D atom chains

Left:

\[ E_{\text{in}} \rightarrow S \rightarrow E_{\text{out}} \quad \text{Right} \]

\[
\begin{pmatrix}
E_{\text{out}} \\
E_{\text{out}}
\end{pmatrix} = S
\begin{pmatrix}
E_{\text{in}} \\
E_{\text{in}}
\end{pmatrix}
\]

(1.1)

Transfer matrix \( M \)

\[
\begin{pmatrix}
E_{\text{out}} \\
E_{\text{in}}
\end{pmatrix} = M
\begin{pmatrix}
E_{\text{in}} \\
E_{\text{out}}
\end{pmatrix}
\]

(1.2)

Right in terms of left

Example: barriers: perfect reflector

\[ S = \begin{pmatrix}
t - r \\
r + t
\end{pmatrix} \]

(1.3)

\[ M = \frac{t}{t + r} \begin{pmatrix}
t - r \\
r + t
\end{pmatrix} \quad \text{note: } \det M = 1 \]
Transfer Matrices
from Lecture 1 Blackboard

1. For a multi-element system:

\[ E_{\text{out}} = M_n \ldots M_2 M_1 (E_{\text{in}}) \]

2. For \( N \) identical elements:

\[ M(N) = \begin{pmatrix} m_1 & m_2 \\ m_1 & m_2 \end{pmatrix} = M_1^N \]

\[ (E_{\text{out}}) = M(N) (E_{\text{in}}) \]

In the case (with elements identical or not), we have:

The amplitude and reflection coefficients are given by:

\[ t = \frac{E_{\text{out}}}{E_{\text{in}}} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \left( e^{i \theta} \right) \]

Transmission amplitude for left to right propagation, \( t \):

Reflection coefficient for left to right propagation, \( r \):

\[ r = \frac{E_{\text{out}}}{E_{\text{in}}} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \]

and

\[ r = \frac{E_{\text{out}}}{E_{\text{in}}} = 6 e^{i \theta} - \frac{m_1}{m_2} \]

for \( \theta = 0 \), \( \theta = \pi \)
Transfer Matrices
from Lecture 1 Blackboard

\[ E_p = \begin{pmatrix} E_{1p} \\ E_{2p} \end{pmatrix} \]

\[ \Gamma_p = \begin{pmatrix} \Gamma_{1p} \\ \Gamma_{2p} \end{pmatrix} \]

\[ \Gamma_0 = \text{constant - A coefficient} \]

\[ \Gamma_{\text{Total}} = \Gamma_0 + \Gamma_p \quad (\text{Total transmission rate for the "new" waveguide}) \]


\[ M_A = \begin{pmatrix} r_{11} & r_{12} \\ -r_{11} & r_{11} \end{pmatrix} \]

\[ r_{11} = \frac{r_0}{1 + r_0} \quad \text{with} \quad r_0 = \frac{\Gamma_{10}}{\Gamma_{1p} + \Gamma_{2p}} \]

\[ r_{12} = 1 + r_0 \]

\[ f = \frac{(\omega_A - \omega_p)}{\frac{1}{2} \Gamma_{1p}} \]

\[ \omega_A - \text{atomic transition frequency} \]

\[ \omega_p - \text{probe input frequency} \]
Transfer Matrices
from Lecture 1 Blackboard

\[ M_1 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \]

Transfer matrix for unit cell

\[ M_N = M_1 \]

Transfer matrix for \( N \) unit cells: \( M_N \)

Use elements of \( M_N \) to deduce reflectance \( R \) and transmission \( T \) coefficients.

\[ e^\pm \text{ (for}\ \text{waveguide)} \]

\[ e^\pm \text{ (for}\ \text{waveguide)} \]

Sample: take \( h = 0 \)

Generic case: \( k_1, k_2, k_3, \ldots \)

Special case: \( k_1 = 0 \)

\[ N(\text{truncation}) \]

\[ N_{\text{trunc}} - \text{symmetrical coupling} \]

\[ \begin{pmatrix} R_1 & T_1 \\ R_2 & T_2 \end{pmatrix} \]

\[ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]
Basic rates for atom-field interaction in 1-d waveguides

\[ r_1(\omega_A) = -\frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} \]

Weisskopf-Wigner to find
\[ \Gamma_{1D}(\vec{r}_A) = \frac{1}{2} \frac{c}{v_g} \frac{\sigma_0}{A_m(\vec{r}_A)} \Gamma_0 \]

\[ \sigma_0 = \frac{3}{2 \pi} \frac{\lambda_A^2}{\omega_A^2} \text{ is the atomic scattering cross section.} \]
\[ \Gamma_0 = \frac{|\mu|^2}{3 \pi \varepsilon_0 \hbar c^3} \omega_A^3 \text{ is the Einstein A coefficient.} \]
\[ v_g \text{ is the group velocity in the waveguide.} \]

\[ A_m(\vec{r}_A): \text{Effective mode area defined for an atom at location } \vec{r}_A \]
\[ A_m(\vec{r}_A) = \int d^2 r \epsilon(\vec{r}) |E(\vec{r})|^2 \]
\[ \epsilon(\vec{r}_A) |E(\vec{r}_A)|^2 \]

Caveat: These results assume \[ |\vec{\mu} \cdot \vec{E}|^2 = |\vec{\mu}|^2 |\vec{E}|^2 \].
The vector character of \( \vec{\mu}, \vec{E} \) generally reduces \( \Gamma_{1D} \).
Examples for atom-field interactions in 1-d waveguides

For Cs atom at surface of SiO$_2$ nanofiber with diameter $d = 400nm$

$\Rightarrow \frac{P_T}{P_{in}} \sim 0.8 \leftarrow 20\%$ absorption / atom

At $\rho - d = 200nm$ from surface $\Rightarrow \frac{P_T}{P_{in}} \sim 0.95 \leftarrow 5\%$ absorption / atom

For Cs atom inside a 1d photonic crystal (Si$_3$N$_4$) with transition frequency near a band edge,

$\Rightarrow \frac{P_T}{P_{in}} \sim 0.003 \leftarrow 99.7\%$ absorption / atom
An Atomic Chain Coupled to a Nano-Fiber Waveguide

Single atom coupling

Atomic Bragg mirror with $N_M$ atoms

Nano-fiber experiment - $r_i \approx 0.07$
Projected PCW - $r_i \approx 0.95$

$$ r_i = \frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} = 0.2 $$

$$ k_{probe} \times d_M = \pi $$

$R_{N_M}$

$F_{N_M}$

$N_M = 10^3$

$N_M = 10^4$

$N_M = 10^5$

$\delta_A = 0$

$\delta_A = 10^3$

$N_M$ vs. $\delta_A$

$F_{N_M}$ vs. $\delta_A$
Cavity QED with Atomic Mirrors

**Quantum protocols**
- Single photon generation
- Entanglement distribution
- Quantum logic
  - atoms
  - photons
  - ...

Mirrors as coherent quantum memories strongly coupled to single impurity atom
Cavity QED with Atomic Mirrors

\[ k_{\text{probe}}(\delta = 0) \times d_M = \pi \]

\[ j = 0 \]

\[ r_1 = \frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} = 0.2 \]
Cavity QED with Atomic Mirrors – Strong Coupling


\[ j = 0 \]

\[ k_{\text{probe}} (\delta_A = 0) \times d_M = \pi \]
Cavity QED with Atomic Mirrors

Cavity QED figures of merit $g, \kappa, \Gamma$

Atom decay: $\Gamma = \Gamma_{1D} + \Gamma'$ total cavity decay rate
Cavity decay: $\kappa = \Gamma'$ for subradiant mode of atomic mirrors
Coherent atom-cavity coupling: $g = \Gamma_{1D} \sqrt{N_A} / 2$

Hence, atomic cooperativity parameter $C$

$$C \equiv \frac{g^2}{\kappa \Gamma} = \frac{\Gamma_{1D}}{\Gamma_{1D} + \Gamma'} \frac{N_A \Gamma_{1D}}{\Gamma'},$$
where cavity finesse $F \propto \frac{N_A \Gamma_{1D}}{\Gamma'}$

A Surprise!

- Strong coupling regime can be reached with very low cavity finesse $F < 10^3$
- Conventional Fabry-Perot cavity with dielectric mirrors requires finesse $F \approx 10^5$
- $\kappa \ll \kappa_c = \nu \pi / L_{\text{eff}} F$
Atoms trapped near waveguide interact with guided modes (GMs) of the structure. The interaction Hamiltonian is of the following form in 1D:

$$H_{\text{int}} = \sum_j \int dk_z g_{k_z} e^{ik_z \cdot x_j} a_{k_z} \sigma_{ge}^j \dagger + \text{H.c.}, \text{ with } \sigma_{ge}^j = |g\rangle_j \langle e| \text{ for atom } j.$$  

Here $$g_{k_z} = \sqrt{\frac{\omega(k_z)}{2\pi \hbar \varepsilon_0 A_m}} \vec{\mu} \cdot \varphi_{k_z}(\vec{r}_j).$$

*$$H_{\text{int}}$$* induces an effective interaction between the atoms. In Born-Markov approximation, a master equation results for the dynamics of the atoms by tracing out the photonic degrees of freedom.

Photon-Mediated Atom-Atom Interactions along a 1D Waveguide*

Master equation for the atomic density matrix $\rho$

$$\dot{\rho} = -i[H_{dd}, \rho] + L_{dd}[\rho],$$

where

$$H_{dd} = \left(\frac{\Gamma_{1D}}{2}\right) \sum_{j,k} \sin k_A |z_j - z_k| \sigma_{eg}^j \sigma_{ge}^k,$$

and

$$L_{dd}[\rho] = -\left(\frac{\Gamma_{1D}}{2}\right) \sum_{j,k} \cos k_A |z_j - z_k| \left(\sigma_{eg}^j \sigma_{ge}^k \rho + \rho \sigma_{eg}^j \sigma_{ge}^k - 2\sigma_{ge}^j \rho \sigma_{eg}^j\right).$$

$H_{dd}$ - coherent dipole-dipole coupling between atoms $j, k$

$L_{dd}$ - cooperative atomic emission (e.g., super- and sub-radiance)

⇒ “Infinite” range spin-spin interactions with sinusoidal coupling set by $\Gamma_{1D}$


Determining the optical fields along the spin chain (D. Chang)*

$E_\pm(z,t)$ and $E^\dagger_\pm(z,t)$ – annihilation and creation operators for right (+) and left (-) propagating quantum fields.

Heisenberg equations for slowly varying envelopes $E_\pm(z,t)$ –

$$\left[ \frac{\partial}{\partial z} \pm \frac{1}{v_g} \frac{\partial}{\partial z} \right] E_\pm(z,t) = \pm \frac{i\sqrt{2\pi} g_k(\omega_A)}{v_g} \sum_j \delta(z - z_j) \sigma_{ge}^j(t),$$

with solutions

$$E_\pm(z,t) = E^\text{in}_\pm(z \pm v_g t) + \frac{i\sqrt{2\pi} g_k(\omega_A)}{v_g} \sum_j \theta(\pm(z - z_j)) \sigma_{ge}^j(t - \frac{z \pm z_j}{v_g}).$$

Extend from two-states (ground & excited electronic states) to “lambda” system

\[ \pi \]

\[ \Omega(t) e^{i\phi_j} \rightarrow \Gamma' \]

\[ \Gamma_{1D} \]

\[ |s\rangle \quad |g\rangle \]

states of atom \( j \)

collective states
Building Blocks for Scalable Quantum Information Processing*

High fidelity quantum bus for state transfer & entanglement distribution

Nano-photonic waveguide

Creation of arbitrary quantum state $\psi$ for the atomic "spin" chain

$$|\psi\rangle = \sum c_{\{j\}} \left| \uparrow_1 \downarrow_2 \ldots \uparrow_N \right\rangle$$

Coherent mapping of atomic spin state $\psi$ to and from propagating optical fields

Photon Transport Along a 1D Atom Chain

Strongly Correlated Two-Photon Transport in a One-Dimensional Waveguide Coupled to a Two-Level System
Jung-Tsung Shen and Shanhui Fan*

Photonic quantum transport in a nonlinear optical fiber
M. Hafezi\textsuperscript{1(a)}, D. E. Chang\textsuperscript{2}, V. Gritsev\textsuperscript{1,3}, E. A. Demler\textsuperscript{1} and M. D. Lukin\textsuperscript{1}

Hybrid quantum system of a nanofiber mode coupled to two chains of optically trapped atoms
Hashem Zoubi\textsuperscript{1} and Helmut Ritsch
Photon-mediated forces can be extremely large in nanophotonic systems. Specifically, $\Gamma_{1D}$ for pairwise interactions in $H_{dd}$ can be comparable to $\Gamma_0$, which corresponds to the Doppler temperature. The long-range nature of interactions can produce forces further enhanced by a factor $\sim N$ over a single pair.

In the semiclassical limit, these equations determine the full atomic dynamics and the possibility of self-organized configurations.
A global perspective – Compression of the lattice with increasing dispersion

\[ N = 150 \text{ atoms} \]

\[ \frac{\Gamma_{1D}}{\Gamma'} = 0.25 \rightarrow r_1 = \frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} = 0.2 \]
Self Organization of Atoms along a 1-D Waveguide

Nano-photonic waveguide

\[ \Gamma_{1D}/\Gamma' = 0.25 \rightarrow r_1 = \frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} = 0.2 \]

\[ N = 150 \text{ atoms} \]

Spectrum of phonon frequencies

\[ \frac{\gamma_{\text{max}}}{\sqrt{\omega_{\text{TSoNT1D}}}} \]

\[ \frac{\omega_{\text{ph}}}{\sqrt{\omega_{\text{TSoNT1D}}}} \times 10 \]
Advances with Atoms and Hollow Core Waveguides

Loading atoms into hollow-core fiber

- Large optical depth
- Small mode volume
- Core diameter = 7 µm

Frequency shift:
\[ \Delta_A / 2\pi \sim 60 \text{ MHz} \]

M. Bajcsy et al., PRL 102, 203902 (2009) [Vuletic & Lukin]

P. Londero et al., PRL 103, 043602 (2009) [Gaeta]

B. Wu et al., Nature Photonics 4, 776 (2010) [Schmidt]
Pioneering Research with Nano-Fiber Traps

Atom trapping and guiding with a subwavelength-diameter optical fiber

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SiO$_2$ nano-fiber
Pioneering Research with Nano-Fiber Traps

Atom trapping and guiding with a subwavelength-diameter optical fiber

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Spontaneous radiative decay of translational levels of an atom near a dielectric surface

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(Received 11 September 2006; revised manuscript received 12 December 2006; published 30 January 2007)

Antibunching and bunching of photons in resonance fluorescence from a few atoms into guided modes of an optical nanofiber

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(Received 17 June 2008; published 11 February 2009)
A Landmark Advance with Nano-Fiber Traps

Optical Interface Created by Laser-Cooled Atoms Trapped in the Evanescent Field Surrounding an Optical Nanofiber

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(Received 4 December 2009; published 20 May 2010)

Two-color FORT (1064nm, 780nm)

Atoms trapped in the evanescent field of the sub-wavelength fiber

$N_A \approx 2000$ trapped atoms
Absorption Spectroscopy for a Nano-Fiber Trap

**Pioneering nano-fiber trap**

Linewidth \(\sim 20\) MHz  
Frequency shift \(\sim 13\) MHz  
Optical depth \(\sim 13\)  
Absorption/atom \(\sim 0.65\%\)

(green curve: MOT cloud spectrum without trap)

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**Is it possible to trap near dielectric surfaces without large frequency shifts & line broadening for electronic transitions?**

*Caltech: Compensated, magic wavelength nanofiber trap*
Demonstration of a State-Insensitive Nanofiber Trap


Nano-fiber

atoms trapped ~ 220nm from fiber surface

~800 atoms

$E_{in}$ $E_{reflected}$ $E_{transmitted}$
Absorption Spectroscopy for a Nano-Fiber Trap

\[ N_A \approx 800 \text{ trapped atoms; } f \approx 0.2 \text{ fraction of sites occupied} \]

\[ T(\delta) = \frac{|E_{\text{in}}|^2}{|E_{\text{transmitted}}|^2} \]

(trap at \( \tau \approx 300 \text{ ms} \))
Linewidth = 5.7 ± 0.1 MHz
(4.94 ± 0.32) MHz ~ 5.26 MHz
Frequency shift < 0.5 MHz

(trap at \( \tau \approx 1 \text{ ms} \))
Optical depth \( \approx 66 \pm 17 \)
OD/atom \( \approx 8\% \)

Recall Vetsch et al. (2010)
OD/atom \( \approx 0.65\% \)
Outline: Strong Atom-Light Interactions in Nano-Photonics

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Lecture 1 - Atom-photon interactions along a 1D waveguide

Lecture 2 - Optical traps in nano-photonics

Lecture 3 - Atom-atom interactions mediated by photons in 1D and 2D photonic crystals

\[ H = \sum_{i,j} \left[ J_{i,j}^{zz} \sigma_i^z \sigma_j^z + J_{i,j}^{xy} \sigma_i^{\dagger} \sigma_j \right] \]
OPTICAL DIPOLE TRAPS FOR NEUTRAL ATOMS

Advances in AMO Physics 42 (2000)

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Atom in optical field $E$

Atomic dipole $\vec{p} \sim \alpha \vec{E}$

$\Rightarrow U_{\text{dipole}} \sim -\vec{p} \cdot \vec{E} \sim -\alpha |\vec{E}|^2$

$\alpha \leftrightarrow$ atomic polarizability
FIG. 3. Illustration of dipole traps with red and blue detuning. In the first case, a simple Gaussian laser beam is assumed. In the second case, a Laguerre-Gaussian $LG_{01}$ “doughnut” mode is chosen which provides the same potential depth and the same curvature in the trap center (note that the latter case requires $e^2$ times more laser power or smaller detuning).
OPTICAL DIPOLE TRAPS FOR NEUTRAL ATOMS

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2 state atom

\[ U_{\text{dip}}(r) = \frac{3\pi c^2}{2\omega_0^3} \frac{\Gamma}{\Delta} I(r) , \]

\[ \Gamma_{\text{sc}}(r) = \frac{3\pi c^2}{2\hbar\omega_0^3} \left( \frac{\Gamma}{\Delta} \right)^2 I(r) . \]

FIG. 1. Light shifts for a two-level atom. Left-hand side, red-detuned light (\( \Delta < 0 \)) shifts the ground state down and the excited state up by same amounts. Right-hand side, a spatially inhomogeneous field like a Gaussian laser beam produces a ground-state potential well, in which an atom can be trapped.
Optical Dipole-Force Trap [FORT] - The real story

OPTICAL DIPOLE TRAPS FOR NEUTRAL ATOMS

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Multilevel atom -

\[
\Delta E_i = \sum_{j \neq i} \left| \frac{\langle j | \mathcal{H}_1 | i \rangle}{\varepsilon_i - \varepsilon_j} \right|^2 .
\]

\[
\mathcal{H}_1 = -\hat{\mu} E \text{ with } \hat{\mu} = -e \mathbf{r}
\]

Cesium atom -


Cesium D₂ Line Data

Daniel A. Steck
Optical dipole-force trap [FORT]

Light-shift Hamiltonian: \( \hat{H}_{1s} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 \)

Scalar
\[
\hat{H}_0 = -\alpha^{(0)} \hat{E}(-) \cdot \hat{E}(+)
\]

\( \alpha^{(0)}, \alpha^{(1)}, \) and \( \alpha^{(2)} \) are the scalar, vector, and tensor atomic dynamic polarizabilities

Vector
\[
\hat{H}_1 = -i\alpha^{(1)} \left( \left( \hat{E}(-) \times \hat{E}(+) \right) \cdot \hat{F} \right) \frac{2 F}{2 F}
\]

Tensor
\[
\hat{H}_2 = -\sum_{\mu, \nu} \alpha^{(2)} \hat{E}^{(-)}_{\mu} \hat{E}^{(+)}_{\nu} \frac{3}{F(2 F - 1)} \left[ \frac{1}{2} (\hat{F}_\mu \hat{F}_\nu + \hat{F}_\nu \hat{F}_\mu) - \frac{1}{3} \hat{F}^2 \delta_{\mu\nu} \right]
\]

\( \hat{E}^{(+)} \) and \( \hat{E}^{(-)} \) are the positive and negative frequency components of the electric field

\( \hat{F} = \hat{I} + \hat{J} \) is the atomic total angular momentum operator, with \( \hat{I} \) and \( \hat{J} \) the nuclear and electronic angular momentum operators

\( \mu, \nu \in \{x, y, z\} \) are Cartesian components
Light-shift Hamiltonian: linear polarization

\[ \hat{H}_{1S} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 \]

Scalar

\[ \hat{H}_0 = -\alpha^{(0)} \hat{E}(-) \cdot \hat{E}(+) \]

For two-level atoms with ground and excited states \(|g\rangle, |e\rangle\), the scalar shift \(U_{\text{scalar}}\) can be approximated by \(U_{\text{scalar}} \propto |E|^2/\delta\) for detunings \(\delta = \omega - \omega_a\) large compared to the excited state decay rate \(\Gamma\), where \(\omega\) is the electric field angular frequency and \(\omega_a\) is the \(|g\rangle \rightarrow |e\rangle\) transition frequency. The ground state will experience a repulsive potential for blue-detuned (\(\delta > 0\)) electric fields, and an attractive potential for red-detuned (\(\delta < 0\)) electric fields. The scalar dynamic polarizability \(\alpha^{(0)}\) is in general different for the states \(|g\rangle\) and \(|e\rangle\) resulting in a differential scalar shift and a mismatch of the ground and excited state potentials. For the typically anti-trapped excited state, near-resonant driving of the transition by an additional beam with frequency \(\omega_2 \approx \omega_a\) can cause significant heating of a trapped atom. This situation can be remedied by the use of “magic” wavelengths for which \(\alpha^{(0)}_{|g\rangle} = \alpha^{(0)}_{|e\rangle}\).
Light-shift Hamiltonian: circular polarization

\[ \hat{H}_{1s} = \hat{H}_0 + \hat{H}_1 + \mathbf{X}_2 \]

Cs: 6S\(_{1/2}\) (green), 6P\(_{1/2}\) (purple), \(\{F,q\} = \{4, 1\}\); \(\lambda_{\text{FORT}} = 0.9356\)

The vector term \(\hat{H}_1\) induces a Zeeman-like splitting proportional to a projection of the total atomic angular momentum \(\mathbf{F}\) and arises from a so-called “fictitious magnetic field” proportional to the ellipticity of the electric field [Cohen-Tannoudji]. In the case of a free-space plane wave propagating along the \(z\) axis, \(\hat{H}_1\) can be expressed in terms of the Stokes operators \(\hat{S} = (\hat{S}_0, \hat{S}_x, \hat{S}_y, \hat{S}_z)\) as:

\[ \hat{H}_1 \propto \alpha^{(1)}(\omega) \epsilon \hat{F}_z / F, \]

where \(\epsilon = \langle \hat{S}_z \rangle / \langle \hat{S}_0 \rangle = \frac{|E_{+1}|^2 - |E_{-1}|^2}{|E_{+1}|^2 + |E_{-1}|^2}\) is the ellipticity of the electric field.
Light-shift Hamiltonian:  
\[ \hat{H}_{1S} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 \]

Tensor shifts from \( \hat{H}_2 \) vanish for atoms with total angular momentum \( F = 1/2 \). In the case of the \( D_2 \) transition of Cs that I will consider here, tensor shifts depend only on the electronic angular momentum \( \hat{J} \) for detunings large compared to the \( 6P_{3/2} \) excited state hyperfine structure, and vanish for \( J = \frac{1}{2} \) [Deutsch2010]. Tensor shifts are therefore important for the excited states of the \( D_2 \) level (but not for \( D_1 \)), inducing shifts on the Zeeman \( m_F \) sublevels proportional to \( m_F^2 \).
Magic wavelengths for atomic Cesium

Cesium D₂ line:

- ground: F=4, 6S½
- excited: F=5, 6P½

Uₙₙ (MHz)

λₜₜ (nm)

852nm

F=5, 6P½
F=4, 6S½
Magic wavelengths for atomic Cesium

Magic wavelengths:
\[ \Delta U^{(0)}_{\text{ground}}(\lambda_{\text{magic}}) = \Delta U^{(0)}_{\text{excited}}(\lambda_{\text{magic}}) \]

- Zero scalar differential shifts for \( \alpha^{(0)} \)
  (i.e., no change in transition frequency as atom moves in trap)
- Reduced decoherence and heating for driven atom
Modes of dielectric waveguides
**HE\textsubscript{11} Mode of a Dielectric Cylinder in Vacuum**

**Fundamental mode:** HE\textsubscript{11}

Electric field external to cylinder:

\[
E_x(r, \phi, z, t) = A \, e^{i(\omega t - \beta_{11} z)}, \quad E_y(r, \phi, z, t) = B \, e^{i(\omega t - \beta_{11} z)}, \quad E_z(r, \phi, z, t) = iC \, e^{i(\omega t - \beta_{11} z)},
\]

\(A, B, C\) are real functions of \(r, \phi\)

\[
\frac{\pi}{2} \text{ phase shift between longitudinal field } E_z \quad \text{and transverse fields } E_x,E_y
\]

\[
E_x^{\text{max}} = 0.892|E|^{\text{max}}, \quad E_y^{\text{max}} = 0.224|E|^{\text{max}}, \quad E_z^{\text{max}} = 0.453|E|^{\text{max}}
\]
Figure 3. Electric field $E(x, y, z, t)$ of a single propagating beam in the plane $y = 0$. The input beam is $x$-polarized. The electric field $\Re[E(x, y, z, t)]$, with $E(x, y, z, t)$ defined as in equation (4), is indicated by the blue arrows. The red arrow indicates the beam propagation direction. The field is shown for (a) $\omega t = 0$, (b) $\omega t = \pi/2$ and (c) $\omega t = \pi$. 

Figure 4. Total electric field $\mathbf{E}(x, y, z, t)$ for two counter-propagating beams in the plane $y = 0$. The input beams are $x$-polarized. The electric field $\text{Re}[\mathbf{E}(x, y, z, t)]$ is indicated by the blue arrows. The red arrows indicate the beam propagation directions. The electric field is shown for (a) $\omega t = 0$, (b) $\omega t = \pi/4$ and (c) $\omega t = \pi$. As opposed to figure 3, the polarization of the electric field is linear at any point $|\mathbf{r}| > a$ (i.e. the polarization vector has no ellipticity and $\mathbf{E}$ does not rotate in time at a given position $\mathbf{r}$ as in 3).
Standing wave for HE_{11} Mode

**Figure 5.** Electric field amplitude after interference $E^{(tot)} = E^{(fwd)} + E^{(bwd)}$ of two 937 nm beams with $\delta_{fb} = 0$, at $t = 0$ and $r = a_+$ as in figure 1(d) (i.e. $x$-polarized inputs with $\phi_0 = 0$). The fields are normalized to the intensity $I_0$ at $r = a_+$, $\phi = 0$, $z = 0$. (a) The axial direction $z$ (at $\phi = 0$). (b) The azimuthal direction $\phi$ (at $z = 0$). In particular, $E^{(tot)}$ has a fixed linear polarization at any given point $r$ which rotates as $r$ is varied.
A State-Insensitive, Compensated Nanofiber Trap

SiO$_2$ nano-fiber diameter = 430nm

Laser parameters

$\lambda_{\text{red}} = 935$ nm
$P_{\text{red}} = 2 \times 0.4$ mW
$\lambda_{\text{blue}} = 687$ nm
$P_{\text{blue}} = 2 \times 5$ mW

Trap frequency

$\nu_{\phi} = 210$ kHz
$\nu_{z} = 292$ kHz
$\nu_{\phi} = 39$ kHz
A state-insensitive nanofiber trap: Magic-compensated scheme*


* Radial
* Axial
* Azimuthal
1-D Photonic Crystal
Joannopoulos, Johnson, Winn, Meade book

Figure 1: The multilayer film, a one-dimensional photonic crystal. The term “one-dimensional” is used because the dielectric function $\varepsilon(z)$ varies along one direction ($z$) only. The system consists of alternating layers of materials (blue and green) with different dielectric constants, with a spatial period $a$. We imagine that each layer is uniform and extends to infinity along the $x$ and $y$ directions, and we imagine that the periodicity in the $z$ direction also extends to infinity.

![Diagram of photonic crystal](image)

Figure 2: The photonic band structures for on-axis propagation, as computed for three different multilayer films. In all three cases, each layer has a width $0.5a$. Left: every layer has the same dielectric constant $\varepsilon = 13$. Center: layers alternate between $\varepsilon$ of 13 and 12. Right: layers alternate between $\varepsilon$ of 13 and 1.
Figure 5: The photonic band structure of a multilayer film with lattice constant \( a \) and alternating layers of different widths. The width of the \( \varepsilon = 13 \) layer is \( 0.2a \), and the width of the \( \varepsilon = 1 \) layer is \( 0.8a \).

Figure 4: The modes associated with the lowest band gap that is shown in the band structure of the right-hand panel of figure 2, at \( k = \pi / a \). The situation is similar to that of figure 3, but the dielectric contrast is larger. The blue and green regions correspond to \( \varepsilon \) of 13 and 1, respectively.
Bloch's theorem:

\[ \vec{E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{\vec{k}}(\vec{r}), \text{ with } u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r} + \vec{R}) \text{ for all lattice vectors } \vec{R}. \]

Bloch wave vector \( \vec{k} \) lies within Brillouin zone, with \( \vec{k} = k_x \vec{b}_x + k_y \vec{b}_y + k_z \vec{b}_z \).

\( \vec{b}_i \) are primitive reciprocal lattice vectors; \( |k_i| \leq 1/2 \).
Band diagrams for TM modes

\( (E_z \text{ polarized with } k_z = 0) \)
$E_z$ field patterns at $k = \pi/a$

$TM$ modes ($k_z = 0$)
**Figure 7:** Schematic depiction of electric field lines (E) for a thin dielectric structure (gray shading) with a $z = 0$ mirror symmetry plane. Modes that are even with respect to this mirror plane (left) are **TE-like:** E is mostly parallel to the mirror plane (and is exactly parallel at $z = 0$). Modes that are odd (right) are **TM-like:** E is mostly perpendicular to the mirror plane (exactly perpendicular at $z = 0$).
Guided modes of photonic crystal waveguide (PCW) for
1) Optical trap - FORT
2) Strong atom-photon interactions
Optical dipole-force trap \textbf{[FORT]}

Light-shift Hamiltonian:  
\[
\hat{H}_{1s} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2
\]

**Scalar**  
\[
\hat{H}_0 = -\alpha^{(0)} \hat{E}(-) \cdot \hat{E}(+)
\]

\(\alpha^{(0)}, \alpha^{(1)}, \text{and } \alpha^{(2)}\) are the scalar, vector, and tensor atomic dynamic polarizabilities

**Vector**  
\[
\hat{H}_1 = -i \alpha^{(1)} \left( \hat{E}(-) \times \hat{E}(+) \right) \cdot \hat{F}
\]

\(\hat{F}\) is the atomic total angular momentum operator, with \(\hat{I}\) and \(\hat{J}\) the nuclear and electronic angular momentum operators

\[
\hat{F} = \hat{I} + \hat{J}
\]

\(\hat{F}_\mu\)\(\hat{F}_\nu\)}} + \hat{F}_\nu\hat{F}_\mu) - \frac{1}{3} \hat{F}^2 \delta_{\mu\nu}\]

**Tensor**

\[
\hat{H}_2 = -\sum_{\mu, \nu} \alpha^{(2)} \hat{E}_{\mu}(-) \hat{E}_{\nu}(+) \left( \frac{3}{F(2F-1)} \left( \frac{1}{2} (\hat{F}_\mu \hat{F}_\nu + \hat{F}_\nu \hat{F}_\mu) - \frac{1}{3} \hat{F}^2 \delta_{\mu\nu} \right) \right)
\]

\(\hat{E}(+)\) and \(\hat{E}(-)\) are the positive and negative frequency components of the electric field

\(\hat{F}_\mu\)\(\hat{F}_\nu\}} + \hat{F}_\nu\hat{F}_\mu) - \frac{1}{3} \hat{F}^2 \delta_{\mu\nu}\]

\(\hat{F}_\mu\)\(\hat{F}_\nu\}} + \hat{F}_\nu\hat{F}_\mu) - \frac{1}{3} \hat{F}^2 \delta_{\mu\nu}\]

\(\alpha^{(0)}, \alpha^{(1)}, \text{and } \alpha^{(2)}\) are the scalar, vector, and tensor atomic dynamic polarizabilities

\(\mu, \nu \in \{x, y, z\}\) are Cartesian components

\(\hat{F} = \hat{I} + \hat{J}\) is the atomic total angular momentum operator, with \(\hat{I}\) and \(\hat{J}\) the nuclear and electronic angular momentum operators

\(\hat{F}_\mu\)\(\hat{F}_\nu\}} + \hat{F}_\nu\hat{F}_\mu) - \frac{1}{3} \hat{F}^2 \delta_{\mu\nu}\]
Nanowire photonic crystal waveguides for single-atom trapping and strong light-matter interactions


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APPLIED PHYSICS LETTERS 104, 111103 (2014)
$|E|^2$ for Alligator Photonic Crystal Waveguide (APCW)

SEM of APCW

Calculated band diagram from SEM
Band diagram calculated from SEM

Air band at $\nu_A$
Probe mode for strong atom-photon interaction

Dielectric band at $\nu_D$
Hybrid optical trap - CP potential and FORT
… and now the hard part

1) Casimir-Polder potential for atom in photonic crystal waveguide
2) Radiative interactions of an atom with guided modes, leaky modes, and free space

Trapped atoms in one-dimensional photonic crystals
C.-L. Lung Hung, S. Meenehan, D. Chang, O. Painter, and J. Kimble

1) G. S. Agarwal, PRA 12 1475 (1975)
A. W. Rodriguez et al., PRA 80, 012115 (2009)

Trapping Cesium atoms in a APCW - Hybrid trap from optical and vacuum forces


- **z direction trapped by vacuum forces – Casimir-Polder**
- **x-y directions trapped by optical forces from guided mode in APCW – blue-detuned FORT**

Casimir-Polder potential
Strong atom-photon interactions at band edge of APCW

Group velocity $v_g = \frac{d\omega}{dk} \to 0$ at $v_{A,D}

\Rightarrow \gamma_1 - d \propto \frac{1}{v_g} \to \infty$ (van Hove singularity)

\[ \frac{\gamma_1 - d}{\gamma'} \approx 20 \]

1-atom reflection:

\[ r_1 = \frac{\gamma_1 - d}{\gamma_1 - d + \gamma'} \approx 0.95 \]

transmission:

\[ T = (1 - r_1)^2 \approx 0.003 \]
An integrated nanophotonic “optical circuit” for atomic physics, quantum optics, and quantum information science
S.-P. Yu, J. D. Hood, J. A. Muniz, M. J. Martin & C.-L. Hung - Kimble Group
R. Norte, S. M. Meenehan, J. D. Cohen - Painter Group

~300x300nm wire spanning 2mm
Atom-Light Interactions in Photonic Crystals*

Nanophotonic Device Challenges –
- Mechanical stability
- Optical absorption and power handling capability
- Tunability
- Testability

*R. Norte, S. M. Meenehan, J. D. Cohen – Painter Group

Alligator Photonic Crystal Waveguide APCW
Cold atom device loading into the Alligator PCW

$N_i \sim 5 \times 10^6$ Cs atoms at $\rho \sim 1 \times 10^{11}/$cm$^3$

$T \sim 10\mu$K

$N_f \sim 10^6$ Cs atoms at $\rho \sim 2 \times 10^{10}/$cm$^3$

$T \sim 20\mu$K

SiN device - 1-d photonic crystal waveguide

Optical fiber butt-coupled to SiN device

Jae Lee
Juan Muniz
Andrew McClung
Mike Martin

Aki Goban
Chen-Lung Hung
Jonathan Hood
Su-Peng Yu
Alligator Photonic Crystal Waveguide – APCW
Model and Measurement for Reflection Spectra
Alligator Photonic Crystal Waveguide – APCW

Low finesse cavity \((F \approx 2)\) formed by impedance mismatch between taper sections and APCW

\[ \omega_{\text{atom}} \approx \omega_{\text{cavity}} + \frac{\Delta_{\text{FSR}}}{2} \]
Estimates for atom-field interaction in APCW

- Reflection spectra with guiding beam
- Fit with matrix transfer model; adjustable parameters $\bar{N}$ and $\Gamma_{1-d}/\Gamma'$

\[ \Rightarrow \bar{N} = 1.1 \pm 0.4 \text{ atoms, 1-atom: } \frac{\Gamma_{1-d}}{\Gamma'} = 0.32 \pm 0.08 \]

\[ \Rightarrow \text{Reflection coefficient for 1 atom } |r_1| \approx 0.24 \]

Transmission coefficient $|t|^2 \sim (1 - |r_1|)^2 \approx 0.57 \rightarrow >40\% \text{ attenuation/atom}$

$\frac{\Gamma_{1-d}}{\Gamma'}$ is unprecedented for all current atom-photon interfaces

(e.g., atoms & nanofiber, 1-atom in free space, 1 molecule on surface)
Consistency check for inference of atomic localization - a) $\Gamma_{1D}$ result from R spectra & b) trajectory simulations

Recall mode area $A_m$:

$$\Gamma_{1D}(\vec{r}_A) = \frac{1}{2} \frac{c}{v_g} A_m(\vec{r}_A) \Gamma_0$$

Ideal: $A_m(0) = 0.24 \, \mu m^2$

Inferred: $A_m(\vec{r}_A) = 0.44 \, \mu m^2$

2-state atom: $\sigma_0 = 0.35 \, \mu m^2$

from $R(\Delta p)$:

$$\frac{\Gamma_{1-d}}{\Gamma'} = 0.32 \pm 0.08$$

SEM of APCW - Alligator Photonic Crystal Waveguide
Coupling a Single Trapped Atom to a Nanoscale Optical Cavity

J. D. Thompson, 1* T. G. Tiecke, 1,2* N. P. de Leon, 1,3 J. Feist, 1,4 A. V. Akimov, 1,5 M. Gullans, 1 A. S. Zibrov, 1 V. Vuletić, 2 M. D. Lukin 1†

Nanophotonic quantum phase switch with a single atom

T. G. Tiecke 1,2*, J. D. Thompson 1*, N. P. de Leon 1,3, L. R. Liu 1, V. Vuletić 2 & M. D. Lukin 1
Modification of transmission for photonic crystal waveguide cavity due to 1 atom
Lukin and Vuletic group - Harvard-MIT
Lukin and Vuletic group - Harvard-MIT
Intensity distribution from side illumination

Trap lifetime measured by free-space imaging

Trap lifetime measured by guided-mode
Atom-guided mode coupling for the nearest trap sites
Strong coupling $\Gamma_{1D}$ for trapped atoms in the APCW

Transmission spectrum of the device

Band gap

1st resonance

Off resonance

At the 1st resonance:
Group index $\approx 10$
Cavity enhancement $\approx 2$

Transmission spectrum with trapped atoms

Slow light and cavity enhancement

$,\frac{\Gamma_{1D}}{\Gamma'} \approx 1, \bar{N} \approx 2$ atoms

$|r_1| = \frac{\Gamma_{1D}}{\Gamma_{1D} + \Gamma'} \approx 0.5 \Rightarrow |t_1|^2 \approx 0.25$
Extension to Two Dimensions
A. Gonzalez-Tudela, C.-L. Hung, D. Chang, I. Cirac & H. J. Kimble
arXiv:1407.7336

Square Lattice of
Cylindrical Holes in a Dielectric Slab
Dimensions
lattice constant \( d = 316 \) nm
hole diameter \( 2R = 253 \) nm

Rb D\(_2\) atoms

- Blue FORT at 740nm
- Rb D\(_2\) line at 780nm

Frequency (c/a) vs. \( k_x \)

Graphical data showing the dispersion relations and other related phenomena.
Guided modes to mediate atom-atom interactions
arXiv:1407.7336

Lambda scheme with Ramna transitions

Trace out photonic modes for $\Delta \beta < 0$

$$H = \sum_{i,j} \left[ J_{i,j}^{z} \sigma_{i}^{z} \sigma_{j}^{z} + J_{i,j}^{xy} \sigma_{i}^{+} \sigma_{j} \right]$$

$$J_{i,j}^{\beta} = h_{\beta} \Gamma_{2d} K_{0} \left[ |r_{ij}|/\xi_{\beta} \right]$$
Guided modes to mediate atom-atom interactions
arXiv:1407.7336

Strength of the interaction determined by $\Gamma_{2d}$

$$\Gamma_{2d} \approx \Gamma_a \frac{c\sigma}{4\pi AL_m(\omega_c, r_a)}$$

Effective cross-section

$$\sigma = \frac{3}{2\pi} \eta \lambda_a^2$$

Effective length

$$L_m(k, r_a) = \frac{\int d^3r \epsilon(r)|\bar{E}_{k,m}(r)|^2}{L^2 \epsilon(r)|E_{k,m}(r_a)|^2}$$

$L_{m,\text{TE}} \sim 0.3 \mu\text{m}$
Casimir-Polder forces for subwavelength vacuum lattices

arXiv:1407.7336

New structure

Horizontal cut of CP potential

\[ d = 50 \text{ nm} \]

\[ V_{\text{CP}}(0, y, z_t) \text{ (mK)} \]

\[ n = 3.2 \text{ (GaP)} \]
\[ W = h = 118.75 \text{ nm} \]
\[ d = 50 \text{ nm} \]

Casimir-Polder forces for subwavelength vacuum lattices
arXiv:1407.7336

Control of vertical position through SI phase: control of trapping potential

\[ \begin{align*}
\lambda_{SI} &= 760 \text{ nm} \\
\Omega_{SI} &= 2\pi \times 130 \text{ GHz} \\
\phi &= 1.7
\end{align*} \]

\[ n = 3.2 \text{ (GaP)} \]
\[ R = 0.2d \]
\[ W = h = 118.75 \text{ nm} \]
\[ d = 50 \text{ nm} \]

A. González-Tudela
Casimir-Polder forces for subwavelength vacuum lattices

arXiv:1407.7336

Control of vertical position through SI phase: control of trapping potential

Trap characteristics
\[
\begin{align*}
\lambda_{SI} &= 760 \text{ nm} \\
\Omega_{SI} &= 2\pi \times 130 \text{ GHz} \\
\phi &= 1.7
\end{align*}
\]

\[
\{V_{d,xy}, V_{d,z}\} \approx 2\pi \times \{3.5, 20.8\} \text{ MHz}
\]

\[
\{\omega_{t,xy}, \omega_{t,z}\} \approx 2\pi \times \{1.7, 4.2\} \text{ MHz}
\]

\[n = 3.2 \text{ (GaP)}
\]
\[R = 0.2d
\]
\[W = h = 118.75 \text{ nm}
\]
\[d = 50 \text{ nm}
\]

A. González-Tudela
Subwavelength lattices: why?
arXiv:1407.7336

Increase on energy scales for Bose-Hubbard model simulation

\[ H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i b_i^\dagger b_i (b_i^\dagger b_i - 1) \]
An atom trap using vacuum forces alone in complex photonic crystal? ➔ “No Go Theorem” forbids stable vacuum trapping near a dielectric structure when the atom is in its electronic ground state.
Trapping atoms using nanoscale quantum vacuum forces

Excited state shift -

$$\frac{\delta \omega_e^{(r)}(z)}{\Gamma_0} \approx -\frac{1}{8(k_0z)^3} \text{Re} \left( \frac{\epsilon_a - 1}{\epsilon_a + 1} \right)$$

Large repulsive potential for excited state for $\epsilon(\omega_{atom}) \rightarrow -1^+$

Example: Drude model with $\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$

$\epsilon(\omega_{atom}) \rightarrow -1^+$ at plasmon resonance $\omega_{pl} = \omega_p / \sqrt{2}$
Properties of quantum vacuum trap - The Simple Story

- Trap depth
- Ground state uncertainty

Experimental implementation?
Atom localized at edge of high Q photonic bandgap
Outline: Strong Atom-Light Interactions in Nano-Photonics

H. Jeff Kimble
Institute for Quantum Information and Matter
California Institute of Technology

Lecture 1 - Atom-photon interactions along a 1D waveguide

Lecture 2 - Optical traps in nano-photonics

~ 300nm

Lecture 3 - Atom-atom interactions mediated by photons in 1D and 2D photonic crystals

\[ H = \sum_{i,j} \left[ J_{ij} z \sigma_i z \sigma_j z + J_{ij} x y \sigma_i x \sigma_j y \right] \]
Quantum Optics & Atomic Physics with 1-d Photonic Crystals

- Large atom-photon interaction
  \[
  \frac{\Gamma_{1D}}{\Gamma'} \approx 20 \rightarrow r_1 = \frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} \approx 0.95
  \]
- Strong coupling in cQED
- Wave-vector “engineering”

- Enhanced atom-photon coupling near the photonic band edge

- Long-range atom-atom interactions mediated by photons
- Quantum many-body physics for internal & external degrees of freedom
- Precision vacuum-force measurements

Single-photon Rabi frequency \( \Omega_1 \gtrsim 10 \text{ GHz} \)
Critical photon number \( n_0 \lesssim 10^{-7} \text{ photons} \)
Basic rates for atom-field interaction in 1-d waveguides

\[ r_1(\omega_A) = -\frac{\Gamma_{1D}}{\Gamma' + \Gamma_{1D}} \]

Weisskopf-Wigner to find

\[ \Gamma_{1D}(\vec{r}_A) = \frac{1}{2} \frac{c}{v_g} \frac{\sigma_0}{A_m(\vec{r}_A)} \Gamma_0 \]

\[ \sigma_0 = \frac{3}{2\pi} \frac{\lambda_A^2}{\Lambda} \] is the atomic scattering cross section.

\[ \Gamma_0 = \frac{|\mu|^2 \omega_A^3}{3\pi \epsilon_0 \hbar c^3} \] is the Einstein A coefficient.

\[ v_g \] is the group velocity in the waveguide.

\[ A_m(\vec{r}_A) : \text{Effective mode area defined for an atom at location } \vec{r}_A \]

\[ A_m(\vec{r}_A) = \int d^2 r e(\vec{r}) |E(\vec{r})|^2 \]

Caveat: These results assume \( |\vec{\mu} \cdot \vec{E}|^2 = |\vec{\mu}|^2 |\vec{E}|^2 \).

The vector character of \( \vec{\mu}, \vec{E} \) generally reduces \( \Gamma_{1D} \).
Spontaneous Emission for an Atom in a Photonic Crystal

Silicon Nitride $Si_3N_4$

---

$a = 366$nm  
$g = 250$nm  
$w = 330$nm  
$d = 100$nm  
$t = 200$nm

---

Group velocity $v_g = \frac{d\omega}{dk} \to 0$ at $\nu_{A,D}$  
$\Rightarrow \gamma_{1-d} \propto \frac{1}{v_g} \to \infty$ (van Hove singularity)
Slow light in photonic crystals

TOSHIHIKO BABA

Figure 1 Waveguides, photonic bands and group-index characteristics. The normalized wavenumber means the wavenumber in units of reciprocal lattice $2\pi/a$, where $a$ is the lattice constant. The normalized frequency is defined as $\omega a/2\pi c = a/\lambda$. a, Scanning electron microscope image and, b, schematic band diagram and group-index spectrum for a silicon PCW. c, Transmission spectrum, group-index spectrum and band diagram with respect to the normalized frequency for a silicon PCW. For the group-index spectrum and band diagram, dots denote experimental results obtained by the modulation phase-shift method, whereas dotted lines denote calculated results with an effective-index approximation. Adapted with permission from ref. 50. d, Scanning electron microscope image and, e, schematic band and group-index spectrum of a silicon PCW with respect to the absolute frequency.
2. Coherent mapping of atomic spin state $\psi$ to and from propagating optical fields
Atom-induced cavities and tunable long-range interactions between atoms trapped near photonic crystals

Coherent evolution

\[ H_{Int} = \frac{g}{2} \sum_{i,j} f(z_j, z_i) \sigma^i_{eg} \sigma^j_{ge} \]

\[ \omega(k) \]

Incoherent evolution

\[ \mathcal{L} \rho \]

\[ \omega_{\text{atom}} \]

\[ k_x \]

Atomic interaction from the virtual exchange of photons along a 1D waveguide. Such resonant dipole-dipole interactions (RDDI) typically are possible via evanescent fields and limited in range.

Consider 2 atoms with transition frequency $\omega_A$ localized within a hollow, metallic waveguide (ideal) with cutoff frequency $\omega_{11}$.

$G_{\alpha\alpha}$ - the single-atom spontaneous emission rate into guided modes

$\sim \Gamma_{1D}$
Nonradiative interaction and entanglement between distant atoms

Ephraim Shahmoon and Gershon Kurizki

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PHYSICAL REVIEW A 87, 033831 (2013)

Initial condition – Atom A in excited state, Atom B in ground state. Plot time evolution from dipole-dipole interaction via virtual photons in waveguide.

\[ \omega_A < \omega_{11} \] atomic frequency within the bandgap
Nonradiative interaction and entanglement between distant atoms

Ephraim Shahmoon and Gershon Kurizki

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Initial condition – Atom A in excited state, Atom B in ground state.
Plot time evolution from dipole-dipole interaction via virtual photons in waveguide.

\( \omega_A < \omega_{11} \) atomic frequency within the bandgap

Tradeoff of fidelity with interaction strength for \( \omega_A \rightarrow \omega_{11} \)

Non-Markovian Dynamics for \( \omega_A \rightarrow \omega_{11} \)
Atom-induced cavities and tunable long-range interactions between atoms trapped near photonic crystals

“Cavity QED” for trapped atoms in photonic crystal waveguides (PCW) with transition frequency lying within the bandgap

Atom-induced cavities and tunable long-range interactions between atoms trapped near photonic crystals

Bound state of atom and photon for atomic frequency within bandgap – S. John circa 1990

\[ \delta = \text{detuning of the bound state resonance between atom and photon} \]

\[ \Delta = \omega_a - \omega_b \]

\[ \beta = \left( \pi g^2 |u_{k_0}(0)|^2 k_0 / \sqrt{\alpha \omega_b} \right)^{2/3} \]
Atom-induced cavities and tunable long-range interactions between atoms trapped near photonic crystals


Photonic and atomic character of bound state -

$P_e$ – atom excitation probability for $|\phi_1\rangle$

$P_p$ – 1 photon probability for $|\phi_1\rangle$

Interaction strength between atoms $(i,j)$ scales as $e^{-2|z_i-z_j|/L}$
Atom-induced cavities and tunable long-range interactions between atoms trapped near photonic crystals


Towards quantum many body physics for trapped atoms in photonic crystal waveguides (PCW)

Cavity QED without mirrors – “all-atom” cQED with dynamic tuning of cavity and atomic interactions
Extend to lambda and butterfly atomic level schemes
- Design diverse spin-spin interaction Hamiltonians
- Tailor functional form for interaction: $H_{\text{Int}} \sim 1/r^\alpha$ (e.g., with $\alpha = 1$ “Coulomb” interaction)
Extension to Two Dimensions

A. Gonzalez-Tudela, C.-L. Hung, D. Chang, I. Cirac & H. J. Kimble
arXiv:1407.7336

Square Lattice of Cylindrical Holes in a Dielectric Slab

Dimensions

lattice constant $d = 316$ nm
hole diameter $2R = 253$ nm

Rb D$_2$ atoms

Frequency (c/a)

Blue FORT at 740nm
Rb D$_2$ line at 780nm

Light line
TE bands
TM bands
D2 line
trap light 740nm
Guided modes to mediate atom-atom interactions
arXiv:1407.7336

Lambda scheme with Ramna transitions

Trace out photonic modes for $\Delta_\beta < 0$

$$H = \sum_{i,j} \left[ J^{z}_i \sigma^z_i \sigma^z_j + J^{xy}_i \sigma^+_i \sigma_j \right]$$

$$J^\beta_{ij} = h_\beta \Gamma_{2d} K_0 \left[ |r_{ij}| / \xi_\beta \right]$$
Atoms close to PCWs interact with the GMs of PhCs:

\[ H_{int} = \sum_i \sum_k (g_k e^{i k \cdot r_i} a_k \sigma_i^\dagger + h.c.) \]

Assume parabolic dispersion for the GMs near bandedge:

\[ \omega(k) = \omega_c + A(k_c - k)^2 \]

\[ \omega_c \quad A > 0 \]

\[ \omega_a \quad k \]


arXiv:1407.7336
Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

\[
\partial_t \rho = \sum_{i,j} \frac{\gamma_{ij}}{2} (2\sigma_i \rho \sigma_j^\dagger - \sigma_j^\dagger \sigma_i \rho - \rho \sigma_j^\dagger \sigma_i) + i \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]
\]

\[
\Gamma_{ij} = \gamma_{ij} + iJ_{ij}
\]
Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

\[
\frac{\partial_t \rho}{2} = \sum_{i,j} \frac{\gamma_{ij}}{2} (2\sigma_i \rho \sigma_j^\dagger - \sigma_j^\dagger \sigma_i \rho - \rho \sigma_j^\dagger \sigma_i) + i \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]
\]

The character of the interaction can be tuned with the detuning: \( \Delta_{ac} \)

\[
\Gamma_{ij} = \gamma_{ij} + iJ_{ij}
\]
Radiative interactions of atoms and guided modes
arXiv:1407.7336

Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

$$
\partial_t \rho = \sum_{i,j} \frac{\gamma_{ij}}{2} (2\sigma_i \rho \sigma_j^\dagger - \sigma_j^\dagger \sigma_i \rho - \rho \sigma_j^\dagger \sigma_i) + i \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]
$$

The character of the interaction can be tuned with the detuning: $\Delta_{ac}$

• If $\Delta_{ac} > 0$, then:
  $$
  \Gamma_{ij} \approx \Gamma_{2d} \frac{\pi}{2} H_0^{(1)} \left[ |a_{ij}|/\xi \right]
  $$

$$
\Gamma_{ij} = \gamma_{ij} + iJ_{ij}
$$

$$
H_0^{(1)}(x) = J_0(x) + iY_0(x)
$$
Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

$$\partial_t \rho = \sum_{i,j} \frac{\gamma_{ij}}{2} (2 \sigma_i \sigma_j^\dagger - \sigma_j \sigma_i^\dagger - \sigma_i \sigma_j) + i \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]$$

The character of the interaction can be tuned with the detuning: $\Delta_{ac}$

- If $\Delta_{ac} > 0$, then: $\Gamma_{ij} \approx \Gamma_2 d \frac{\pi}{2} H_0^{(1)} \left[ |r_{ij}| / \xi \right]$\
  $$\Gamma_{ij} = \gamma_{ij} + iJ_{ij}$$  
  $$H_0^{(1)}(x) = J_0(x) + iY_0(x)$$

- If $\Delta_{ac} < 0$, then: $\Gamma_{ij} = iJ_{ij}$\
  $$J_{ij} \approx \Gamma_2 d K_0 \left( r_{ij} / \xi \right)$$
Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

$$\partial_t \rho = \sum_{i,j} \frac{\gamma_{ij}}{2} \left( 2\sigma_i \rho \sigma_j^\dagger - \sigma_j^\dagger \sigma_i \rho - \rho \sigma_j^\dagger \sigma_i \right) + \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]$$

The character of the interaction can be tuned with the detuning: $\Delta_{ac}$

- If $\Delta_{ac} > 0$, then: $\Gamma_{ij} \approx \Gamma_{2d} \frac{\pi}{2} H_0^{(1)} \left[ |r_{ij}|/\xi \right]$
  $$\Gamma_{ij} = \gamma_{ij} + iJ_{ij}$$

- If $\Delta_{ac} < 0$, then: $\Gamma_{ij} = iJ_{ij}$
  $$J_{ij} \approx \Gamma_{2d} K_0 \left( r_{ij}/\xi \right)$$

The range of interaction can be controlled through detuning and curvature: $\xi = \sqrt{A/\Delta_{ac}}$
Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

\[
\frac{\partial \rho}{\partial t} = \sum_{i,j} \frac{\gamma_{ij}}{2} \left( 2\sigma_i \rho \sigma_j^\dagger - \sigma_j^\dagger \sigma_i \rho - \rho \sigma_j^\dagger \sigma_i \right) + i \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]
\]

The character of the interaction can be tuned with the detuning: \(\Delta_{ac}\)

- If \(\Delta_{ac} > 0\), then:
  \[
  \Gamma_{ij} \approx \Gamma_{2d} \frac{\pi}{2} H_0^{(1)} \left[ |r_{ij}|/\xi \right] J_0(x) \propto 1/\sqrt{x}
  \]

  \[
  H_0^{(1)}(x) = J_0(x) + iY_0(x)
  \]

- If \(\Delta_{ac} < 0\), then:
  \[
  \Gamma_{ij} = iJ_{ij}
  \]
  \[
  J_{ij} \approx \Gamma_{2d} K_0(r_{ij}/\xi)
  \]

The range of interaction can be controlled through detuning and curvature: \(\xi = \sqrt{A/\Delta_{ac}} \ll 1\)
Eliminate guided modes to arrive at master equation describing atomic spin-spin interactions (mediated by photons):

$$\partial_t \rho = \sum_{i,j} \frac{\gamma_{ij}}{2} (2\sigma_i \rho \sigma_j^\dagger - \sigma_j^\dagger \sigma_i \rho - \rho \sigma_j^\dagger \sigma_i) + i \sum_{ij} \frac{J_{ij}}{2} [\rho, \sigma_j^\dagger \sigma_i]$$

The character of the interaction can be tuned with the detuning: $\Delta_{ac}$

- If $\Delta_{ac} > 0$, then: $\Gamma_{ij} \approx \Gamma_{2d} \frac{\pi}{2} H_0^{(1)} \left[ |r_{ij}| / \xi \right] J_0(x) \propto 1$

$$H_0^{(1)}(x) = J_0(x) + iY_0(x)$$

$$\Gamma_{ij} = \gamma_{ij} + iJ_{ij}$$

- If $\Delta_{ac} < 0$, then: $\Gamma_{ij} = iJ_{ij}$

$$J_{ij} \approx \Gamma_{2d} K_0 \left( r_{ij} / \xi \right)$$

$$K_0(x) \propto \log(1/x)$$

The range of interaction can be controlled through detuning and curvature: $\xi = \sqrt{A/\Delta_{ac}} \gg 1$
We use a two-photon Raman transition:

\[ H_{xy} = \sum_k g_k \frac{\Omega_2}{2\Delta_2} (a_k^\dagger \sigma^x e^{i(\omega_k - \omega_{g2} + \omega_{g1} - \omega_{L,1})t} + h.c.) \]

\[ H_z = \sum_k g_k \frac{\Omega_1}{2\Delta_1} (a_k^\dagger \sigma^z e^{i(\omega_k - \omega_{L,1})t} + h.c.) \]

This defines two effective detunings respect to band cut-off:

\[ \Delta_z = \omega_{L,1} - \omega_c \]
\[ \Delta_{xy} = \omega_{g,2} - \omega_{g,1} + \omega_{L,2} - \omega_c \]

Tracing out photonic modes & when \( \Delta_\beta < 0 \)

\[ H = \sum_{i,j} [J_{ij}^z \sigma_i^z \sigma_j^z + J_{ij}^{xy} \sigma_i^\dagger \sigma_j^\dagger] \quad J_{ij}^\beta = h_\beta \Gamma_{2d} K_0 \left( |r_{ij}|/\xi_\beta \right) \]
Guided modes to mediate atom-atom interactions
arXiv:1407.7336

Strength of the interaction determined by $\Gamma_{2d}$

$$\Gamma_{2d} \approx \Gamma_a \frac{c\sigma}{4\pi A L_m(\omega_c, r_a)}$$

Effective cross-section

$$\sigma = \frac{3}{2\pi} \eta \lambda_a^2$$

Effective length

$$L_m(k, r_a) = \frac{\int d^3r \epsilon(r)|\tilde{E}_{k,m}(r)|^2}{L^2 \epsilon(r_a)|\tilde{E}_{k,m}(r_a)|^2}$$

$L_{m,TE} \sim 0.3 \mu m$
Range of the interaction determined by $J_{ij}^\beta = h_\beta \Gamma_{2d} K_0 \left( \frac{|r_{ij}|}{\xi_\beta} \right)$
"A Quantum Adventure" featuring Dr. C.-L. Hung
IQIM – http://quantumfrontiers.com/2014/02/10/a-quantum-adventure/
YouTube – https://www.youtube.com/watch?v=1zD1U1sIPQ4
Outline: Strong Atom-Light Interactions in Nano-Photonics

H. Jeff Kimble
Institute for Quantum Information and Matter
California Institute of Technology

Lecture 1 - Atom-photon interactions along a 1D waveguide

Lecture 2 - Optical traps in nano-photonics

~ 300nm

Lecture 3 - Atom-atom interactions mediated by photons in 1D and 2D photonic crystals

\[ H = \sum_{i,j} \left[ J_{ij}^{zz} \sigma_i^z \sigma_j^z + J_{ij}^{xy} \sigma_i^x \sigma_j^y \right] \]
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